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CIRCUIT ANALYSIS OF A-C POWER SYSTEMS

VOLUME I

Symmetrical and Related Components

By
EDITH CLARKE
CENTRAL STATION ENGINEERING DEPARTMENT
GENERAL ELECTRIC COMPANY

One of a Series Written in the Interest of
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Engineering Program

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PREFACE

This book is a compilation of notes and lectures given over a period of years to members of the Central Station Engineering Department of the General Electric Company in Schenectady, New York. Beginning in 1928, the notes were revised and extended for new groups of men entering the department, practical problems in power system performance with numerical solutions being added from time to time as they were presented by operating engineers. As the notes were helpful to members of the department and others receiving them, it was suggested that they be put in book form. In 1932, with Professor H. W. Bibber as co-author, a book on symmetrical components was undertaken. Parts of that unfinished book are included in Chapters I–IV of this one.

In answer to the repeated request that the methods of symmetrical and related components be presented very simply, the methods of solving unbalanced power system problems by means of components are analyzed and discussed in detail. The book has been divided into two volumes. Volume I deals largely with the determination of currents and voltages of fundamental frequency in power systems during unbalanced conditions by means of symmetrical and related components. Included in this volume are the electrical characteristics of overhead transmission circuits and information and data on transformers and synchronous machines which permit them to be represented by equivalent circuits in the solution of practical problems. Volume II will give additional characteristics of synchronous machines, equivalent circuits for types of transformers not included in Volume I, characteristics of insulated cables, induction machines, and other electrical equipment encountered in a-c power systems. Overvoltages from various causes and the effects of saturation in transformers and of amortisseur windings in synchronous machines will also be included in Volume II. In both volumes special attention is given to equivalent circuits and the solution of practical problems.

The author wishes to express appreciation to her associates who have assisted in the preparation of this book by helpful suggestions or critical reviews of completed chapters; especially to Mr. Charles Concordia for his willingness to act as consultant in regard to arrangement and presentation of material and to Miss Amelia De Lella for her patience and good nature in typing and retyping the various editions and revisions of the notes, drawing preliminary figures, and proofreading.

Edith Clarke

Schenectady, New York
June, 1943

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CIRCUIT ANALYSIS OF A–C POWER SYSTEMS

VOLUME I — SYMMETRICAL AND RELATED COMPONENTS

By Edith Clarke

John Wiley and Sons, Inc., Publishers

Page 52

Line 10. "$V_t = V_t + \cdots$" should read "$V_t = V_s + \cdots$."

Line 11. "$E_g = V_g + \cdots$" should read "$E_g = V_t + \cdots$."

Line 13. "G" should read "the generator."

Line 14. "$I_t = I_g = \cdots$" should read "$I_p = I_r = I_g \cdots$."

Line 15. "$I_t = \cdots$" should read "$I_r = \cdots$."

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Fig. 14(*g*). The vector labeled "I_{ab2}" (between I_{a2} and I_{b2}) should read "I_{ac2}."

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Fig. 2(*a*). Add "$x_n = 0.30$" to constants of generator G.

Page 170

Equation [21] should read "$E_r = \cdots = E_sD - I_sB$."

Equation [22] should read "$I_r = \cdots = I_sA - E_sC$."

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INTRODUCTION

The problems of the power transmission engineer at any given time may be divided roughly into three classes:

1. Problems which can be solved analytically by well-known methods in general use. The methods are satisfactory, because it is thought that all the factors influencing the problem are understood and can be evaluated, and the time required is not considered unduly long.
2. Problems which can be solved analytically and the various factors evaluated, but the time and labor required are excessive.
3. Problems for which there is no known analytic method of evaluating all the factors involved. This is not intended to imply that, for a given problem with all conditions specified, the engineer given sufficient time cannot provide a workable solution; but rather that, the effect of the various influences not being thoroughly understood, a different and independent problem is encountered with each change in given conditions.

The extensive use of mechanical and electrical calculating devices, factory tests on equipment, laboratory tests on miniature electrical systems, and field tests on actual systems have made it unnecessary to include a fourth class of problems — those for which there is no known solution.

The types of problems appearing under the three classifications change with time. Before the period of interconnection of power systems, many of the problems connected with the normal operation of a substantially balanced system were thought to be in class 1. Some, however, were placed in class 2, one example being the determination by calculation of load currents in balanced networks with multiple paths. The use of the a-c network analyzer[1] to determine current and voltage distribution in a balanced system moved this problem from class 2 into class 1. But when it was discovered that, under certain conditions of balanced system operation, a generating station at the distant end of a long transmission line could not be operated at its rated output and remain in synchronism with the rest of the system, i.e., the system as designed was not stable under steady state, problems previously thought to be in class 1 dropped back into class 3. For a time the problems of system stability under both steady-state and transient conditions were in class 3; but after a few years they could be moved up into class 2. At the present time they probably still remain in class 2. Although the various factors involved are understood and the use of the a-c network analyzer materially reduces the time required for a given solution, the methods used are essentially step-by-step ones. They are the best available at present, but improvements are to be expected.

---

[1] For numbered references see the list at end of the introduction and at end of each chapter.

The application of the method of symmetrical components[2] to the solution of power system problems during unbalanced conditions, within a few years moved a long list of problems from classes 2 and 3 into class 1, the d-c calculating table[3] or the a-c network analyzer reducing the time required for solution.

There are certain problems which, although solvable by symmetrical components, can be solved more readily by a different set of components.[4,5] These components are simply related to symmetrical components and can be derived from them. By providing a different approach to the problem, they frequently allow a simpler solution.

Outstanding examples of problems in class 3 at the present time are those which involve non-linear relations. The effect upon system voltages and currents of unequal saturation in the phases of a transformer bank because of unsymmetrical system conditions is an example of a class 3 problem. Although solutions have been obtained for certain problems involving non-linear relations by making use of a differential[6] analyzer or a transient[7] analyzer, it will probably be some time before all problems of this type are removed from class 3.

The purpose of this book is to help the power transmission engineer solve some of his problems. Since it is expected that many of these problems will deal with systems during unbalanced conditions, where the use of symmetrical components and their related components will materially aid him, the greater part of the book is devoted to these components and their applications. But as he will also be expected to determine system conditions during normal operation, tables and charts are given to assist him in the solution of such problems.

Except for an occasional integral or differential equation, introduced for a better understanding of the fundamental principles involved, a knowledge of the elementary principles of alternating currents, algebra, plane geometry, trigonometry, and familiarity with electrical equipment are the only prerequisites for an understanding of this book.

In an endeavor to simplify the application of the method of symmetrical components to unbalance power system problems, those concepts necessary for its application, but not an integral part of the method itself, are discussed in Chapter I. Among these are vectors, complex quantities, operators, vector representation of sinusoidal currents and voltages, per unit quantities, one-line impedance diagrams, equivalent circuits, and the principle of superposition.

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CHAPTER I

DEFINITIONS AND FUNDAMENTAL CONCEPTS

A **vector** has magnitude and direction. A vector is represented by a straight line, of length proportional to the magnitude of the vector, extending in the direction of the vector and terminated by an arrowhead. The starting point of a vector is called the origin and the terminal point the terminus. In Fig. 1 (*a*), O is the origin of the vector V and T is the terminus. Vectors having the same magnitude and direction are equal wherever their origins may be. In this book, unless specifically stated to the contrary, a vector, V, will be written with no distinguishing mark. When the magnitude alone of the vector V is specified, V will be enclosed in bars, thus, $|V|$.

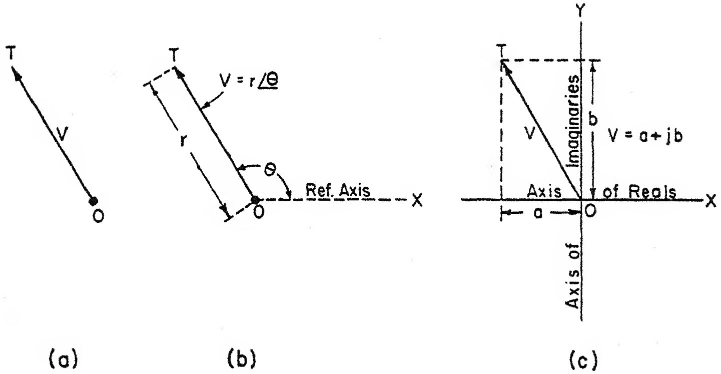


FIG. 1. (*a*) Vector. (*b*) In polar coordinates. (*c*) In rectangular coordinates.

A plane vector can be defined by two coordinates. If *polar coordinates* are used, the two coordinates are the magnitude of the vector and the angle it makes with the axis of reference, the angle being positive when measured in the counterclockwise direction. In Fig. 1 (*b*), if OX is the reference axis, the vector V is specified if written $r\underline{/\theta}$, where r, a scalar quantity called the modulus, represents the magnitude of the vector, and θ, called the argument, is the angle measured in the counterclockwise direction from the reference axis to the positive direction of the vector. A positive angle θ is written $\underline{/\theta}$, a negative angle θ is written $/\overline{\theta}$ or $\underline{/-\theta}$. In *Cartesian coordinates* the vector is defined by its projections upon two reference axes intersecting at a known angle. The simplest Cartesian coordinates are rectangular, with one axis horizontal and the other vertical. In Fig. 1(*c*) the horizontal and vertical axes are OX and OY, respectively. There will be a positive or negative projection on the horizontal axis and similarly on the vertical axis, projections to the right of the origin on the horizontal axis and above the origin on the vertical axis being positive. The axes are commonly referred to as the axes of *reals* and *imaginaries*, respectively. The letter j is used in electrical engineering to designate the projection on the vertical or imaginary axis. The vector expressed in this manner is written $a + jb$, where the signs of a and b may be either positive or negative. In Fig. 1(*c*) the real part of V is negative. A vector in the form $a + jb$ is said to be expressed in complex form. The magnitude of a vector in polar form is independent of the reference axis, but its argument is not. When expressed in complex form, both coordinates depend upon the choice of reference axes.

A **complex quantity,** as here used, is a complex number whose real and imaginary parts correspond to physical quantities. Complex quantities differ from vectors in that they have no direction of their own and consequently are independent of the reference axes. A complex quantity of magnitude different from unity attached to a vector rotates the vector through an angle and also changes its magnitude, producing a new vector of a different kind. The impedance of an electric circuit is a complex quantity. When this complex quantity is multiplied by a current vector the result is a voltage vector which, in the general case, differs in magnitude and phase from the current vector.

To change a vector or complex number expressed in the polar form $r\underline{/\theta}$ to the complex form $a + jb$, or vice versa, the reference axis for θ being the axis of reals, the following relations are employed (see Fig. 1):

$$a = r\cos\theta$$

$$b = r\sin\theta$$

$$a + jb = r(\cos\theta + j\sin\theta)$$

$$r = \sqrt{a^2 + b^2}$$

$$\theta = \tan^{-1}\frac{b}{a}$$

$$r\underline{/\theta} = \sqrt{a^2 + b^2}\ \underline{/\tan^{-1}\frac{b}{a}}$$

where the signs of a and b and of the trigonometric functions should be observed.

Addition and subtraction of vectors or complex numbers can be performed either graphically or algebraically.

To find the *sum* of two or more vectors graphically, the origin of the second vector is placed at the terminus of the first vector, and so on with the others successively, the proper direction being given to each. The sum is the vector drawn from the origin of the first vector to the terminus of the last vector. (See Fig. 2(a).) The algebraic addition of vectors is readily performed if they are expressed in the complex form $a + jb$. The sum of all the real components gives the real part of the resultant, whereas the sum of all the j components gives the imaginary part of the resultant.

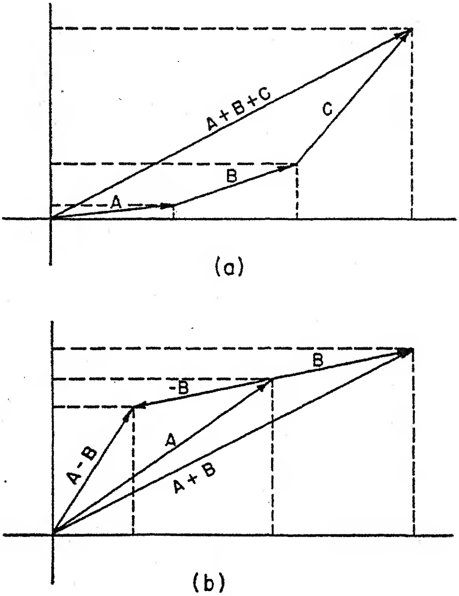


FIG. 2. (a) Sum of vectors $A, B,$ and C. (b) Sum and difference of two vectors, A and B.

The *difference* of two vectors is defined as the result of adding to the first vector the second vector drawn 180° from its given direction. Figure 2(b) shows the graphical operations of adding the vectors A and B and subtracting B from A. The algebraic difference of two vectors expressed in complex form $a + jb$ is obtained by reversing the signs of both components of the second vector, then adding the real components and the imaginary components, just as in the addition of two given vectors.

When addition or subtraction of vectors is required, manipulations are more easily carried out with the vectors in complex form. When a vector is multiplied or divided by a complex number it may be preferable to express both in polar form.

Multiplication and Division of Vectors and Complex Numbers. By the use of Maclaurin's theorem it can be shown that

$$e^{j\theta} = \cos\theta + j\sin\theta$$
$$e^{-j\theta} = \cos\theta - j\sin\theta$$

where e is the base of Napierian logarithms

$$\left(e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots = 2.718 +\right)$$

The vector V can therefore be written

$$V = r\underline{/\theta} = r(\cos\theta + j\sin\theta) = re^{j\theta} \qquad [1]$$

In [1], θ is an angle expressed in either degrees or radians. If the vector is written $r\underline{/\theta}$, θ will, in general, be expressed in degrees; if written $re^{j\theta}$, θ will be in radians; if written $r(\cos\theta + j\sin\theta)$, θ may be in either degrees or radians but is usually in degrees. (π radians = 180°, where $\pi = 3.14159 +$.)

With vectors and complex numbers expressed in the form $re^{j\theta}$, multiplication and division may be performed by applying the rules for exponentials developed in algebra. Given a vector V_1 and a complex number Z_2, where

$$V_1 = r_1e^{j\theta_1} = r_1\underline{/\theta_1}$$
$$Z_2 = r_2e^{j\theta_2} = r_2\underline{/\theta_2}$$

Then, adding exponents of e when multiplying and subtracting when dividing,

$$V_1 \times Z_2 = r_1r_2e^{j(\theta_1+\theta_2)} = r_1r_2\underline{/\theta_1 + \theta_2} \qquad [2]$$

$$V_1 \div Z_2 = \frac{r_1e^{j\theta_1}}{r_2e^{j\theta_2}} = \frac{r_1}{r_2}e^{j(\theta_1-\theta_2)} = \frac{r_1}{r_2}\underline{/\theta_1 - \theta_2}(r_2 \neq 0) \qquad [3]$$

From [2] it follows that: *To obtain the direct product of a vector and a complex number expressed in polar form, multiply their magnitudes (moduli) and add their angles (arguments)*, obtaining as a result a new vector. And, from [3]: *To obtain the quotient of a vector and a complex number expressed in polar form, divide the modulus of the dividend by the modulus of the divisor and subtract the argument of the divisor from the argument of the dividend*, obtaining a new vector.

Take, for example,

$$V_1 = 2.000\,\underline{/60^\circ} = 1.000 + j1.732$$

$$Z_2 = 1.000\,\underline{/120^\circ} = -0.500 + j0.866$$

Then, applying the rules for multiplying and dividing,

$$V_1 \times Z_2 = 2.000\,\underline{/180^\circ} = -2.000 + j0$$

$$V_1 \div Z_2 = 2.000\overline{/60^\circ} = 1.000 - j1.732$$

A complex number may be raised to any power by multiplying it by itself a sufficient number of times. Thus,

$$(Z)^n = (r\,\underline{/\theta})^n = r^n\,\underline{/n\theta} \qquad [4]$$

Any desired root of a complex number may be obtained by extracting the given root of its modulus and dividing its argument by the given root. Thus,

$$\sqrt[n]{Z} = \sqrt[n]{r\,\underline{/\theta}} = \sqrt[n]{r}\ \underline{\Big/\frac{\theta + 2\pi k}{n}}, \text{ where } k = 0,\ 1,\ \cdots\ (n-1) \qquad [5]$$

An **operator** is a complex number of unit magnitude. An operator is used to rotate a vector through an angle without changing its magnitude. A familiar operator is $-1 = 1\,\underline{/180^\circ}$. Attached to a vector, -1 rotates it through 180°. Another common operator is $j = 1\,\underline{/90^\circ}$ which rotates the vector to which it is attached through 90° in the positive direction, while $-j = 1\,\overline{/90^\circ}$ will rotate it 90° in the negative (clockwise) direction.

If the vector $V_1 = 300\,\underline{/120^\circ}$ is multiplied by the operator j, the resultant vector, V_2, is

$$V_2 = 300\,\underline{/120^\circ} \times 1\,\underline{/90^\circ} = 300\,\underline{/210^\circ} = -259.8 - j150$$

If V_1 is multiplied by $-j$, the resultant vector, V_2 is

$$V_2 = 300\,\underline{/120^\circ} \times 1\,\overline{/90^\circ} = 300\,\underline{/30^\circ} = 259.8 + j150$$

The Operator j. Powers of the operator j may be written in several ways, as indicated below:

$$j = 1\,\underline{/90^\circ} = 1\,\overline{/270^\circ} = 0 + j = j$$

$$j^2 = 1\,\underline{/180^\circ} = 1\,\overline{/180^\circ} = -1 + j0 = -1$$

$$j^3 = 1\,\underline{/270^\circ} = 1\,\overline{/90^\circ} = 0 - j = -j$$

$$j^4 = 1\,\underline{/360^\circ} = 1\,\underline{/0^\circ} = 1 + j0 = 1$$

$$j^5 = 1\,\underline{/450^\circ} = 1\,\underline{/90^\circ} = 0 + j = j$$

The Operator a. In dealing with balanced three-phase vectors, an operator which rotates vectors through 120° is very useful. It is commonly accepted practice to use a to represent this operator. Thus,

$$a = 1\underline{/120^\circ}$$

In the complex form, using the transformation relations given previously,

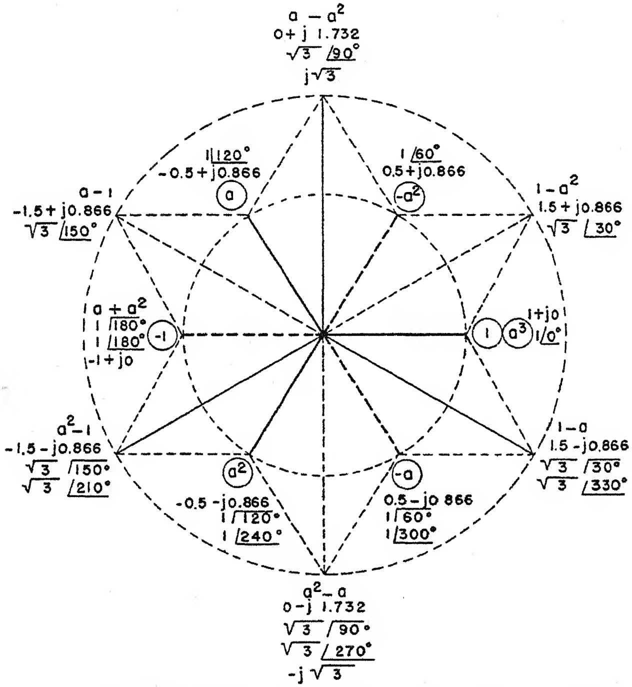
$$a = 1(\cos\theta + j\ \sin\theta) = \cos 120^\circ + j\ \sin 120^\circ = -0.500 + j0.866$$

The square of a is obtained by multiplying a by itself:

$$a^2 = (-0.500 + j0.866)^2 = -0.500 - j0.866$$

or

$$a^2 = a \times a = 1\underline{/120^\circ + 120^\circ} = 1\underline{/240^\circ} = 1\overline{/120^\circ}$$



a − a²
0+ j 1.732
√3 /90°
j√3
1/120°
−0.5+j0.866
a
1 /60°
0.5+j0.866
−a²
a − 1
−1.5+ j0.866
√3 /150°
1 − a²
1.5 + j0.866
√3 /30°
a + a²
1 /180°
1 /180°
−1+j0
−1
1
a³
1+j0
1/0°
a² − 1
−1.5 − j0.866
√3 /150°
√3 /210°
a²
−0.5 −j0.866
1 /120°
1 /240°
−a
0.5 − j0 866
1 /60°
1 /300°
1 − a
1.5 −j0.866
√3 /30°
√3 /330°
a² − a
0 − j 1.732
√3 /90°
√3 /270°
−j √3

FIG. 3. Graphical representation of functions of the operator a.

Functions of the operator a occur frequently in work with symmetrical components. Table I and Fig. 3 give some of these in convenient form. Where functions more complicated than those given in the table are encountered, they may always be reduced to a single complex number by replacing the powers of a by their equivalents in complex form from Table I or Fig. 3. If a function of a is a fraction with a in the denominator, either an algebraic operation such as factoring followed by division may be performed, or else the numerator and denominator may be multiplied by a quantity which will make the denominator a real number. The quantity which will accomplish this elimination of all but real numbers in the denominator is an operator capable of rotating the denominator to the axis of reals.

Examples.

(a)
$$\begin{aligned} 2a^2 + 3a + 5 &= 2(-0.5 - j0.866) + 3(-0.5 + j0.866) + 5 = 2.5 + j0.866 \\ &= 2.648\,\underline{/19.1^\circ} \end{aligned}$$

(b)
$$\frac{1 - a^2}{a - a^2} = \frac{(1 - a)(1 + a)}{a(1 - a)} = \frac{1 + a}{a} = \frac{-a^2}{a} = -a$$

or

$$\frac{1 - a^2}{(a - a^2)} = \frac{(1 - a^2)}{(a - a^2)} \times \frac{(a^2 - a)}{(a^2 - a)} = \frac{a^2 - a^4 - a + a^3}{a^3 - a^4 - a^2 + a^3} = \frac{a^2 + 1 - 2a}{2 - (a^2 + a)}$$

$$= \frac{-3a}{3} = -a$$

(c)
$$\frac{1 - a}{1 + a^2} = \frac{(1 - a)}{(-a)} \times \frac{(-a^2)}{(-a^2)} = \frac{1 - a^2}{1} = 1 - a^2$$

TABLE I

Functions of the Operator a

$$\begin{aligned}
a &= 1\,\underline{/120^\circ} = -0.5 + j0.866 \\
a^2 &= 1\,\underline{/240^\circ} = -0.5 - j0.866 \\
a^3 &= 1\,\underline{/360^\circ} = 1.0 + j0 \\
a^4 &= 1\,\underline{/120^\circ} = -0.5 + j0.866 = a \\
-a &= 1\,\overline{/60^\circ} = 0.5 - j0.866 \\
-a^2 &= 1\,\underline{/60^\circ} = 0.5 + j0.866 \\
1 + a + a^2 &= 0 \\
a + a^2 &= -1 \\
a - a^2 &= 0 + j1.732 = \sqrt{3}\,\underline{/90^\circ} \\
a^2 - a &= 0 - j1.732 = \sqrt{3}\,\overline{/90^\circ} \\
1 - a &= 1.5 - j0.866 = \sqrt{3}\,\overline{/30^\circ} \\
1 - a^2 &= 1.5 + j0.866 = \sqrt{3}\,\underline{/30^\circ} \\
a - 1 &= -1.5 + j0.866 = \sqrt{3}\,\underline{/150^\circ} \\
a^2 - 1 &= -1.5 - j0.866 = \sqrt{3}\,\overline{/150^\circ}
\end{aligned}$$

Sinusoidal Quantities. Sinusoidal currents and voltages have magnitudes which vary sinusoidally with time. The ordinate in Fig. 4 gives, at any time t measured from an initial time t_0, the instantaneous value of the current i, of oscillation frequency f. The equation for the current i may be written

$$i = I_m \sin(2\pi ft + \theta) = I_m \sin(\omega t + \theta) \tag{6}$$

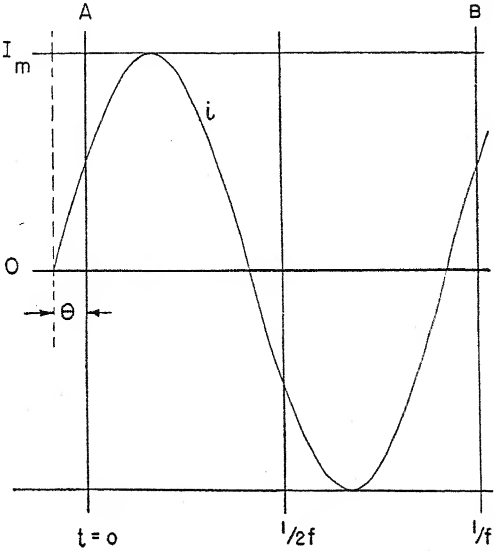


FIG. 4. Graph of instantaneous current, $i = I_m \sin(2\pi \mathrm{ft} + \theta)$. Ordinates are the magnitude of i, and abscissas are time t in seconds.

In [6], I_m is the maximum or crest value of the current i, f the frequency in cycles per second, t the time in seconds, and θ the electrical angle between the point at which the current wave is zero and increasing in a positive direction and the point from which time is measured. θ is the *phase angle* and may be either positive or negative. At time $t = 0$ the magnitude of i is $I_m \sin \theta$.

The *period*, or the time required for the current to complete one cycle, is $1/f$ second and is indicated by T. A 60-cycle current, for example, requires $\frac{1}{60}$ second to complete one cycle. One cycle is shown in Fig. 4 between A and B. In [6], $(2\pi ft + \theta)$ is in radians. If θ is expressed in degrees and 2π is replaced by 360°, [6] becomes

$$i = I_m \sin\left(360° \frac{t}{T} + \theta\right) \tag{7}$$

where t/T is number of periods in time t.

Vector Representation of Sinusoidal Currents and Voltages. The instantaneous values of the current i in Fig. 4 may be obtained by the projection on a fixed reference line of a plane vector of magnitude I_m revolving in the positive or counterclockwise direction at a constant angular velocity of $\omega = 2\pi f$ radians ($360f$ degrees) per second. In Fig. 5, the vector OA makes one revolution in time $T = 1/f$ second. Angle is measured from OX and represents the instantaneous phase; projection is on OY and represents the instantaneous magnitude. At time $t = 0$, the vector $OA = I_m$ makes the angle θ with OX, and at any time t the angle is $(\omega t + \theta)$. The position of OA shown in Fig. 5 corresponds to $t = 0$.

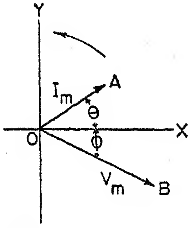


FIG. 5. Vector diagram at time $t = 0$ of sinusoidal quantities i and v.

Sinusoidal currents and voltages of the same frequency may be represented in the same vector diagram, each current and voltage being represented by a revolving vector whose length is proportional to the maximum magnitude of the quantity and whose displacement from the OX axis at time $t = 0$ is its phase angle. In Fig. 5 the projection on OY of the revolving vector OB of magnitude V_m, angular velocity $2\pi f$ radians per second, and phase angle ϕ (ϕ is negative in this case) represents the magnitude of the voltage v at any time t. The angle which OB makes with OX represents the phase of v at time t. The position of OB shown in Fig. 5 corresponds to $t = 0$.

At $t = 0$, i leads v by an angle, $\theta - \phi$, equal to the algebraic difference of the phase angles θ and ϕ; and, since OA and OB revolve at the same rate, i and v retain this relative phase displacement.

In those problems dealing with sinusoidal currents and voltages in which effective rather than crest values are considered, it is found more convenient to represent the voltages and currents by vectors whose magnitudes are proportional to effective values and to select a value of t such that one of the vectors lies along OX. The vector chosen to coincide with OX is called the *reference vector*, and the phase displacements of the other currents and voltages are then their relative phase displacements from the current or voltage selected as reference vector.

The *root mean square* value of an alternating current, written rms current, is the effective value of the current. The power loss, I^2R, in a resistance circuit with an alternating current of rms value I, is the same as the loss in the same circuit with a direct current I. The rms value of an alternating current during any given time is the square root of the average value of the square of the current during that time. This definition applies to non-sinusoidal as well as to sinusoidal currents.

If the current is sinusoidal, the rms current in terms of crest current is given by the following equation:

$$I_{\text{rms}} = \left[\int_0^{2\pi} \frac{(I_m \sin x)^2\, dx}{2\pi}\right]^{1/2} \tag{8}$$

Integration of [8], with the use of a table of integrals,* gives

$$I_{\text{rms}} = \left[\frac{I_m^2}{2\pi}\left[\tfrac{1}{2}x - \tfrac{1}{4}\sin 2x\right]_0^{2\pi}\right]^{1/2} = \frac{I_m}{\sqrt{2}} \tag{9}$$

With a sinusoidal voltage, the rms voltage is likewise the crest value of the voltage divided by $\sqrt{2}$:

$$V_{\text{rms}} = \frac{V_m}{\sqrt{2}} \tag{10}$$

A-c ammeters and voltmeters read rms amperes and volts, respectively, whereas an oscillograph records instantaneous values versus time. Since the ratio of the rms to the crest value of sinusoidal currents and voltages is constant, one can readily be obtained from the other.

In the design of generators for polyphase systems, care is taken to secure sinusoidal (or approximately sinusoidal) generated voltages of equal rms values in all phases and of equal crest values, which occur successively in the phases in equal time intervals. For an n-phase generator, the time interval or *time phase displacement* in degrees is $360°/n$. For a three-phase generator it is 120°.

The *phase order* of the phase voltages in a polyphase circuit is the order in which they reach their crest values. The *positive-sequence phase order* for a polyphase system is the phase order of the generated voltages. Since the naming of the phases is arbitrary, it will be assumed that the phases are named so that the phase order of the generated voltages is $a, b, c, d, \cdots$. It should be pointed out, however, that in some three-phase power systems the phases are indicated by x, y, z(a, b, c, or 1, 2, 3) and the generated voltages have the phase order x, z, y(a, c, b or 1, 3, 2). In calculations of such systems, the phases can be renamed to conform to the chosen standard.

The phase voltages and line currents of a polyphase circuit are *balanced*, or *symmetrical*, if they are sinusoidal, equal in magnitude, and displaced from each other by equal phase angles. Balanced voltages and currents of positive-sequence phase order are called *positive-sequence voltages* and *currents*, respectively.

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* Equation 261, *A Short Table of Integrals*, B. O. Peirce, Second Revised Edition, Ginn and Co.

A polyphase power system is balanced if the phase voltages and line currents at every point in the system are balanced and of the same frequency and phase order as the generated voltages, that is, if the voltages and currents are positive-sequence voltages and currents, respectively. For a polyphase system to be balanced, it is necessary that the generated voltages be balanced and the impedance to positive-sequence currents be the same in all phases. Figure 6(a) shows the vector diagram of the rms currents and voltages at a point of a balanced three-phase power system, with V_a as reference vector. In determining rms voltages and currents, and average power in a balanced polyphase system, only one phase of the system need be considered, because impedances to neutral (positive-sequence impedances) and rms currents and voltages are the same in all phases. Instantaneous currents and voltages in the phases will differ in time phase, but the difference in time phase between the current and voltage of any one phase will be the same as that in all other phases. A balanced polyphase power system can therefore be represented for purposes of calculation by a *one-line diagram* in which the applied voltages are voltages to neutral and the impedances are impedances to neutral. The one-line diagram of a balanced polyphase power system is a *positive-sequence diagram*, so called because all applied voltages are of positive-sequence phase order (i.e., of the phase order of the generated voltages), and the impedances are those met by balanced currents of positive-sequence phase order.

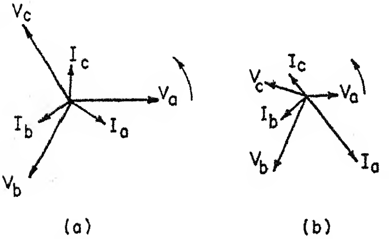

FIG. 6. Vector diagram of fundamental-frequency rms currents and voltages at a point in a three-phase system, with V_a as reference vector. (a) Balanced system. (b) Unbalanced system.

An *unbalanced polyphase power system* is one in which the voltages and currents are unbalanced. A system will be unbalanced if the generated voltages are unbalanced, or if the circuits which compose the system are unsymmetrical. When an unsymmetrical fault occurs on a system which was previously balanced, the currents and voltages become unbalanced because the phases have become unsymmetrical. The fundamental-frequency currents and voltages of an unbalanced system, being sinusoidal, can be represented by vectors. Fundamental frequency is impressed frequency. Figure 6(b) illustrates the vector representation of the fundamental-frequency currents and voltages at a point of an unbalanced three-phase power system with V_a, the voltage of phase a, as reference vector.

Voltage Rise, Voltage Drop, Direction of Current Flow, Average Power. In calculations in which sinusoidal voltages and currents are represented by vectors, it is important to distinguish between voltage rise and voltage drop and to adopt a convention for indicating positive direction of current flow. Since many of the conventions used in d-c calculations are also used in a-c calculations, these conventions and the laws governing d-c circuits will be discussed before those for a-c systems.

D-C Systems. According to established convention, the voltage generated in an open-circuited d-c dynamo is a rise or increase in voltage from negative to positive terminal; it may also be considered a drop or decrease in voltage from positive to negative terminal. If the d-c dynamo supplies an external resistance load connected across its terminals, positive direction of current flow is taken from the negative to the positive terminal through the machine and from the positive to the negative terminal through the load. By Ohm's law, a current flowing through a resistance causes a voltage drop (or a negative voltage rise) in the direction of current flow equal to the product of the current and the resistance. It also causes a voltage rise through the resistance in the opposite direction. Voltage rise and voltage drops have opposite signs when taken in the same direction through an element of the circuit, but the same signs when taken in opposite directions.

The voltage at a point in the system is the difference in potential between that point and some other point used as reference. Unless otherwise stated, the voltage to ground, or to some other reference point, is understood to mean the *rise* in voltage in going from the reference point to the given point.

In d-c circuits, where the direction of current flow is not known, an arrow is used to indicate the direction assumed as positive and the current is indicated by a symbol. (See Fig. 7(a).) Calculations are then based on the assumption of positive current flowing in the direction indicated by the arrow. If calculations give a negative current flowing in the indicated direction, it can be concluded that positive current flows in the opposite direction or the current flowing in the indicated direction is a negative current. Negative current flowing in a given direction is equal in magnitude to positive current flowing in the opposite direction.

When the voltage rise through a d-c machine from the negative to the positive terminal is positive and the current in the same direction is also positive, power *out* of the machine is positive and it is a generator; but, if the current in the same direction as the positive voltage rise is negative, power *out* of the machine is negative and it is a motor. Power into the motor, however, is positive. A generator sends out positive power or receives negative power; a motor sends out negative power or receives positive power.

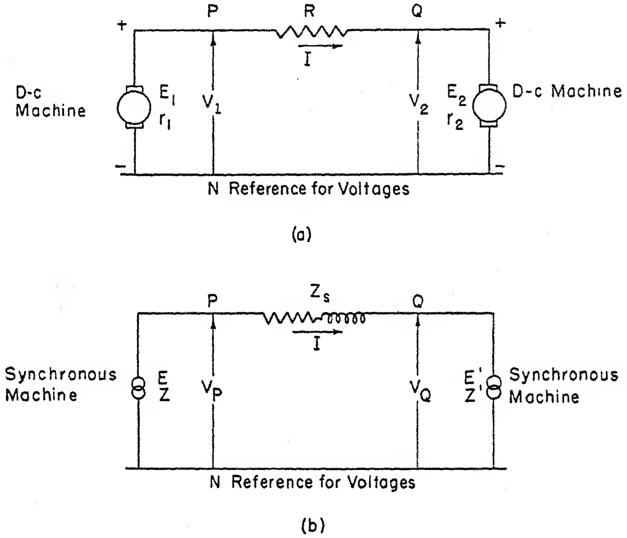

Fig. 7. (*a*) D-c system. (*b*) A-c system — single-phase or one-phase of balanced polyphase system.

A-C Systems. The conventions used in d-c systems will be applied to a-c systems in which fundamental frequency currents and voltages are represented by vectors. The main difference is that in d-c systems currents and voltages can be only positive or negative, that is, can differ in phase only by exactly 0° or 180°, whereas in a-c systems, with a particular voltage or current selected as reference vector, the other currents and voltages may have any phase relation with the reference vector between 0° and 360°. If this distinction is borne in mind, and resistance in Ohm's law replaced by impedance, the conventions and principles used in d-c systems can be applied also to a-c systems.

In a-c systems, as in d-c systems, the voltage at a given point is the difference in potential between that point and some other point used as reference. Unless otherwise stated, it is the voltage rise in going from the reference point to the given point. This is a convention in general use. When the voltages at all points in a system are referred to the same point, and this point is of zero potential relative to the system, a single subscript will be used to indicate the voltage at a given point. Thus, the voltage at point a will be written V_a, where the magnitude of V_a is the rms voltage at a referred to a point of zero potential, and the phase angle of V_a depends upon the voltage or current vector selected as reference vector. If V_a is selected, its phase angle is zero and $V_a = |V_a| \underline{/0^\circ}$.

The voltage at any point a, referred to a second point b, will be written

$$V_{ba} = V_a - V_b$$

where the magnitudes of V_a and V_b are the rms voltages at points a and b, respectively, referred to the zero-potential reference point for the system, and their phase angles are determined by the vector selected as reference vector. V_{ba} indicates the voltage rise in going from point b (indicated by the first subscript) to point a (indicated by the second subscript). This notation, which is arbitrary, is used in many books, but not in all. V_{ba} is an rms vector voltage of magnitude independent of the reference vector; its phase angle is the angle it makes with the reference vector.

Figure 7(b) shows two single-phase synchronous machines, or one phase of each of two polyphase machines. Let machines 1 and 2 have constant impedances Z and Z' and rms generated voltages E and E', respectively, as indicated. Terminal P of machine 1 is connected through an external impedance Z_s to terminal Q of machine 2; their other terminals are connected directly to N, a point of zero potential.

Following the convention that the positive direction of voltage rise through a machine is from the reference point to the terminal, E represents the rms voltage rise from N to P on open circuit in machine 1, and E' the rms voltage rise from N to Q in machine 2 on open circuit. If E' is equal in magnitude to E and also in phase, the generated voltages of the two machines, as connected in Fig. 7(b), oppose each other and no current will flow in the circuit. If, however, E' is different in magnitude or in phase from E there will be a current flowing in the loop circuit. To calculate this current I when the magnitudes and relative phases of E and E' and the impedances Z, Z', and Z_s are known, a positive direction for current flow will be assumed. The choice of the direction assumed as positive is arbitrary, but, having been assumed, the calculated current will be the current flowing in the assumed direction. Let the positive direction of current flow be from N to P through machine 1, as indicated by arrow in Fig. 7(b). As there is only one loop, the current I will flow from P to Q through the impedance Z_s, and from Q to N through the second machine. When there is more than one loop, it may be necessary to assign symbols to several currents and to indicate all assumed directions by arrows. The *voltage drop* around the loop in Fig. 7(*b*) in the assumed direction of current flow is given by the following equation, obtained by applying Ohm's law, which states that the *voltage drop in the direction of current flow is the product of the current and the impedance through which it flows*, and Kirchhoff's law, which states that *the sum of the voltage drops* (*or voltage rises*) *taken in the same direction around a closed loop is zero:*

$$-E + IZ + IZ_s + E' + IZ' = 0 \tag{11}$$

This is also the equation for voltage rise in the opposite direction around the loop.

The *voltage rise* around the closed loop in the direction of current flow is

$$E - IZ - IZ_s - E' - IZ' = 0 \tag{12}$$

This is also the equation for voltage drop in the opposite direction around the loop. Either equation may be written

$$E - IZ = E' + IZ_s + IZ' \tag{13}$$

In [13] the voltage rise through machine 1 to its terminal P is equated to the voltage rise through machine 2 and the impedance Z_s to the point P. The rise in voltage from the reference point to point P is the same no matter which path is taken.

$$V_P = E - IZ = E' + IZ_s + IZ' \tag{14}$$

where V_P is the voltage rise from the reference point N to point P.

From [14],

$$I = \frac{E - E'}{Z + Z' + Z_s} \tag{15}$$

The magnitude and phase, relative to the reference vector, of the current I in the direction assumed can be obtained from [15] when E and E' are referred to the same reference vector.

If the current had been assumed to flow in the opposite direction to that indicated in Fig. 7(*b*), its magnitude would be unchanged but its sign would be reversed, giving a current 180° out of phase with that given by [15]. Therefore, a known current flowing in a given direction can be replaced by a current of equal magnitude flowing in the opposite direction and 180° out of phase with it.

The *average power* P at any point in a single-phase system or in each phase of a balanced polyphase system is

$$P = |V| \times |I| \cos \theta \qquad [16]$$

where $|V|$ and $|I|$ are the scalar values of the effective voltage and current at the point and θ is the phase angle between V and I.

If the point is at the terminals of a machine, V represents the voltage rise through the machine. If I is the current flowing from the machine, P in [16] is the power out of the machine. If this power is positive, the machine is generating power; if negative, it is absorbing power. If I is the current flowing into the machine, P in [16] is the power into the machine. If this power is positive, the machine is receiving power; if negative, it is generating power.

PER CENT AND PER UNIT QUANTITIES

The numerical per unit value of any quantity is its ratio to the chosen base quantity of the same dimensions, expressed as a decimal. For example, if base voltage is taken as 110 kv, voltages of 99 kv, 110 kv, and 115 kv will be 0.90, 1.00, and 1.045, respectively, when expressed in per unit on the given base voltage. The chosen base voltage, 110 kv, is referred to as base voltage, 100% voltage, or unit voltage. The numerical value of a quantity in per unit is equal to its value in per cent divided by 100, when the base quantities are the same for the per unit as for the per cent system. Per unit values are more convenient to use in calculations than per cent values. When quantities expressed in per cent are multiplied, it is necessary to divide the product by some power of 100 to obtain the correct result in per cent. The confusion caused by the introduction of powers of 100 is avoided in per unit calculations.

If *base impedance in ohms* is defined as that impedance which will have a voltage drop across it of 100% of base voltage when 100% current flows through it, base impedance will be determined when base voltage and current have been specified. Defined in per unit terms, *base or unit impedance in ohms* is that value of impedance which will have unit voltage drop across it when unit current flows through it. Because of the definition of base impedance and the fundamental relation between current, voltage, and kva in electrical circuits, any two of the four quantities, current, voltage, kva, and impedance, may be selected as the independent base quantities and the other two will then be determined. In most calculations, it is found convenient to select voltage and kva as the independent base quantities and to define **base power in kilowatts** as numerically equal to base kva.

Single-Phase Systems. With voltage and kva as the two independent base quantities and base power numerically equal to base kva, the following relations exist in a single-phase circuit:

$$\text{Base current in amperes} = \frac{\text{base kva}}{\text{base voltage in kv}} = \frac{\text{base kva}}{\text{base kv}} \qquad [17]$$

$$\text{Base impedance in ohms} = \frac{\text{base voltage in volts}}{\text{base current in amperes}}$$

$$= \frac{(\text{base kv})^2 \times 10^3}{\text{base kva}} \qquad [18]$$

$$\text{Base power in kw} = \text{numerical value of base kva} \qquad [19]$$

An impedance Z, given in ohms, can be expressed in per cent or per unit of base impedance, without first calculating base impedance, when voltage and kva are the two independent base quantities:

$$Z\ (\text{in } \%) = 100\,\frac{Z\ (\text{in ohms})}{\text{base ohms}} \qquad [20]$$

$$Z\ (\text{in per unit}) = \frac{Z\ (\text{in ohms})}{\text{base ohms}} \qquad [21]$$

Replacing base ohms in [20] and [21] by $\dfrac{(\text{base kv})^2 \times 10^3}{\text{base kva}}$ from [18],

$$Z\ (\text{in } \%) = \frac{Z\ (\text{in ohms}) \times \text{base kva}}{(\text{base kv})^2 \times 10} \qquad [22]$$

$$Z\ (\text{in per unit}) = \frac{Z\ (\text{in ohms}) \times \text{base kva}}{(\text{base kv})^2 \times 1000} \qquad [23]$$

Solving [22] and [23] for Z in ohms,

$$Z\ (\text{in ohms}) = \frac{Z\ (\text{in } \%) \times (\text{base kv})^2 \times 10}{\text{base kva}} \qquad [24]$$

$$Z\ (\text{in ohms}) = \frac{Z\ (\text{in per unit}) \times (\text{base kv})^2 \times 1000}{\text{base kva}} \qquad [25]$$

where

Base kva = the single-phase base kva

Base kv = the single-phase base voltage in kv

Example. If base kva and voltage are given as 5000 kva and 10,000 volts, respectively, from [17]–[19]:

$$\text{Base current} = \tfrac{5000}{10} = 500 \text{ amp}$$

$$\text{Base impedance} = \frac{10{,}000}{500} = 20 \text{ ohms}$$

$$\text{Base power} = 5000 \text{ kw}$$

A given impedance Z of 2 ohms is 10%, or 0.1 per unit of the base impedance 20 ohms; or, directly from [23] without first calculating base impedance,

$$Z \text{ (in per unit)} = \frac{2 \times 5000}{(10)^2 \times 1000} = 0.1$$

Polyphase Systems. Since a polyphase system under balanced conditions can be represented by a one-line diagram with impedances and voltages to neutral, [17]–[25], developed for a single-phase system, apply also to the one-line diagrams of balanced polyphase systems, base voltage and kva being line-to-neutral voltage and kva per phase, respectively.

Three-Phase Systems. For three-phase equipment, rated kva is customarily given for the three phases and rated voltage is the line-to-line voltage. In calculations of balanced three-phase circuits made on a line-to-neutral basis, base voltage will be a line-to-neutral voltage, and base kva the kva per phase. Base kva per phase is one-third the three-phase base kva and the base line-to-neutral voltage is the base line-to-line voltage divided by $\sqrt{3}$. With these values of base kva and voltage, [17]–[25], developed for a single-phase circuit, apply as well for a line-to-neutral circuit.

Equations [17]–[25] apply also to a three-phase delta-connected circuit, if base voltage is a line-to-line voltage and base kva one-third the three-phase base kva. In balanced three-phase power systems, line-to-line voltages at any point in the system are $\sqrt{3}$ times the line-to-neutral voltages at that point, and currents in delta-connected windings are $1/\sqrt{3}$ times the line currents flowing from the delta. Expressed in per unit of their respective base values, line-to-line and line-to-neutral voltages at any point in the system are the same in magnitude; likewise per unit line currents flowing from a delta and the per unit delta currents, when referred to their respective base currents, are the same in magnitude.

In a line-to-neutral circuit, an impedance to neutral Z, given in ohms, may be expressed in per cent or per unit of base impedance to neutral by equations involving three-phase base kva and line-to-line

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base voltage. It can be seen that the use of

$$\frac{\text{Three-phase base kva}}{3} \quad \text{and} \quad \frac{\text{line-to-line base voltage}}{\sqrt{3}}$$

in [22]–[25] will give a factor 3 in both numerator and denominator; so that, for three-phase circuits, [22]–[25] can be rewritten

$$Z \text{ (in \%)} = \frac{Z \text{ (in ohms)} \times \text{base kva}}{(\text{base kv})^2 \times 10} \qquad [26]$$

$$Z \text{ (in per unit)} = \frac{Z \text{ (in ohms)} \times \text{base kva}}{(\text{base kv})^2 \times 1000} \qquad [27]$$

$$Z \text{ (in ohms)} = \frac{Z \text{ (in \%)} \times (\text{base kv})^2 \times 10}{\text{base kva}} \qquad [28]$$

$$Z \text{ (in ohms)} = \frac{Z \text{ (in per unit)} \times (\text{base kv})^2 \times 1000}{\text{base kva}} \qquad [29]$$

where

Z = impedance to neutral
Base kva = the three-phase base kva
Base kv = the line-to-line base voltage in kv

Equations [26]–[29] are given for convenience in terms of three-phase base kva and line-to-line base voltage. In stating the base quantities selected for the solution of a three-phase system problem, three-phase kva and line-to-line voltage are usually given as the base quantities. It should be remembered, however, that in a line-to-neutral diagram, base or unit voltage is *line-to-neutral voltage* and base or unit kva is *kva per phase*. Thus, if it is stated that base kva is 100,000 kva and base voltage 110 kv, it is understood that these base quantities are three-phase kva and line-to-line voltage; therefore, in the line-to-neutral impedance diagram base kva is one-third of 100,000 kva = 33,333 kva, base kv is $\frac{110}{\sqrt{3}} = 63.5$ kv.

Change in Base Quantities. In electric systems, some impedances are ordinarily given in ohms; transmission line impedances, for example. Some are given in per cent or per unit based on their kva and voltage ratings; machine reactances, for example. It is often necessary therefore to convert ohms to per unit, or vice versa, and to transform per unit impedances from their own kva and voltage bases to the kva and voltage bases selected as base values for the system.

From [22], [23], [26], and [27], it is apparent that impedances in per cent and per unit vary directly as their kva bases and inversely as the squares of their voltage bases. Impedances in per unit on any given kva base may be expressed on a new kva base by the equation,

$$\begin{matrix}\text{Per unit impedance}\\ \text{on new kva base}\end{matrix} = \begin{matrix}\text{per unit impedance}\\ \text{on given kva base}\end{matrix} \times \frac{\text{new kva base}}{\text{given kva base}} \qquad [30]$$

Impedances in per unit on a given voltage base may be expressed on a new voltage base by the equation,

$$\begin{matrix}\text{Per unit impedance}\\ \text{on new voltage base}\end{matrix} = \begin{matrix}\text{per unit impedance}\\ \text{on given voltage base}\end{matrix} \times \left(\frac{\text{given base voltage}}{\text{new base voltage}}\right)^2 \qquad [31]$$

Advantages of Per Unit or Per Cent Impedances. It is customary to express the reactances of electrical equipment in per cent or per unit based on their ratings. Ordinarily the rated kva and voltage constitute the given base quantities. The characteristics of electrical apparatus of various types and ratings can be readily compared when their reactances are expressed in this manner. Such comparisons show that the reactances of synchronous machines of different ratings, but of the same general type, fall within definite limits and that the reactances of transformers of the same rated voltages do not differ by a wide margin. It is frequently possible, therefore, when certain reactances are unknown, to assume values for them which will be satisfactory for use in calculations. This is especially true when the quantities to be determined are but slightly affected by variations in the unknown reactances.

Per unit quantities can be used to great advantage in determining currents, voltages, etc., throughout a three-phase system in which there are many circuits connected by transformers, operating at different voltages. If impedances in ohms are used, one circuit is selected as reference and all impedances are referred to that circuit. To refer an impedance in ohms on one side of a transformer to the other side, it is necessary to multiply this impedance by the square of the transformer turn ratio. This is a relatively simple procedure when there are but few circuits. As the number of circuits is increased, the advantages of per unit impedances are more pronounced. Impedances expressed in per unit on a chosen kva base, with the ratio of base voltages on the two sides of a transformer equal to the transformer turn ratio, are the same referred to either side of the transformer. For example, consider a single-phase transformer rated 5000 kva, 13,800 volts–138,000 volts, connecting circuits 1 and 2. The turn ratio, determined from the rating, is $n_2/n_1 = 138{,}000/13{,}800 = 10$. From [23], 4 ohms in the low voltage circuit in per unit on 5000 kva and 13,800 volts is

$$\frac{4 \times 5000}{(13.8)^2 \times 10^3} = 0.105 \text{ per unit, referred to circuit 1}$$

Referred to the high voltage circuit, 4 ohms in the low voltage circuit is

$$4 \times (10)^2 = 400 \text{ ohms}$$

From [23], 400 ohms referred to the high voltage circuit in per unit on 5000 kva and 138 kv is

$$\frac{400 \times 500}{(138)^2 \times 10^3} = 0.105 \text{ per unit, referred to circuit 2}$$

The impedance in per unit is 0.105 referred to either circuit.

In the illustration above, base impedances in the two circuits were not calculated, since per unit impedances were determined directly from the given base quantities, kva and voltage. An alternate method is first to determine base impedances in the two circuits by [18] and then to express the impedances referred to circuits 1 and 2 in per unit of their respective base impedances. From [18],

$$\text{Base impedance in ohms in circuit 1} = \frac{(13.8)^2 \times 10^3}{5000} = 38 \text{ ohms}$$

$$\text{Base impedance in ohms in circuit 2} = \frac{(138)^2 \times 10^3}{5000} = 3800 \text{ ohms}$$

4 ohms is $\frac{4}{38} = 0.105$ per unit of (38 ohms) base ohms in circuit 1

400 ohms is $\frac{400}{3800} = 0.105$ per unit of (3800 ohms) base ohms in circuit 2

Transformer impedances are usually given in per cent based on the transformer rating. In a two-winding transformer, the kva ratings of the windings are the same and the ratio of rated voltages is usually the same as the turn ratio. In a two-winding transformer, the impedance of the transformer with exciting current neglected, expressed in per cent or per unit on the kva rating, is the same referred to either winding of the transformer, if base voltages in the two windings are in proportion to the turns. This follows directly from the illustration above.

In the one-line per unit impedance-to-neutral diagram (positive-sequence impedance diagram) of a balanced three-phase power system, all impedances are expressed in per unit of the base impedance of the circuit in which they are located; or, with kva and voltage as the independent base quantities, all per unit impedances are based on the same kva and on the base voltages of their respective circuits. In a system consisting of several circuits connected by transformers, if base voltage is arbitrarily chosen for some particular circuit, base voltages in the other circuits are not independent but are determined by the transformer turn ratios. With transformer exciting currents neglected, they are the voltages in these circuits at no load when the arbitrarily chosen base voltage is the voltage of the selected circuit. This is illustrated in the following problem.

Problem 1. Given a three-phase, 60-cycle transmission system, consisting of: a three-phase transmission line; a three-phase generator rated 15,000 kva, 4 kv, with a reactance of 25%; a transformer bank of three single-phase transformers connected Δ–Y, each transformer rated 10,000 kva, 4200–66,500/115,000 Y volts with low voltage taps. The operating tap is 3900–66,500 volts, with a leakage reactance of 10% based on rated kva and tap voltages. Base three-phase kva and base line-to-line voltage have been arbitrarily chosen as 100,000 kva and 110 kv, respectively, in the transmission line. What is base line-to-line voltage in the generator circuit? What are the per unit positive-sequence reactances of generator and transformer on the chosen base quantities?

Solution. Base line-to-line voltage in the generator circuit is determined by the arbitrarily chosen base line-to-line voltage in the transmission circuit and the transformer turn ratio of the operating tap. With 110-kv line-to-line voltage in the transmission circuit, base line-to-line voltage in the generator circuit is $3.9 \times \frac{110}{115} = 3.72$ kv. On a 30,000-three-phase-kva base and a base line-to-line voltage of 115 kv (10,000 kva per phase, $115/\sqrt{3} = 66.5$-kv line-to-neutral voltage), the transformer bank has a reactance of 10%. On a three-phase-kva base of 100,000 kva and a line-to-line voltage base of 110 kv (33,330 kva per phase, $110/\sqrt{3} = 63.5$-kv line-to-neutral voltage), the transformer leakage reactance in per unit from [30] and [31] is

$$0.10 \times \frac{100{,}000}{30{,}000} \times \left(\frac{115}{110}\right)^2 = 0.364$$

The per unit generator reactance on 100,000 kva and 3.72 kv, the base voltage of the generator circuit, is

$$0.25 \times \frac{100{,}000}{15{,}000} \times \left(\frac{4.0}{3.72}\right)^2 = 1.93$$

COMPONENTS OF CURRENT AND VOLTAGE AND SUPERPOSITION

The division of currents and voltages into components is not unfamiliar. Balanced currents and voltages are often divided into two components, one component in phase with a particular voltage or current used as a reference vector and the other component 90° out of phase with it. The voltage drop caused by a current divided into components is obtained by adding vectorially the drops due to its components.

Validity of the Use of Components. The calculation of currents and voltages in electric circuits by adding currents due to components of voltage, and voltages due to components of current, is justified by the *principle of superposition.* This principle states that the response to a force can be determined by adding the responses to the components of the force, provided the responses vary directly with the forces applied, i.e., if the equations involved are *linear*. Superposition can be rigorously applied to electric circuits only when the values of the circuit constants or parameters (resistance, leakance, inductance, capacitance) are independent of the current, voltage, or frequency which may be associated with them. Although the usual resistances, leakances, and capacitances in power circuits are not strictly constant, they may be approximately so within certain limts. This is also true of inductances where the flux paths are in non-magnetic mediums. Inductances associated with iron, where the flux density is not high over the current range of the circuit, may in many cases be considered to have essentially linear characteristics. When saturation must be taken into consideration, constant values of reactance *adjusted* to allow for saturation can frequently be used to advantage. Cases in which the effect of saturation upon reactance must be taken into account are pointed out in later chapters.

IMPEDANCE NETWORKS

The principle of superposition will be applied to impedance networks in which the self-impedances of the circuits and the mutual impedances between circuits at constant frequency are assumed constant.

Self- and Mutual Impedances. The *self-impedance* of a circuit is the ratio of the voltage drop in the circuit in the direction of current flow to the current when all other circuits are open. The *mutual impedance* between two circuits is the ratio of the voltage drop induced in one of the circuits to the current in the other circuit which induces it. Self-impedance will be indicated by Z with two subscripts, both referring to the circuit. The self-impedance of circuit A is Z_{aa}. The mutual impedance between two circuits will be indicated by Z with two subscripts which refer to the two circuits. Let $I_a, I_b, I_c, \cdots I_n$ and $V_a, V_b, V_c, \cdots V_n$ represent currents and voltage drops in circuits A, B, C, $\cdots N$, respectively, in the positive direction. With I_a flowing in circuit A and all other circuits open, $Z_{ba} = V_b/I_a$, $Z_{na} = V_n/I_a$, etc.

With I_b flowing in circuit B and all other circuits open, $Z_{ab} = V_a/I_b$, $V_{nb} = V_n/I_b$, etc. With all circuits closed, the voltage drops are

$$\begin{aligned} V_a &= I_aZ_{aa} + I_bZ_{ab} + I_cZ_{ac} + \cdots + I_nZ_{an} \\ V_b &= I_aZ_{ba} + I_bZ_{bb} + I_cZ_{bc} + \cdots + I_nZ_{bn} \\ V_c &= I_aZ_{ca} + I_bZ_{cb} + I_cZ_{cc} + \cdots + I_nZ_{cn} \\ &\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ V_n &= I_aZ_{na} + I_bZ_{nb} + I_cZ_{nc} + \cdots + I_nZ_{nn} \end{aligned} \qquad [32]$$

In [32] the first subscript of Z refers to the voltage and the second to the current associated with it. If the mutual impedances are the same viewed from either circuit, $Z_{ab} = Z_{ba}$, $Z_{cn} = Z_{nc}$, etc. As the voltage drop in any circuit is equal to the voltage applied to the circuit, $V_a, V_b, V_c, \cdots V_n$ in [32] can be equated to the applied voltages. If no voltage is applied to a closed circuit, the total voltage drop in it is zero. If any circuit is opened, its current becomes zero.

Driving-Point and Transfer Impedances. Let $E_1, E_2, \cdots E_n$ represent the voltages at terminal points 1, 2, $\cdots n$ of a network, where voltages are applied between the terminals and the zero-potential bus, and let $I_1, I_2, \cdots I_n$ be the corresponding currents. Positive direction of current flow is assumed to be *into the network* from the point at which voltage is applied. The resultant current which flows into the network from point 1 will be the difference between the component current sent into the network by E_1 acting alone and the sum of the component currents which the voltages $E_2, E_3, \cdots E_n$, each acting alone, send out the network at point 1. When a voltage is assumed to act alone, the other voltages of the network are reduced to zero and their points of application connected to the zero-potential bus through zero impedance. When the voltage E_1 is applied between point 1 and the zero-potential bus, with the other voltages of the system reduced to zero, there will be a current entering the network at point 1 and currents leaving the network at points 2, 3, $\cdots n$. The current entering the network at point 1 will depend upon the applied voltage and the impedance offered to it. This impedance is the *driving-point* impedance of point 1 and is defined as the ratio of the voltage at point 1 to current entering the network at 1, with no other voltages applied to the network. The current leaving the network at point 2 will likewise depend upon the voltage applied at point 1 and an impedance which takes into account the impedances of the other paths to the zero-potential bus as well as those in the direct path between points 1 and 2. This impedance is called the *transfer* impedance between points 1 and 2, and is defined as the ratio of the voltage at point 1 to the current at point 2 with the other voltages of the system reduced to zero. It can be shown[1] that in a static network composed of linear, bilateral elements without internal sources of energy, the transfer impedance between points 1 and 2 is the same as that between points 2 and 1, provided the applied voltages are of the same frequency. Such a network is called a *reciprocal* network. A bilateral element is a two-terminal circuit which has the same impedance viewed from either terminal. Any two-terminal rectifying device is a unilateral element.

The currents flowing into the network at the various points may be obtained by adding the currents produced by the various applied voltages, each acting alone, provided the network is a reciprocal one and superposition can be applied. Adding the currents due to each voltage acting alone,

$$\begin{aligned} I_1 &= \frac{E_1}{A_{11}} - \frac{E_2}{A_{21}} - \cdots - \frac{E_n}{A_{n1}} \\ I_2 &= -\frac{E_1}{A_{12}} + \frac{E_2}{A_{22}} - \cdots - \frac{E_n}{A_{n2}} \\ &\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ I_n &= -\frac{E_1}{A_{1n}} - \frac{E_2}{A_{2n}} + \cdots + \frac{E_n}{A_{nn}} \end{aligned} \qquad [33]$$

where

$A_{11}, A_{22}, \cdots, A_{nn}$ = driving-point impedance at points 1, 2, and n, respectively

$A_{12}, \cdots, A_{1n}$ = transfer impedance between points 1 and 2, $\cdots$, between points 1 and n

Driving-point and transfer impedances are designated by double subscripts, the first subscript referring to the point at which the voltage is applied and the second to the point where current is measured. The letter A is used here rather than the usual Z to avoid confusing driving-point and transfer impedance with self- and mutual impedance, which are likewise designated by double subscripts. Equation [33] can also be written in terms of transfer and driving-point admittances, where admittances are reciprocals of the corresponding impedances.

ONE-LINE IMPEDANCE DIAGRAMS AND EQUIVALENT CIRCUITS

Two-Conductor Single-Phase Power System. When the problem is to determine the rms voltage between conductors at any point in the system or the line currents during normal operation or with faults between conductors, the two-conductor, single-phase system can be represented by a one-line impedance diagram in which the impedances are lumped impedances, equal to the sum of the equivalent impedances met by the outgoing and return current. The return path for current in the one-line diagram is a conductor, or bus, of zero impedance at zero potential. The current at any point in the one-line diagram represents the single-phase current at that point, and the voltage to the neutral bus represents the voltage between conductors at that point.

Balanced Three-Phase Power System. When the problem is to determine rms line currents and voltages to neutral, during normal operations or during three-phase balanced faults, the three-phase system can be represented by a one-line impedance diagram in which the impedances are impedances to neutral. The one-line diagram may be considered the diagram of one phase of the balanced system with a return path for the currents in a neutral conductor or bus of zero impedance at zero potential. The currents in the diagram are line currents and the voltage between any point and the neutral bus is the voltage to neutral at that point. The diagram of a balanced three-phase power system is called a positive-sequence diagram because the phase order of the balanced voltages at any point in the system is the same as the phase order of the generated voltage, and therefore positive.

In the one-line impedance diagram of a single-phase or a balanced three-phase power system, each piece of apparatus and each transmission circuit has its own *equivalent circuit*. These equivalent circuits will depend upon the purpose for which the one-line impedance diagram is to be used and the degree of precision required in the calculations to be made from it; therefore, under different conditions, the same piece of apparatus may have different equivalent circuits.

An oscillogram of the currents or voltages at a given point in a system during a disturbance may be shown, in addition to fundamental-frequency components, components of other frequencies. These components may include d-c components, one or more natural-frequency components determined by the system inductances and capacitances, even and odd harmonics, and sometimes subharmonics. Subharmonic frequencies are frequencies less than fundamental. For example, a third subharmonic in a 60-cycle system has a frequency of 20 cycles per second. The magnitude of a fundamental frequency component, in general, does not remain constant throughout the disturbance but decreases from its initial or subtransient value to its transient value and eventually reaches its steady-state or sustained value. With d-c components, natural-frequency components, harmonics, and subharmonics neglected, equivalent circuits for determining fundamental-frequency currents and voltages will depend upon whether subtransient, transient, or sustained currents and voltages are required. In transmission lines, transformers, and other static equipment the transition from subtransient to sustained impedance usually takes place within a fraction of a cycle of fundamental frequency. It is only in rotating equipment, that a distinction is made between subtransient, transient, and synchronous reactances when sinusoidal currents and voltages of fundamental frequency are to be determined.

An *equivalent circuit* for use in analytic calculations is a static network composed of self-impedance branches without mutual impedances between them which under the same operating conditions will produce, to the required degree of precision, the same conditions electrically at its terminals as the actual circuit (with or without mutual impedances) which it replaces. It should be noted that it is only at the *terminals* of the actual and equivalent circuits that the currents and voltages must be the same.

The number of branch impedances in an equivalent circuit, in its simplest form, will depend upon the number of terminals in the circuit it is to replace. A two-terminal equivalent circuit will have but one impedance. A three-terminal equivalent circuit will have three branch impedances, which may be connected either in Y or in Δ. A four-terminal equivalent circuit, in the general case, will have six branch impedances. The number of branch impedances is determined by the number of connections which can be made between terminals. The problem of determining the equivalent circuit for a given actual circuit (or combination of actual circuits) is that of determining the branch impedances of the equivalent circuit. One method of procedure is:

1. Draw the desired equivalent circuit with the required number of terminals, indicating the unknown branch impedances by symbols.
2. Test the given circuit and the assumed equivalent circuit by applying a voltage between the terminals taken two at a time, with the other terminals free, and determining the currents.
3. Equate the ratios of the applied voltages to the resultant currents in the given and equivalent circuits.

A second method of determining equivalent circuits is:

(1) Draw an equivalent circuit with the same number of terminals as the actual circuit, with each terminal connected to every other terminal through an impedance. (A three-terminal equivalent circuit will be a Δ.)

(2) With all terminals in the actual circuit connected to a common point by leads, insert a voltage in each lead in turn and measure the currents in the other leads.

(3) The voltage inserted in the lead to any terminal A divided by the current flowing in the lead from a second terminal B gives the impedance between terminals A and B to be used in the equivalent circuit. This is the transfer impedance between terminals A and B.

As the transfer impedance between any two terminals is the same measured from either terminal in a reciprocal static network, it is unnecessary to make all the measurements described in (2) above.

For networks which can be set up on a d-c calculating table or on an a-c network analyzer the second method for determining equivalent circuit is the more direct. Some of the branch impedances of an equivalent circuit determined on the a-c network analyzer may contain negative resistances; this makes the equivalent circuit unsuitable for subsequent use on the a-c network analyzer unless special negative resistance devices are used. Negative resistance offers no difficulty in an analytic solution.

In determining an equivalent circuit of a specified number of terminals, either of the methods may be used, or a combination of the two methods. Driving-point impedance is not required in the second method, but it may often be used to advantage in connection with the first method.

The following discussion of equivalent circuits assumes constant frequency and constant circuit parameters (the effect of saturation is not included).

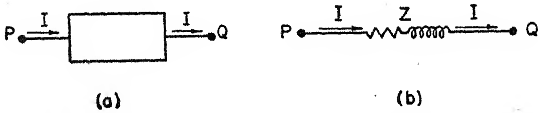
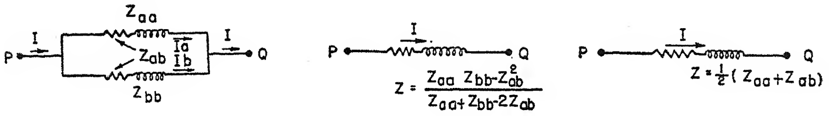


FIG. 8. (*a*) Two-terminal circuit. (*b*) Equivalent circuit to replace (*a*).

Two-Terminal Equivalent Circuit. A two-terminal circuit is shown in Fig. 8(*a*) and its assumed equivalent circuit in Fig. 8(*b*). The given circuit in Fig. 8(*a*) is represented as a box with two terminals, P and Q. The box may represent any impedance network between P and Q, with or without mutual impedances between the branches. Since there are only two terminals, the equivalent circuit will have but a single impedance branch, Z. For the circuit (*b*) to be the equivalent of (*a*) with the same voltage V applied between its terminals, the current I entering and leaving the equivalent circuit must be the same as that entering and leaving the given circuit. The impedance Z of the equivalent circuit may be determined by applying a voltage V between terminals P and Q in the given circuit and measuring the current I; then $Z = V/I$.

Two Circuits with Self-Impedances Z_{aa} and Z_{bb} Connected at Both Ends with Mutual Impedance Z_{ab} between Them. The given circuit for this case is shown in Fig. 9(a) and the desired equivalent circuit in Fig. 9(b), with impedance Z to be determined. The impedance Z of the equivalent circuit can be obtained as follows:



(a) (b) (c)

FIG. 9. (a) Two circuits connected at their terminals, with self-impedances Z_{aa} and Z_{bb}, and mutual impedance Z_{ab} between them. (b) Equivalent circuit for (a), $Z_{aa} \neq Z_{bb}$. (c) Equivalent circuit for (a), $Z_{aa} = Z_{bb}$.

Applying a voltage V between terminals P and Q of the given circuit,

$$V = I_a Z_{aa} + I_b Z_{ab}$$

$$V = I_a Z_{ab} + I_b Z_{bb}$$

where

I_a and I_b = the currents in circuits a and b, respectively

and

$I = I_a + I_b$ = total current entering the circuit at terminal P and leaving at terminal Q

Solving for I_b in terms of I_a,

$$I_b = I_a \frac{Z_{aa} - Z_{ab}}{Z_{bb} - Z_{ab}} \qquad [34]$$

Substituting [34] in the original equations,

$$V = I_a \frac{Z_{aa}Z_{bb} - Z_{ab}^2}{Z_{bb} - Z_{ab}} \qquad [35]$$

From [34] and [35],

$$I_a = V \frac{Z_{bb} - Z_{ab}}{Z_{aa}Z_{bb} - Z_{ab}^2}$$

$$I_b = V \frac{Z_{aa} - Z_{ab}}{Z_{aa}Z_{bb} - Z_{ab}^2}$$

Adding I_a and I_b,

$$I = I_a + I_b = V\frac{Z_{aa} + Z_{bb} - 2Z_{ab}}{Z_{aa}Z_{bb} - Z_{ab}^2}$$

The impedance of the equivalent circuit is

$$Z = \frac{V}{I} = \frac{Z_{aa}Z_{bb} - Z_{ab}^2}{Z_{aa} + Z_{bb} - 2Z_{ab}} \qquad [36]$$

For identical circuits, $Z_{aa} = Z_{bb}$ and [36] becomes

$$Z = \frac{Z_{aa}^2 - Z_{ab}^2}{2(Z_{aa} - Z_{ab})} = \frac{Z_{aa} + Z_{ab}}{2} \qquad [37]$$

The equivalent circuit for two identical circuits with equal self-impedances Z_{aa}, connected at both ends, with mutual impedance Z_{ab} between them is shown in Fig. 9(*c*).

Three-Terminal Circuits. A three-terminal circuit can be represented by either a Y or a Δ. Figure 10(*a*) shows three terminals *a*, *b*, and *c*, connected by a Δ with self-impedance branches Z_{ab}, Z_{ac}, Z_{bc}.

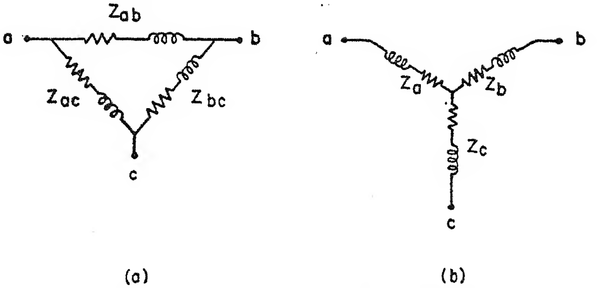


FIG. 10. (*a*) Δ-connected circuit between terminals *a*, *b*, and *c*. (*b*) Y-connected circuit between terminals *a*, *b*, and *c*.

Figure 10(*b*) shows the same terminals connected by a Y with self-impedance branches Z_a, Z_b, and Z_c. The Δ and Y must be exact equivalents of each other for conditions at their terminals. The relations[2] between the branch impedances of the Y and Δ circuits, determined by applying a voltage between the terminals taken two at a time with the third terminal open in both the Δ and the Y, and equating V/I for the two circuits, are

$$\begin{aligned} Z_a + Z_b &= \frac{Z_{ab}(Z_{ac} + Z_{bc})}{Z_{ab} + Z_{ac} + Z_{bc}} \\ Z_a + Z_c &= \frac{Z_{ac}(Z_{ab} + Z_{bc})}{Z_{ab} + Z_{ac} + Z_{bc}} \qquad [38] \\ Z_b + Z_c &= \frac{Z_{bc}(Z_{ab} + Z_{ac})}{Z_{ab} + Z_{ac} + Z_{bc}} \end{aligned}$$

Subtracting each of the equations of [38] in turn from the sum of the other two, and dividing the resulting equation by 2,

$$\begin{aligned} Z_a &= \frac{Z_{ab}Z_{ac}}{Z_{ab} + Z_{ac} + Z_{bc}} \\ Z_b &= \frac{Z_{ab}Z_{bc}}{Z_{ab} + Z_{ac} + Z_{bc}} \\ Z_c &= \frac{Z_{ac}Z_{bc}}{Z_{ab} + Z_{ac} + Z_{bc}} \end{aligned} \tag{39}$$

The branch impedances of the Y in terms of the Δ impedances are given by [39].

Equating the transfer impedances between terminals in the Δ to those in the Y,

$$\begin{aligned} Z_{ab} &= Z_a + Z_b + \frac{Z_aZ_b}{Z_c} = \frac{Z_aZ_b + Z_aZ_c + Z_bZ_c}{Z_c} \\ Z_{ac} &= Z_a + Z_c + \frac{Z_aZ_c}{Z_b} = \frac{Z_aZ_b + Z_aZ_c + Z_bZ_c}{Z_b} \\ Z_{bc} &= Z_b + Z_c + \frac{Z_bZ_c}{Z_a} = \frac{Z_aZ_b + Z_aZ_c + Z_bZ_c}{Z_a} \end{aligned} \tag{40}$$

The branch impedances of the Δ in terms of the Y impedances are given by [40].

Equal Impedances in the Three Branches of the Δ or Y. If $Z_{ab} = Z_{ac} = Z_{bc}$, from [39],

$$Z_a = Z_b = Z_c = \frac{Z_{ab}}{3} \tag{41}$$

If $Z_a = Z_b = Z_c$, from [40],

$$Z_{ab} = Z_{ac} = Z_{bc} = 3Z_a \tag{42}$$

From [41] and [42] it follows that the branch impedances of an equivalent Y are one-third those of a given symmetrical Δ; and the branch impedances of the equivalent Δ are three times those of a given symmetrical Y. These relations hold when impedances are expressed in *ohms* or in *per unit on the same kva and voltage bases*.

If the impedances in [41] and [42], given in ohms, are expressed in per unit on the same base kva per phase, with Δ impedances based on line-to-line voltage and those of the Y on line-to-neutral voltage, from [23] they will be the same. The Δ impedances in ohms are three times the Y impedances, but the square of base line-to-line voltage is three times the square of base line-to-neutral voltage; therefore the impedances of the Y and Δ are equal when expressed in per unit on the same kva base and the base voltages of their respective circuits. This makes it possible to replace a symmetrical three-phase self-impedance circuit connected in Δ by an equivalent Y of the same per unit impedance per phase when conditions *outside* the circuit are considered, and per unit quantities are used.

A Circuit Consisting of Two Self-Impedances Z_{aa} and Z_{bb} Connected at One End, with Mutual Impedance Z_{ab} between Them. Figure 11(*a*) shows the given circuit and Fig. 11(*b*) the equivalent Y. The branches

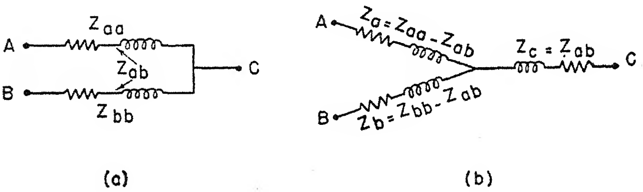


FIG. 11. (*a*) Two circuits connected at one set of terminals, with self-impedances Z_{aa} and Z_{bb} and mutual impedance Z_{ab} between them. (*b*) Equivalent circuit for (*a*).

Z_a, Z_b, and Z_c of the equivalent Y are determined by applying a voltage between terminals *A* and *C* with *B* isolated, between *B* and *C* with *A* isolated, and between *A* and *B* with *C* isolated, then equating V/I in each case.

The first two equations can be determined by inspection. The impedance between terminals *A* and *C* in the given circuit with terminal *B* isolated is Z_{aa}, and in the equivalent circuit $Z_a + Z_c$. Similarly, the impedance between terminals *B* and *C* with *A* isolated is Z_{bb} in the given circuit and $Z_b + Z_c$ in the equivalent circuit. With a voltage *V* applied between terminals *A* and *B* in the equivalent circuit, the impedance is $Z_a + Z_b$; in the given circuit, the current *I* enters at *A* and leaves at *B*, flowing in opposite directions through the self-impedances Z_{aa} and Z_{bb} in series. The voltage drop between terminals *A* and *B* is therefore

$$V = IZ_{aa} - IZ_{ab} + IZ_{bb} - IZ_{ab} = I(Z_{aa} + Z_{bb} - 2Z_{ab})$$

The three equations expressing Z_a, Z_b, and Z_c in terms of Z_{aa}, Z_{bb}, and Z_{ab} are

$$Z_a + Z_c = Z_{aa}$$

$$Z_b + Z_c = Z_{bb}$$

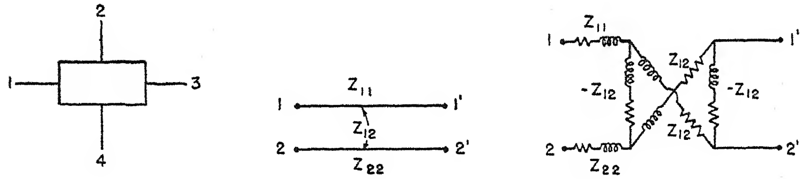
$$Z_a + Z_b = Z_{aa} + Z_{bb} - 2Z_{ab}$$

Solving these equations,

$$\begin{aligned} Z_a &= Z_{aa} - Z_{ab} \\ Z_b &= Z_{bb} - Z_{ab} \\ Z_c &= Z_{ab} \end{aligned} \qquad [43]$$

The values of Z_a, Z_b, and Z_c are indicated in Fig. 11(*b*). If terminals *A* and *B* of Fig. 11(*b*) are now connected, the circuit becomes a two-terminal circuit and checks Fig. 9(*b*) with $Z_{aa} \neq Z_{bb}$ and Fig. 9(*c*) if $Z_{aa} = Z_{bb}$.

Four-Terminal Circuits. A four-terminal circuit is shown in Fig. 12(*a*) as a box with four terminals. The general equivalent circuit will have a minimum of six branch impedances which can be evaluated when the nature of the given circuit is known. Two four-terminal circuits will be considered.



(a) (b) (c)

FIG. 12. (*a*) General four-terminal circuit. (*b*) Four-terminal circuit consisting of two circuits with self-impedances Z_{11} and Z_{22} and mutual impedance Z_{12} between them. (*c*) Equivalent circuit for (*b*).

A Circuit Consisting of Two Self-Impedances Z_{11} and Z_{22}, Not Connected at Either End, with Mutual Impedance Z_{12} between Them. Figure 12(*b*) shows the given four-terminal circuit. The impedances directly connecting the four terminals of the equivalent circuit, determined from the transfer impedances in accordance with the second method, are[3,4]

$$Z_{11} - \frac{Z_{12}^2}{Z_{22}} \quad \text{between terminals 1 and 1}'$$

$$Z_{22} - \frac{Z_{12}^2}{Z_{11}} \quad \text{between terminals 2 and 2}' \qquad [44]$$

$$\frac{Z_{11}Z_{22} - Z_{12}^2}{Z_{12}} \quad \text{between terminals 1 and 2 and also between terminals 1}' \text{ and 2}'$$

$$-\frac{Z_{11}Z_{22} - Z_{12}^2}{Z_{12}} \quad \text{between terminals 1 and 2}' \text{ and also between terminals 2 and 1}'$$

A simplification[5] of the equivalent circuit obtained by using the impedances given in [44] is shown in Fig. 12(*c*). In this equivalent circuit, the self-impedances Z_{11} and Z_{22} are placed in the terminal links outside the mesh. The impedances in the four-terminal mesh are then obtained from [44] for circuits with zero self-impedances: $Z_{11} = Z_{22} = 0$. For this condition, the impedances given by the first two equations in [44] are infinite; the other two equations give $-Z_{12}$ and Z_{12}, respectively. The self-impedance Z_{11} may be placed in a terminal link at either 1 or at 1′, and likewise Z_{22} may be placed at either 2 or 2′. If terminals 1′ and 2′ are connected, Fig. 12(*c*) becomes Fig. 11(*b*); if 1 and 2 are also connected, Fig. 12(*c*) becomes Fig. 9(*b*).

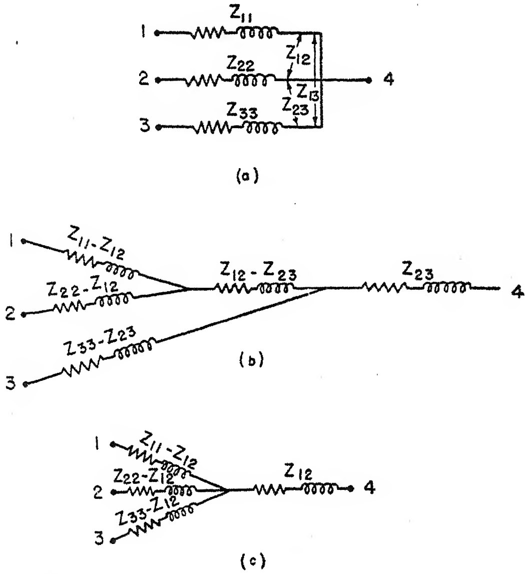


Fig. 13. (*a*) Four-terminal circuit consisting of three circuits connected at one set of terminals with self-impedances Z_{11}, Z_{22}, and Z_{33} and mutual impedances Z_{12}, Z_{13}, and Z_{23} between circuits. (*b*) Equivalent circuit for (*a*), if $Z_{13} = Z_{23} \neq Z_{12}$. (*c*) Equivalent circuit for (*a*), if $Z_{13} = Z_{23} = Z_{12}$.

A Circuit Consisting of Three Self-Impedances Z_{11}, Z_{22}, *and* Z_{33}, *Connected at One End, with Mutual Impedances between Them* — (*a*) *Two Equal Mutual Impedances:* $Z_{13} = Z_{23} \neq Z_{12}$. Figure 13(*a*) shows the given four-terminal circuit. Figure 13(*b*) is the equivalent circuit. That (*b*) is the equivalent of (*a*) can be shown by applying voltages between the four terminals in (*a*) and (*b*) taken two at a time with the other two terminals open and each time equating V/I in the two circuits.

(*b*) *Three Equal Mutual Impedances:* $Z_{13} = Z_{23} = Z_{12}$. With $Z_{23} = Z_{12}$ in Fig. 13(*b*), the equivalent circuit becomes that shown in Fig. 13(*c*).

Special Equivalent Circuits for Use in the One-Line Impedance Diagrams

Two-Winding Transformer. The two-winding transformer has four terminals. In most system calculations, however, it may be represented by a two- or a three-terminal equivalent circuit. An equivalent circuit has the same number of terminals as the equivalent of the circuit it replaces, but not necessarily the same number of terminals as the actual circuit itself. This is allowable, because the equivalent circuit is part of a one-line impedance diagram of the system to be used for certain specified calculations. Therefore certain connections can be made in the actual circuit which do not really exist, but, if they did exist, they would in no way affect the calculations to be made for the *given problem*.

Figure 14(*a*) shows a two-winding transformer with terminals A, A', B, and B'. A single-phase voltage E is applied to the primary winding AA'. The secondary winding BB' supplies a single-phase load through a two-conductor transmission line. The problem is to determine the equivalent circuit for the transformer to be used in a one-line impedance diagram of the single-phase system for calculating currents and voltages between conductors for various load conditions, or the currents during short circuits between conductors.

If the transformer in Fig. 14(*a*) with the four terminals A, A', B, B' is represented as in 14(*b*) with A' and B' connected, the given problem is in no way affected. The equivalent circuit, however, is required to replace a three-terminal equivalent of the actual circuit and not a four-terminal one. Since there are but three terminals, the equivalent circuit may be either an equivalent Y or Δ. An equivalent Y will be used, as shown in Fig. 14(*c*), with branch impedances Z_1, Z_2, and Z_3 to be determined.

Let the per unit self-impedance of winding AA' be Z_{11} and that of winding BB' be Z_{22} with mutual impedance Z_{12} between windings. There are three unknown impedances in the assumed equivalent circuit, Z_1, Z_2, and Z_3; therefore three tests will be required on the actual transformer to determine the desired equivalent circuit. Positive directions for currents are indicated by arrows in both circuits. Applying a voltage between A and A' with B isolated in Figs. 14(*b*) and (*c*), and then between B and B' with A isolated, reading the currents in both cases, and equating V/I in the transformer and in the equivalent circuit, two equations are obtained:

$$Z_{11} = Z_1 + Z_3 \qquad [45]$$

$$Z_{22} = Z_2 + Z_3 \qquad [46]$$

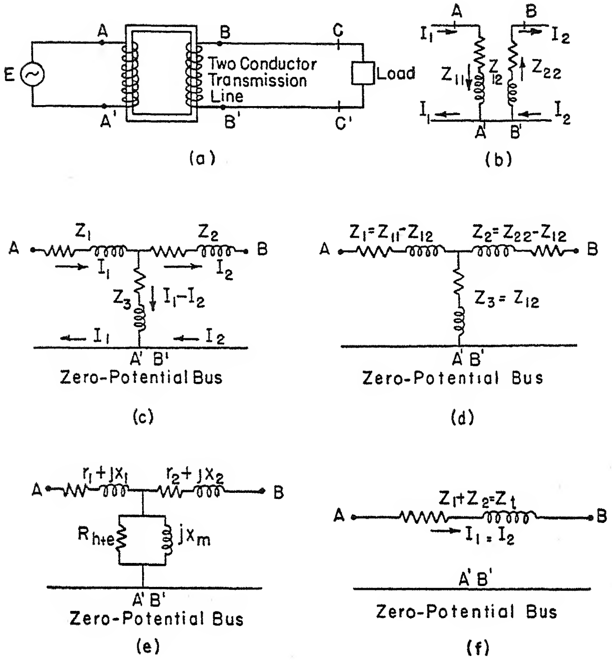

A
B
C
E
Two Conductor Transmission Line
Load
A'
B'
C'
(a)
A
B
I_1
I_2
Z_{11}
Z_{12}
Z_{22}
I_1
I_2
A' B'
(b)
Z_1
Z_2
A
B
I_1
I_2
Z_3
I_1-I_2
I_1
I_2
A' B'
Zero-Potential Bus
(c)
$Z_1=Z_{11}-Z_{12}$
$Z_2=Z_{22}-Z_{12}$
A
B
$Z_3=Z_{12}$
A' B'
Zero-Potential Bus
(d)
r_1+jx_1
r_2+jx_2
A
B
R_{h+e}
jx_m
A' B'
Zero-Potential Bus
(e)
$Z_1+Z_2=Z_t$
A
B
$I_1=I_2$
A' B'
Zero-Potential Bus
(f)

FIG. 14 (*a*) Two-line diagram of a single-phase two-conductor system with single-phase transformer. (*b*) Points A' and B' connected to give an equivalent of the transformer in (*a*) which satisfies a given problem. (*c*) Assumed equivalent circuit for (*b*) with Z_1, Z_2, and Z_3 to be determined. (*d*), (*e*) Three-terminal equivalent circuits for (*b*) with exciting current included. (*f*) Two-terminal equivalent circuit for (*b*) with exciting current neglected.

The third test can be made on the transformer by applying a voltage V to one winding AA' with the other winding BB' short-circuited, and in the equivalent circuit by applying a voltage between A and the zero-potential bus, with B connected to this bus. In the transformer,

$$V = I_1 Z_{11} - I_2 Z_{12}, \quad \text{in winding } AA'$$

$$0 = -I_1 Z_{12} + I_2 Z_{22}, \quad \text{in winding } BB'$$

Eliminating I_2 in these equations,

$$V = I_1\left(Z_{11} - \frac{Z_{12}^2}{Z_{22}}\right), \quad \text{and} \quad \frac{V}{I_1} = Z_{11} - \frac{Z_{12}^2}{Z_{22}}$$

In the equivalent circuit,

$$\frac{V}{I_1} = Z_1 + \frac{Z_2 Z_3}{Z_2 + Z_3}$$

Therefore

$$Z_{11} - \frac{Z_{12}^2}{Z_{22}} = Z_1 + \frac{Z_2 Z_3}{Z_2 + Z_3} \qquad [47]$$

Subtracting [47] from [45],

$$\frac{Z_{12}^2}{Z_{22}} = Z_3 - \frac{Z_2 Z_3}{Z_2 + Z_3} = \frac{Z_3^2}{Z_2 + Z_3} \qquad [48]$$

Equations [45], [46], and [48] are satisfied if

$$\begin{aligned} Z_1 &= Z_{11} - Z_{12} \\ Z_2 &= Z_{22} - Z_{12} \\ Z_3 &= Z_{12} \end{aligned} \qquad [49]$$

The equivalent circuit for the transformer is shown in the one-line impedance diagram of Fig. 14(*d*), in which

$Z_1 = r_1 + jx_1 = Z_{11} - Z_{12} =$ leakage impedance of winding AA'

$Z_2 = r_2 + jx_2 = Z_{22} - Z_{12} =$ leakage impedance of winding BB'

$Z_3 = Z_{12} =$ mutual impedance between windings. Z_{12} is usually represented by x_m, the magnetizing reactance, paralleled with R_{h+e}, representing the hysteresis and eddy current losses, as in Fig. 14(*e*).

In transformers, the sum of the primary and secondary leakage reactances, $x_1 + x_2$, may be of the order of 10% and x_m at normal voltage about 3000%. For this reason, there is no measurable difference between the per unit self-impedance of the windings at normal voltage and the exciting or mutual impedance; therefore tests 1 and 2 described above would give but one per unit impedance. Usually but two tests are made, either 1 or 2, which gives the mutual impedance, and 3, which (with Z_3 large with respect to Z_1 and Z_2) gives $Z_1 + Z_2$, the sum of the resistances and leakage reactances of the two windings. In many problems the mutual impedance is considered infinite and the transformer replaced by a series impedance, $Z_t = Z_1 + Z_2$. It is then a two-terminal circuit without connection to the zero-potential bus, as shown in Fig. 14(*f*).

When Fig. 14(*d*) is compared with 11(*b*), it is seen that the equivalent circuit for the two-winding transformer for use in the one-line impedance diagram could have been obtained directly from the general three-terminal equivalent circuit representing two self-impedances connected at one end, with mutual impedance between them.

Two-Winding Transformer Banks in Balanced Three-Phase Systems. In three-phase power systems, the windings of the three transformers comprising the bank may be connected either in Δ or in Y. There are four possible arrangements. The primary windings may be connected either in Δ or in Y, and with either arrangement the secondary windings may likewise be connected either in Δ or in Y. With identical transformers in the bank, the circuits are symmetrical, and therefore the bank if connected Δ–Δ, Δ–Y, or Y–Δ can be replaced by an equivalent bank connected Y–Y, when conditions *outside* the bank are to be determined. Expressed in per unit on the rating of the transformer, the self-impedances of the windings and the mutual impedance between them are the same in the equivalent Y–Y bank as in the given bank. The *difference in phase* of the currents and voltages on opposite sides of a Δ–Y transformer bank is discussed in Chapter III.

With a Y–Y bank (or an equivalent Y–Y bank) in a balanced three-phase power system, the neutral points of the Y's may be connected without affecting the balanced system. Then the equivalent circuit for the bank, to be used in the one-line impedance-to-neutral diagram of the balanced system for determining currents and voltages outside the bank, will be a three-terminal circuit, one terminal to be connected to the primary circuit, one to the secondary circuit, and the third to the neutral bus. Since per unit impedances in a line-to-neutral diagram are based on line-to-neutral voltage and the kva per phase, the per unit equivalent circuit for use in the one-line diagram will be the same as that of any one of the three single-phase units. See Fig. 14(*e*) if exciting current is considered, and Fig. 14(*f*) if neglected. Figure 15 gives the equivalent circuit of a Δ–Y connected transformer bank for use in a one-line diagram of a balanced three-phase system.

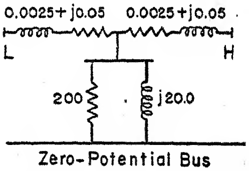


FIG. 15. Per unit equivalent circuit based on rating of a Δ–Y transformer bank on an equivalent Y–Y basis. Each unit rated 10,000 kva, 4000–66,500/115,000 Y volts. Leakage reactance 10%, resistance 0.5%, magnetizing current 5%, hysteresis and eddy current 0.5%. One-half the total leakage reactance and copper resistance for the two windings has been assumed for each winding.

Three-Winding Transformer Banks in Balanced Three-Phase Systems.[6] With three identical transformers in the bank and balanced currents and voltages in the system, any of the windings connected in Δ can be replaced by an equivalent Y of the same per unit impedances.

If exciting current is neglected, the equivalent circuit for the three-winding transformer bank for use in the single-phase one-line diagram for determining balanced currents and voltages outside the bank is a three-terminal circuit.

The three windings of each single-phase unit are called primary, secondary, and tertiary. These windings are shown in Fig. 16(a). Let

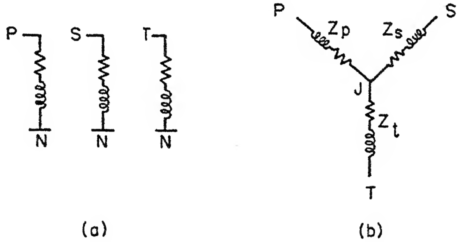


FIG. 16. (a) One phase of a three-winding transformer bank (if connected in Δ, replaced by its equivalent Y). (b) Equivalent circuit of (a) for use in the positive-sequence diagram. With exciting current neglected, there is no connection to the neutral bus N. Terminals P, S, and T are to be connected to the primary, secondary, and tertiary circuits, respectively.

Z_{ps} = per unit leakage impedance in primary and secondary windings with the tertiary winding open

Z_{pt} = per unit leakage impedance in primary and tertiary windings with the secondary winding open

Z_{st} = per unit leakage impedance in secondary and tertiary windings with the primary winding open

Impedances are based on voltages which are proportional to the number of turns in the windings, and therefore the given impedance between two windings with the third open is the same referred to either of the two windings. Since the kva ratings of the three windings are not usually equal, Z_{ps}, Z_{pt}, and Z_{st} as defined above are expressed on the same kva base.

Figure 16(b) shows the assumed equivalent circuit with branch impedances Z_p, Z_s, and Z_t to be evaluated. Following the procedure given above,

$$Z_p + Z_s = Z_{ps}$$

$$Z_p + Z_t = Z_{pt}$$

$$Z_s + Z_t = Z_{st}$$

Solving the three simultaneous equations,

$$\begin{aligned} Z_p &= \tfrac{1}{2}(Z_{ps} + Z_{pt} - Z_{st}) \\ Z_s &= \tfrac{1}{2}(Z_{ps} + Z_{st} - Z_{pt}) \\ Z_t &= \tfrac{1}{2}(Z_{pt} + Z_{st} - Z_{ps}) \end{aligned} \qquad [50]$$

Z_p, Z_s, and Z_t in Fig. 16(*b*) are replaced by their values given in [50]. Equivalent circuits for transformers are further discussed in this volume and in Vol. II.

Three-Phase Synchronous Machines. In the design of synchronous machines, care is taken to have the phases symmetrical and the generated voltages approximately sinusoidal and balanced. The windings of the machine may be connected in either Y or Δ. If they are connected in Δ, the Δ can be replaced by an equivalent Y for determining currents and voltages of fundamental frequency outside the machine. As a first approximation, the machine can be represented by an equivalent circuit consisting of a generated voltage between the neutral (or equivalent neutral) of the machine and a lumped impedance, representing the impedance to neutral of the machine under balanced loads. The generated voltage in the equivalent circuit represents the generated voltage of one phase. Figure 17 shows the equivalent circuit of a synchronous machine for use in the positive-sequence diagram. N is the neutral of the machine, Z the impedance to neutral, T the terminal of one phase, and E the generated voltage in that phase. The values to be assigned to Z and E depend upon the problem. The resistance component of the positive-sequence impedance is relatively small and can usually be neglected. If initial short-circuit current of fundamental frequency is to be determined, subtransient reactance will be used in Z, and E will be the voltage behind subtransient reactance. In transient stability studies, or in short-circuit studies after subtransient effects have disappeared, transient reactance and the voltage behind transient reactance may be used in Z and E. For steady-state operation, equivalent steady-state reactance and the voltage behind this reactance are used. These reactances and the corresponding voltages are discussed further in Vol. II.

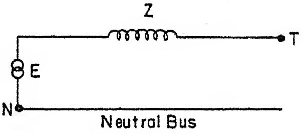


FIG. 17. Equivalent circuit for a synchronous machine with generated voltage, E, and impedance to neutral, Z, for use in the positive-sequence one-line diagram.

Transmission Circuits. If capacitance and leakance are neglected, a symmetrical three-phase transmission circuit (or one assumed symmetrical) has equal impedances in the three phases under balanced operation. It can therefore be represented in the positive-sequence diagram as a lumped self-impedance Z.

$$Z = \ell(r + jx) = \ell(r + j2\pi fL) \tag{51}$$

where ℓ is the length of line in miles, r the resistance in ohms per mile, L the inductance in henries per mile, and f the frequency in cycles per second. Equivalent circuits for transmission circuits with distributed constants are discussed in Chapter VI.

THREE-PHASE FAULTS

A three-phase fault, since it is symmetrical in the three phases, does not unbalance the system. The fundamental-frequency currents and voltages resulting from the fault are of the same phase order as the generated voltages, and can be calculated from the positive-sequence one-line impedance diagram and the operating conditions previous to the occurrence of the fault.

D-C Components. The d-c component of fault current in any phase will depend upon the point of the voltage wave at which the fault is applied. If the phase voltage is at its crest value, which corresponds to zero flux linkages in the phase, the current wave with resistance neglected will be a sinusoidal wave symmetrical about the current axis. If the phase voltage is at its zero value, which corresponds to maximum flux linkages in the phase, the current wave with resistance neglected will be a completely offset wave. Its initial crest value will be equal to twice that of the symmetrical current wave. The rms value of the symmetrical current wave is $(1/\sqrt{2})$ times its crest value. Let this rms value be I. Then the d-c component of the completely offset wave is $\sqrt{2}I$. The initial rms value of the completely offset wave with resistance neglected is

$$\sqrt{I^2 + (\sqrt{2}I)^2} = \sqrt{3}I$$

When the d-c component has decayed to zero, the wave becomes symmetrical about the current axis. In three-phase short-circuit calculations, initial symmetrical rms fault currents are calculated by using subtransient reactances and allowance is made for the d-c component if the maximum possible rms current is required. As the currents and voltages of the three phases are displaced from each other by 120°, the current of only one phase can have the approximate maximum rms value given above. Also, only one phase could have an initial symmetrical current wave.

Transient currents of fundamental frequency are currents which exist after d-c components and subtransient effect have disappeared. They are symmetrical about the current axis and their rms values are determined by using transient reactances. Sustained short-circuit currents are symmetrical about the current axis and are determined by using steady-state reactances.

Fundamental Frequency Components. Two methods are available for determining three-phase short-circuit currents of fundamental frequency.

1. Currents in the fault and throughout the system can be expressed in terms of the applied (generated) voltages and the transfer and driving-point impedances with the fault on the system by [33]. This method requires the determination of the generated voltages from given operating conditions previous to the occurrence of the fault.

2. Any fault point F in the network may be considered a terminal point and the voltage V_f at this point determined from the given operating conditions previous to the occurrence of the fault. V_f is applied through zero impedance between F and the zero-potential bus. The initial currents may then be expressed in terms of the driving-point and transfer impedances of the network and the voltages at all terminal points, F included. When initial currents are expressed with the point F at which the fault will occur as one of the terminal points, none of the transfer and driving-point impedances will be changed by the fault. The only change in [33] resulting from the fault will be in the voltage V_f at the fault, which becomes zero. Before the fault the voltage at F was V_f; after the fault it is zero. The change in voltage is therefore $-V_f$. Applying the principle of superposition, the currents in the system can be determined by adding to the load currents the currents resulting from the voltage $-V_f$ applied at F, with all other applied voltages equated to zero. Because of the fault, the changes in the currents flowing *into the network* at terminal points 1, 2, $\cdots n$, and F are

$$\begin{aligned}
\Delta I_1 &= \frac{V_f}{A_{f1}} \\
\Delta I_2 &= \frac{V_f}{A_{f2}} \\
&\cdots\cdots\cdots\cdots \\
\Delta I_n &= \frac{V_f}{A_{fn}} \\
\Delta I_f &= -\frac{V_f}{A_{ff}}
\end{aligned} \tag{52}$$

where A_{ff} is the driving-point impedance at point F, and A_{f1}, A_{f2}, $\cdots$ A_{fn} are the transfer impedances between F and 1, 2, $\cdots$ n, respectively.

ΔI_f represents the change in the current flowing from the fault into the network. Since there was no current flowing into the network from the fault before the fault occurred, ΔI_f is the fault current flowing *into the network*. If I_f is the current flowing *from the network* into the fault,

$$I_f = -\Delta I_f = \frac{V_f}{A_{ff}} = \frac{V_f}{Z_1} \qquad [53]$$

From [53], the current flowing into the fault can be determined from the prefault voltage and the driving-point impedance at the fault. In a positive-sequence network, the driving-point impedance at any point is the positive-sequence impedance viewed from that point and is designated by Z_1. To determine the three-phase fault current by method 2, it is necessary to know only V_f, the prefault voltage, and Z_1, the positive-sequence impedance viewed from the fault. The currents in the network due to the fault can be determined from the fault current and the positive-sequence impedance diagram of the system, or the changes in the currents entering the network at terminal points can be determined from [52]. The current at any point in the network due to the fault is added to the load current to obtain the resulting current at that point.

The fault and system currents calculated by the two methods given above are the **initial symmetrical rms values,** which exist for only a short time after the occurrence of the fault and are determined by the use of subtransient reactances. At the instant the fault occurs, the voltages in the positive-sequence system behind the subtransient reactances of the synchronous machines on the system will remain fixed in magnitude and retain their relative phase angles. The fluxes linking the rotor circuits cannot change instantly and the rotors cannot change their relative angular positions instantly; therefore the magnitudes and phases of the voltages behind the subtransient reactances, being proportional to the fluxes, will remain fixed at the first instant. If currents and voltages at subsequent intervals are to be determined, changes in relative angular positions of the rotors should be taken into account, if their effect is appreciable, as well as the change of machine reactances from subtransient to transient and finally to steady-state reactances.

The simplest way to determine three-phase short-circuit currents in a large three-phase system is by means of a d-c calculating table, if resistances and capacitances can be neglected. On the d-c table, a voltage V_f is applied between the neutrals of the machines, connected at a common point, and the zero-potential bus of the table. This corresponds to the condition of no load when all per unit generated voltages are equal and in phase. The procedure with the d-c table would be just the same if the system were assumed to be operating under load and the currents due to the fault were determined by applying a voltage $-V_f$ at the point of fault, equal in magnitude and opposite in sign to the voltage which existed there before the fault, following the second method given above. Load current can be added to short-circuit current to obtain resultant currents. Since load currents, in general, are not in phase with short-circuit currents, the error in neglecting them is small.

If an a-c network analyzer is used, the positive-sequence network is set up and the generated voltages adjusted in magnitude and phase until the operating conditions previous to the occurrence of the fault are obtained. Connection is then made between the point of the system where the fault occurs and the neutral bus. The currents in the system will include load currents as well as currents resulting from the fault. Currents are initial symmetrical rms values when subtransient reactances and the voltages behind these reactances are used. Solution on the a-c network analyzer corresponds to the first method given above.

To illustrate the procedure for determining the currents and voltages during a three-phase fault by the two methods given above, a simple problem will be solved.

Problem 2. A synchronous generator supplies power to a synchronous motor connected directly to its terminals. The positive-sequence subtransient reactances of the generator and motor on a certain kva base and the rated voltage of the machines are 40 and 200%, respectively. Resistance is neglected. The current supplied by the generator is 50% of base current. The power factor at the bus is unity and the voltage is 98% of rated voltage. Find the initial symmetrical rms current in the fault, the generator, and the motor when a three-phase fault occurs on the bus.

Solution by Second Method. Figure 18 (*a*) shows a one-line diagram of the system, part (*b*) the positive-sequence diagram. With V_t, the voltage at the bus before the fault occurred as reference vector,

$$V_f = V_t = 0.98 + j0$$
$$I_g = 0.5 + j0$$
$$I_m = -0.5 + j0$$

where I_g and I_m denote currents in the generator and motor, respectively, positive direction of current flow being from the neutrals of the machines towards the fault.

The positive-sequence impedance viewed from the fault in Fig. 18 (*b*) is

$$Z_1 = \frac{j0.40 \times j2.00}{j2.40} = j0.333$$

The current flowing into the fault, from [53], is

$$I_f = \frac{V_f}{Z_1} = \frac{0.98}{j0.333} = -j2.94$$

The distribution of the fault current in the system can be obtained from the impedance diagram. In the simple system of Fig. 18(*b*), I_f divides inversely as the parallel impedances in its paths. Thus,

I_f (from generator)

$$= -j2.94 \times \frac{2.00}{2.40} = -j2.45$$

I_f (from motor)

$$= -j2.94 \times \frac{0.40}{2.40} = -j0.49$$

Adding the currents due to the fault and the load currents in generator and motor,

$$I_g = 0.5 - j2.45$$
$$I_m = -0.5 - j0.49$$

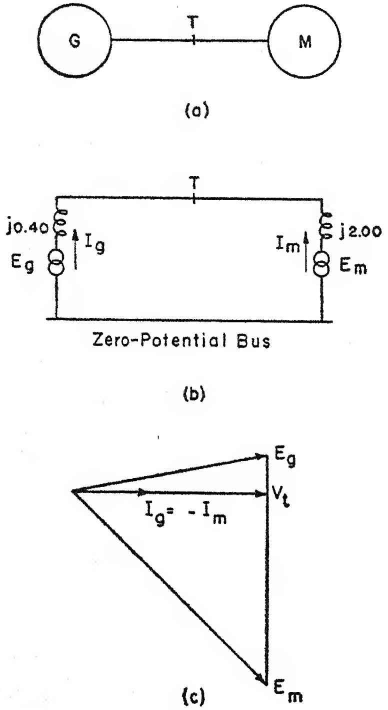


FIG. 18. (*a*) One-line diagram of system of Problem 2. (*b*) Positive-sequence diagram. (*c*) Voltage and current vector diagram before the fault.

Solution by First Method. The internal voltages of both machines must be calculated from given operating conditions before this method can be applied. With V_t, the voltage at the bus before the fault occurs, as reference vector, the voltages behind subtransient reactances are

$$E_g = V_t + I_gZ_g = 0.98 + (0.5 \times j0.40)$$
$$= 0.98 + j0.20 = 1.00\ \underline{/11.5^\circ}$$

$$E_m = V_t + I_mZ_m = 0.98 + (-0.5 \times j2.00)$$
$$= 0.98 - j1.00 = 1.40\ \overline{/45.6^\circ}$$

where the subscripts *g* and *m* refer to generator and motor, respectively. Figure 18(*c*) gives the voltage and current vector diagram for the given operating conditions. After the fault occurs,

$$I_g \text{ (in generator)} = \frac{E_g}{Z_g} = \frac{0.98 + j0.20}{j0.40} = 0.5 - j2.45$$

$$I_m \text{ (in motor)} = \frac{E_m}{Z_m} = \frac{0.98 - j1.00}{j2.00} = -0.5 - j0.49$$

$$I_f \text{ (in fault)} = \frac{E_g}{Z_g} + \frac{E_m}{Z_m} = -j2.94$$

Problem 3. A three-phase, 60-cycle generator, rated 5000 kva, 11 kv, with a transient reactance of 30% on its rating, supplies power through a three-phase current limiting reactor in series with two miles of three-phase transmission line. The reactor is rated 5% in a three-phase, 60-cycle, 13.2-kv, 350-ampere circuit. The reactance to neutral of the transmission line is 0.61 ohm per mile and the resistance is 0.27 ohm per mile.

(*a*) Draw the positive-sequence diagram with impedances (1) in per unit based on the generator rating, (2) in ohms.

(*b*) If the generator has a field current which will give rated terminal voltage on open circuit, what will be the transient current when a three-phase fault occurs at the terminals of the transmission line?

Note: The kva and voltage ratings of the generator are three-phase kva and line-to-line voltages, respectively. With generator reactance given in per cent it is not necessary to know whether the windings are connected in Δ or in Y unless currents in the windings are required in amperes. The rated three-phase kva of the reactor is $\sqrt{3} \times 13.2 \times 350$.

Solution. Base line-to-neutral voltage $= \dfrac{11{,}000}{\sqrt{3}} = 6350$ volts

$$\text{Base kva per phase} = \frac{5000}{3} = 1667 \text{ kva}$$

$$\text{Base line current} = \frac{5000}{\sqrt{3} \times 11} = 262 \text{ amperes}$$

With no-load field current, the generator voltage behind transient reactance is 6350 in volts and 1.00 in per unit.

Generator reactance based on rating $= 0.30$ per unit

Reactor reactance

$$\text{from [30] and [31]} = 0.05 \times \frac{5000}{\sqrt{3} \times 13.2 \times 350} \times \left(\frac{13.2}{11}\right)^2 = 0.045 \text{ per unit}$$

$$\text{Line resistance from [27]} = \frac{2 \times 0.27 \times 5000}{(11)^2 \times 1000} = 0.0223 \text{ per unit}$$

$$\text{Line reactance from [27]} = \frac{2 \times 0.61 \times 5000}{(11)^2 \times 1000} = 0.0504 \text{ per unit}$$

$$\text{Generator reactance from [28]} = \frac{30 \times (11)^2 \times 10}{5000} = 7.26 \text{ ohms}$$

$$\begin{array}{l}\text{Reactor reactance by}\\ \text{definition from rating}\end{array} = \frac{0.05 \times 13{,}200/\sqrt{3}}{350} = 1.09 \text{ ohms}$$

$$\text{Line resistance} = 2 \times 0.27 = 0.540 \text{ ohm}$$

$$\text{Line reactance} = 2 \times 0.61 = 1.22 \text{ ohms}$$

The positive-sequence impedance diagrams between generator neutral *N* and line terminals *T* in per unit and in ohms are given in Fig. 19(*a*) and (*b*), respectively. Although base kva in this one-line diagram is the kva per phase and base voltage is line-to-neutral voltage, it is customary to give base three-phase kva and base line-to-line voltage.

The transient short-circuit current in per unit of rated generator current with the voltage behind transient reactance as reference vector is

$$I = \frac{1.00 + j0}{0.0223 + j0.395} = \frac{1.00 \,\underline{/0^\circ}}{0.396 \,\underline{/86.7^\circ}} = 2.52 \,\overline{/86.7^\circ}$$

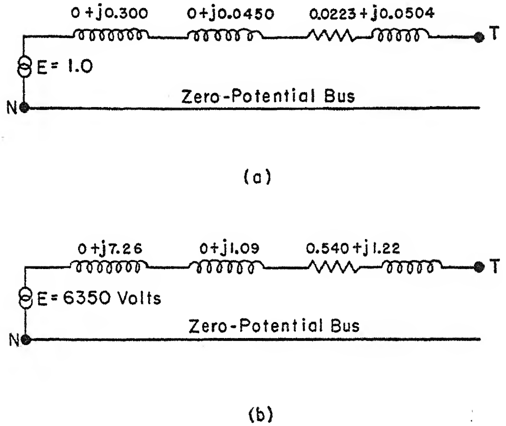


FIG. 19. Positive-sequence diagrams of system of Problem 3. (*a*) Impedances in per unit; base three-phase kva = 5000 kva, base line-to-line voltage = 11 kv. (*b*) Impedances in ohms.

Expressing current in amperes by multiplying its value in per unit by base current,

$$I = 262 \times 2.52 \,\overline{/86.7^\circ} = 661 \,\overline{/86.7^\circ} = 37 - j660 \text{ amperes}$$

Calculating current with volts and ohms,

$$I = \frac{6350}{0.540 + j9.57} = \frac{6350}{9.58 \,\underline{/86.7^\circ}} = 661 \,\overline{/86.7^\circ} = 37 - j660 \text{ amperes}$$

Problem 4. Given a three-phase, 60-cycle transmission system consisting of:

A three-phase, Y-connected synchronous generator, rated 30,000 kva, 13.8 kv, with subtransient reactance 15%.

Step-up transformer bank of three single-phase units connected Δ–Y, each unit rated 10,000 kva, 13.2–66.4/115 Y kv, leakage reactance 7.7%, resistance 0.5%.

Transmission line, 40 miles long with positive-sequence reactance of 0.8 ohm per mile, resistance 0.2 ohm per mile.

Step-down transformer bank of three single-phase units connected Y–Δ, each unit rated 8,333 kva, 110–3.98/6.9 Y kv, leakage reactance 8%, resistance 0.8%.

Connected load consisting of three-phase synchronous motor rated 25,000 kva, 6.6 kv, with subtransient reactance of 25% and a three-phase shunt impedance load of 10,000 kw at 70% lagging power factor.

(*a*) With 6.6 kv at the load and 10,000 kw at unity power factor delivered to the synchronous motor, find the current in the transmission line and the voltages at its terminals.

(*b*) If a three-phase fault occurs at the motor terminals, what is the initial symmetrical rms line current in the generator?

Solution. The choice of base kva and base voltage is entirely arbitrary, as is also the circuit in which base voltage is specified. They should be chosen so that calculations are reduced to a minimum. Assume a three-phase base kva of 25,000 kva and a line-to-line base voltage of 6.6 kv in the motor circuit. Then base line-to-line voltage in the transmission line circuit is

$$6.6 \times \frac{110}{6.9} = 105.2 \text{ kv}$$

Base line-to-line voltage in the generator circuit is

$$6.6 \times \frac{110}{6.9} \times \frac{13.2}{115} = 12.1 \text{ kv}$$

On a three-phase kva base of 25,000 kva and a base line-to-line voltage of 6.6 kv in the motor circuit, the per unit positive-sequence impedances are

$$Z_g = \text{impedance of generator} = j0.15 \times \frac{25{,}000}{30{,}000} \times \left(\frac{13.8}{12.1}\right)^2 = j0.163$$

$$Z_{t1} = \begin{matrix}\text{impedance of}\\ \text{step-up}\\ \text{transformer}\end{matrix} = (0.005 + j0.077) \times \frac{25{,}000}{30{,}000} \times \left(\frac{115}{105.2}\right)^2 = 0.0050 + j0.0768$$

$$Z_L = \begin{matrix}\text{impedance of}\\ \text{transmission}\\ \text{line}\end{matrix} = 40(0.2 + j0.8) \times \frac{25{,}000}{(105.2)^2 \times 10^3} = 0.018 + j0.072$$

$$Z_{t2} = \begin{matrix}\text{impedance of step-}\\ \text{down transformer}\end{matrix} = (0.008 + j0.08)\left(\frac{6.9}{6.6}\right)^2 = 0.009 + j0.088$$

$$Z_m = \text{impedance of motor} = j0.25$$

$$Z = \begin{matrix}\text{impedance}\\ \text{of shunt}\end{matrix} = \frac{V}{I} = \frac{V^2(\text{power factor})\underline{/\cos^{-1} \text{power factor}}}{\text{kw}} = \frac{1(0.7)}{0.4}\underline{/\cos^{-1} 0.7}$$

$$= 1.750\,\underline{/45.6^\circ} = 1.225 + j1.250$$

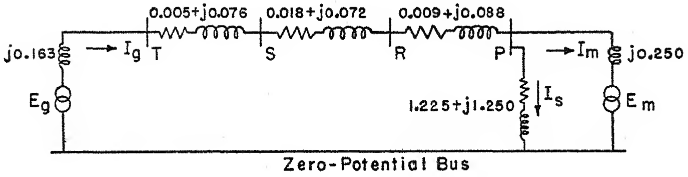


FIG. 20. Per unit positive-sequence diagram for Problem 4. Base three-phase kva = 25,000 kva. Base line-to-line voltage = 6.6 kv in motor circuit.

The positive-sequence impedance diagram with a three-phase base kva of 25,000 kva and 6.6 kv base line-to-line voltage in the motor circuit is shown in Fig. 20. The voltage at point P, the terminals of the motor, will be used as reference vector. V and I indicate voltage and current, respectively, with subscripts referring to the points indicated in Fig. 20. E is used instead of V for the voltages behind machine reactances.

$$V_p = 1 + j0$$

$$I_{(\text{motor})} = 0.400 + j0$$

$$E_m = 1 - (0.4)(j0.25) = 1.00 - j0.10 = 1.005\,\overline{/5.7^\circ}$$

$$I_{(\text{shunt})} = 0.400 - j0.408$$

$$I_p = I_{(\text{motor})} + I_{(\text{shunt})} = 0.800 - j0.408$$

$$V_r = V_p + (0.800 - j0.408)(0.009 + j0.088) = 1.043 + j0.067 = 1.045\,\underline{/3.6^\circ}$$

$$V_s = V_r + (0.800 - j0.408)(0.018 + j0.072) = 1.087 + j0.117 = 1.093\,\underline{/6.2^\circ}$$

$$V_t = V_t + (0.800 - j0.408)(0.005 + j0.076) = 1.122 + j0.176 = 1.138\,\underline{/8.9^\circ}$$

$$E_g = V_g + (0.800 - j0.408)(0 + j0.163) = 1.188 + j0.307 = 1.225\,\underline{/14.5^\circ}$$

(*a*) The current in *per unit* of base current in the generator and transmission line is the same as at the load, since there are no shunts between P and G.

$$I_t = I_g = (0.800 - j0.408) = 0.900\,\overline{/27.0^\circ} \text{ in per unit}$$

$$I_t = 0.900 \times \frac{25{,}000}{\sqrt{3} \times 105.2} = 123 \text{ amperes}$$

$$V_s = 1.093 \times 105.2 \text{ kv} = 115 \text{ kv line-to-line voltage}$$

$$V_r = 1.045 \times 105.2 \text{ kv} = 110 \text{ kv line-to-line voltage}$$

(*b*) With a three-phase fault at P,

$$I_g = \frac{E_g}{Z} = \frac{1.188 + j0.307}{0.032 + j0.399} = \frac{1.225\,\underline{/14.5^\circ}}{0.400\,\underline{/85.4^\circ}}$$

$$= 3.06\,\overline{/70.9^\circ} \text{ in per unit}$$

$$= 3.06 \times \frac{25{,}000}{\sqrt{3} \times 12.1} = 3660 \text{ amperes}$$

Problem 5. Solve Problem 4, using 100,000 kva as three-phase base kva and 115 kv as base line-to-line voltage in the transmission circuit.

Problem 6. Derive the equivalent Δ, without first deriving the equivalent Y, to replace a three-winding transformer with exciting current neglected in the positive-sequence one-line impedance diagram.

Problem 7. Derive the equivalent circuit to replace three circuits with self-impedances Z_{11}, Z_{22}, and Z_{33}, bussed at one end with unequal mutual impedances Z_{12}, Z_{13}, and Z_{23} between them. (If difficulties are encountered, see Fig. 5(*a*), Chapter VI or reference 5.)

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CHAPTER II

SYMMETRICAL COMPONENTS — BASIC EQUATIONS FOR THREE-PHASE SYSTEMS

The first paper indicating the possibilities of resolving an unbalanced system of currents into positive- and negative-sequence components of current was published in 1912 by L. G. Stokvis.[1] A second paper[2] dealing with third-harmonic voltages in alternators was presented under the sponsorship of André Blondel at a meeting of the French Academy of Science in 1914. It is interesting to note that positive- and negative-sequence currents as they are now known were a by-product of Stokvis's main endeavor, which was to find a means of determining the magnitude of the third-harmonic voltage produced by unbalanced line-to-line loads. A more detailed treatment of the resolution into positive- and negative-sequence currents of the unbalanced currents in three-phase ungrounded systems was given[3] in 1915.

In 1918, Dr. C. L. Fortescue presented before the American Institute of Electrical Engineers a paper[4] which introduced the concept of zero-sequence currents and voltages and provided a general method for the solution of unbalanced polyphase systems. In this paper Dr. Fortescue proved that "*a system of n vectors or quantities may be resolved when n is prime into n different symmetrical groups or systems, one of which consists of n equal vectors and the remaining* (*n* − 1) *systems consist of n equi-spaced vectors which with the first mentioned groups of equal vectors forms an equal number of symmetrical n-phase systems* . . ."

Many unbalanced problems hitherto solved only with difficulty are now solved as routine problems by the method of symmetrical components. Chief among these is the determination of currents and voltages of fundamental frequency in systems during unsymmetrical short circuits. This important application of the method of symmetrical components was made available by two papers[5, 6] in 1925 and another[7] in 1926. The application of the method to the ever present problem of unsymmetrical short circuits indicated to power engineers its value as a tool in the study of system performance, and it was but a short time before the method was widely used. The publication of a series[8] of articles and courses in engineering schools and commercial organizations further stimulated interest in symmetrical components. Many engineers have contributed in the application of the method to various problems encountered in power system operation.

The method of symmetrical components is a general one, applicable to any polyphase system. Because of the widespread use of three-phase systems and the greater familiarity which electrical engineers have with them, symmetrical component equations will be developed for them first.

THREE-PHASE SYSTEMS

In three-phase power systems, sinusoidal currents and voltages of fundamental frequency are represented for purposes of calculation by vectors revolving at an angular velocity, $\omega = 2\pi f$ radians per second. (See Chapter I.) The components which replace them must therefore be sinusoidal quantities of the same frequency, represented by vectors revolving at the same angular velocity. Since the angles between vectors revolving at the same rate are fixed, a set of three voltage or current vectors, V_a, V_b, and V_c, and the components which are to replace them, can be represented in the same vector diagram, with any current or voltage vector revolving at the same rate as reference vector.

Choice of Components. Any three co-planar vectors V_a, V_b, and V_c can be expressed in terms of three new vectors V_1, V_2, and V_3 by three simultaneous linear equations with constant coefficients. Thus,

$$V_a = c_{11}V_1 + c_{12}V_2 + c_{13}V_3 \qquad [1]$$

$$V_b = c_{21}V_1 + c_{22}V_2 + c_{23}V_3 \qquad [2]$$

$$V_c = c_{31}V_1 + c_{32}V_2 + c_{33}V_3 \qquad [3]$$

where the choice of coefficients is arbitrary, except for the restriction that the determinant* made up of the coefficients[9] must not be zero.

Each of the original vectors has now been replaced by three vectors, making a total of nine vectors. The nine vectors consist of three groups or systems of three vectors each. These systems are as follows:

1st system: $c_{11}V_1, \quad c_{21}V_1, \quad c_{31}V_1$

2nd system: $c_{12}V_2, \quad c_{22}V_2, \quad c_{32}V_2$

3rd system: $c_{13}V_3, \quad c_{23}V_3, \quad c_{33}V_3$

When values are assigned to the coefficients, the relations between the vectors of each system are fixed. It follows therefore that the three original vectors can be determined when the three new vectors are known. The purpose of expressing the three original vectors in terms of three new vectors is to simplify calculations, and thereby to gain a better understanding of a given problem and its related problems. With this thought in mind, two conditions should be satisfied in selecting systems of components to replace three-phase current and voltage vectors:

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* For a discussion of determinants, see Appendix A.

1. Calculations should be simplified by the use of the chosen systems of components. This is possible only if the impedances (or admittances) associated with the components of current (or voltage) can be obtained readily by calculation or test.
2. The systems of components chosen should have physical significance and be an aid in determining power system performance.

It will be seen that symmetrical components satisfy both these requirements.

Symmetrical Components. Although there are many ways of choosing the coefficients in [1]–[3] so that a system of three vectors can be replaced by three systems of vectors, consisting of three vectors each, there is only one way in which it can be replaced by three systems, each consisting of three *symmetrical* vectors. A system of three symmetrical vectors is one in which the three vectors are equal in magnitude and displaced from each other by equal angles. If V_a, V_b, and V_c are a set of voltage or current vectors, referring to phases a, b, and c, respectively, of a three-phase system, the three systems of three symmetrical vectors replacing V_a, V_b, V_c are the following:

1. A system of three vectors equal in magnitude displaced from each other by 120°, with the component of phase b lagging the component of phase a by 120°, and the component of phase c lagging the component of phase b by 120°, as in Fig. 1 (a).
2. A system of three vectors equal in magnitude displaced from each other by 120°, with the component of phase b lagging the component of phase a by 240°, and the component of phase c lagging the component of phase b by 240°, as in Fig. 1 (b).
3. A system of three vectors equal in magnitude displaced from each other by 0° or 360°, as in Fig. 1 (c).

In the first two systems of revolving vectors there is a sequence of phases; in the third system there is none, the three vectors being in phase. In the first system, Fig. 1 (a), taking counterclockwise direction as positive, the time order of arrival of the component vectors at a fixed axis of reference is *abc*. In the second system, Fig. 1 (b), the time order of arrival at the fixed axis of reference is *acb*. In both systems the vectors are displaced from each other by 120°, but the *phase order* of one system is the reverse of the other.

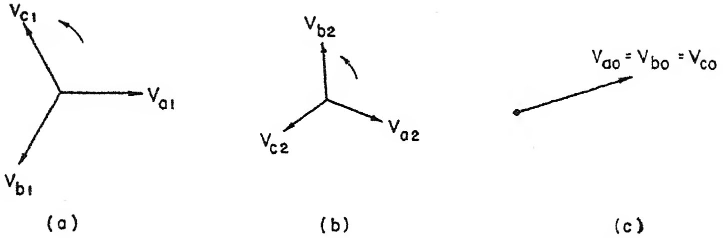

FIG. 1. (*a*) Positive-, (*b*) negative- and, (*c*) zero-sequence systems of vectors.

Meaning of Positive, Negative, and Zero Sequence. By a *positive-phase-sequence*, or *positive-sequence*, system of vectors is meant a system of three vectors equal in magnitude and 120° apart in phase, in which the time order of arrival of the phase vectors at a fixed axis of reference is the same as that of the generated voltages. The phases have been arbitrarily named so that the phase order of the generated voltages is *abc*. (See Chapter I.) In Fig. 1(*a*), the system of vectors is a positive-sequence system. The vectors in Fig. 1(*b*), which are also equal in magnitude and 120° apart in phase, arrive at a fixed axis in the phase order *acb* instead of *abc*. The system of vectors in Fig. 1(*b*) has consequently been called a *negative-sequence* system. If the vectors representing currents or voltages in the three phases are not separated in time phase, there will be no sequence of phases, and the currents or voltages, as the case may be, will vary simultaneously in each phase. A vector diagram for this system, which has been called *zero sequence*, is shown in Fig. 1(*c*). The vectors which form the zero-sequence system are equal in magnitude and in phase.

Notation. In Fig. 1 the double subscript notation has been employed, the first subscript indicating the phase, the second the sequence. Small letters *a*, *b*, and *c* are used to indicate the phases, while numerals 0, 1, and 2 are employed to designate respectively zero, positive, and negative sequence.

Symmetrical Component Equations. Three given vectors V_a, V_b, and V_c are expressed in terms of their symmetrical components by the equations

$$V_a = V_{a1} + V_{a2} + V_{a0} \tag{4}$$

$$V_b = V_{b1} + V_{b2} + V_{b0} \tag{5}$$

$$V_c = V_{c1} + V_{c2} + V_{c0} \tag{6}$$

where known relations exist between the vectors of each of the three sets of vectors, V_{a1}, V_{b1}, V_{c1}; V_{a2}, V_{b2}, V_{c2}; V_{a0}, V_{b0}, V_{c0}.

Phase *a* as Reference Phase. The choice of the phase to be regarded as the reference phase is arbitrary. With phase a as reference phase, and making use of the operator a (see Chapter I), the following relations exist:

For positive-sequence vectors,

$$V_{b1} = a^2 V_{a1}; \quad V_{c1} = a V_{a1}$$

For negative-sequence vectors,

$$V_{b2} = a V_{a2}; \quad V_{c2} = a^2 V_{a2}$$

For zero-sequence vectors,

$$V_{b0} = V_{a0}; \quad V_{c0} = V_{a0}$$

Substituting these relations in [4]–[6], there results

$$V_a = V_{a1} + V_{a2} + V_{a0} \qquad [7]$$

$$V_b = a^2 V_{a1} + a V_{a2} + V_{a0} \qquad [8]$$

$$V_c = a V_{a1} + a^2 V_{a2} + V_{a0} \qquad [9]$$

Comparing [7]–[9] with [1]–[3], the constant coefficients required to express a set of three vectors in terms of their symmetrical components are 1, a^2, a for the positive-sequence components, 1, a, a^2 for the negative-sequence components, and 1, 1, 1 for the zero-sequence components.

The three vectors V_a, V_b, and V_c are expressed in terms of their symmetrical components by [7]–[9]. When the symmetrical components are known, V_a, V_b, and V_c can be obtained either algebraically or graphically from [7]–[9]. A graphical combination of the sequence components is given in Fig. 2. The zero-, positive-, and negative-sequence components are shown in parts (*a*), (*b*), and (*c*), respectively. The combination of the components to produce the set of unbalanced vectors is shown in Fig. 2(*d*), and in Fig. 2(*e*) the three unbalanced vectors without their components.

Resolution of Unbalanced Vectors into Their Symmetrical Components. The resolution of a given unbalanced system of three vectors into symmetrical components can be carried out either graphically or analytically. As the most common procedure is the analytical one, this will be given first, and the graphical method, which follows readily from it, will be explained afterwards.

Analytical Method. The three symmetrical component vectors V_{a1}, V_{a2}, and V_{a0} can be expressed in terms of the original vectors V_a, V_b, and V_c by a solution of the simultaneous linear vector equations [7]–[9]. This will be done by two methods.

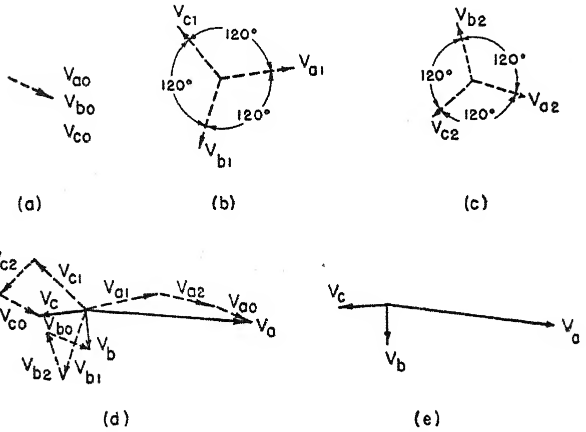
Vao
Vbo
Vco
Vc1
120°
Va1
120°
120°
Vb1
Vb2
120°
120°
Va2
120°
Vc2
(a)
(b)
(c)
Vc2
Vc1
Va1
Va2
Vc
Vao
Vco
Vbo
Va
Vb
Vb2
Vb1
(d)
Vc
Va
Vb
(e)

FIG. 2. Symmetrical component systems and their synthesis into three vectors.

Adding [7]–[9], remembering that $(1 + a + a^2) = 0$,

$$\begin{aligned} V_a + V_b + V_c &= (1 + a + a^2)V_{a1} + (1 + a + a^2)V_{a2} + 3V_{a0} \\ &= 3V_{a0} \end{aligned}$$

Therefore

$$V_{a0} = \tfrac{1}{3}(V_a + V_b + V_c) \tag{10}$$

Multiplying [7], [8], and [9] by 1, a, and a^2, respectively, and adding,

$$\begin{aligned} V_a + aV_b + a^2V_c = (1 + a^3 + a^3)V_{a1} &+ (1 + a^2 + a^4)V_{a2} \\ &+ (1 + a + a^2)V_{a0} = 3V_{a1} \end{aligned}$$

Therefore

$$V_{a1} = \tfrac{1}{3}(V_a + aV_b + a^2V_c) \tag{11}$$

Multiplying [7], [8], and [9] by 1, a^2, and a, respectively, and adding,

$$V_a + a^2V_b + aV_c = 3V_{a2}$$

Therefore

$$V_{a2} = \tfrac{1}{3}(V_a + a^2V_b + aV_c) \tag{12}$$

An alternate method of solving [7]–[9] is by means of determinants. The solution of vector equations, except in special cases such as the present one, is usually most easily performed by determinants. Since determinants will be used extensively in this book, a few simple rules for their use are given in Appendix A. Using [A–5], Appendix A, the following equations may be written directly from [7]–[9]:

$$V_{a1} = \frac{1}{D}[(a - a^2)V_a - (1 - a^2)V_b + (1 - a)V_c]$$

$$V_{a2} = \frac{1}{D}[-(a^2 - a)V_a + (1 - a)V_b - (1 - a^2)V_c]$$

$$V_{a0} = \frac{1}{D}[(a - a^2)V_a - (a^2 - a)V_b + (a - a^2)V_c]$$

where $D = (a - a^2) - (a^2 - a) + (a - a^2) = 3(a - a^2)$. Simplifying the above equations, they reduce to [10]–[12].

Graphical method. The graphical method of analyzing three unbalanced vectors follows directly from [10], [11], and [12]. Let the given vectors be those of Fig. 2(*e*). The zero-sequence component V_{a0} is found by adding the three given vectors and then dividing the vector closing the polygon by three. Figure 3 shows this process, a graphical method of solving [10].

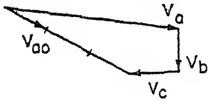

FIG. 3. Graphical method of securing zero-sequence component V_{a0}.

The positive-sequence component V_{a1} is determined by solving [11] graphically. To V_a is added aV_b, i.e., V_b rotated through 120°. To the sum of V_a and aV_b is added a^2V_c, i.e., V_c rotated through 240°. The positive-sequence component is secured by dividing the vector which forms the closing side of the resulting polygon by three. Figure 4 illustrates this process.

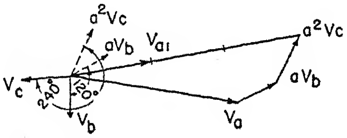

FIG. 4. Graphical method of securing positive-sequence component V_{a1}.

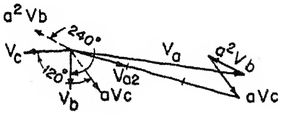

FIG. 5. Graphical method of securing negative-sequence component V_{a2}.

By solving [12] graphically, the negative-sequence component V_{a2} is obtained as pictured in Fig. 5.

Special Case — Zero-Sequence Components Absent. When the sum of the three vectors V_a, V_b, and V_c is zero, it follows from [10] that there are no zero-sequence components, and the three vectors will be expressed in terms of positive- and negative-sequence components only.

With $V_a + V_b + V_c = 0$, any one of the three vectors can be expressed in terms of the other two. Replacing V_c by $-V_a - V_b$ in [11] and [12],

$$V_{a1} = \tfrac{1}{3}[(1 - a^2)V_a + (a - a^2)V_b]$$

$$V_{a2} = \tfrac{1}{3}[(1 - a)V_a + (a^2 - a)V_b]$$

Replacing $(1 - a^2)$, $(a - a^2)$, $(1 - a)$, and $(a^2 - a)$ by their values in polar form given in Table I, Chapter I,

$$V_{a1} = \tfrac{1}{3}[\sqrt{3}V_a \underline{/30^\circ} + \sqrt{3}V_b \underline{/90^\circ}] = \frac{1}{\sqrt{3}}[V_a + V_b \underline{/60^\circ}]\, \underline{/30^\circ} \qquad [13]$$

$$V_{a2} = \tfrac{1}{3}[\sqrt{3}V_a /\overline{30^\circ} + \sqrt{3}V_b /\overline{90^\circ}] = \frac{1}{\sqrt{3}}[V_a + V_b /\overline{60^\circ}]\, /\overline{30^\circ} \qquad [14]$$

Using the complex form instead of the polar form for $(1 - a)$, etc.,

$$\begin{aligned} V_{a1} &= \tfrac{1}{3}\left[V_a\left(\frac{3}{2} + j\frac{\sqrt{3}}{2}\right) + V_b(j\sqrt{3})\right] \\ &= \frac{V_a}{2} + \frac{1}{\sqrt{3}}\left(\frac{V_a}{2} + V_b\right)\underline{/90^\circ} \qquad [15] \end{aligned}$$

$$\begin{aligned} V_{a2} &= \tfrac{1}{3}\left[V_a\left(\frac{3}{2} - j\frac{\sqrt{3}}{2}\right) + V_b(-j\sqrt{3})\right] \\ &= \frac{V_a}{2} - \frac{1}{\sqrt{3}}\left(\frac{V_a}{2} + V_b\right)\underline{/90^\circ} \qquad [16] \end{aligned}$$

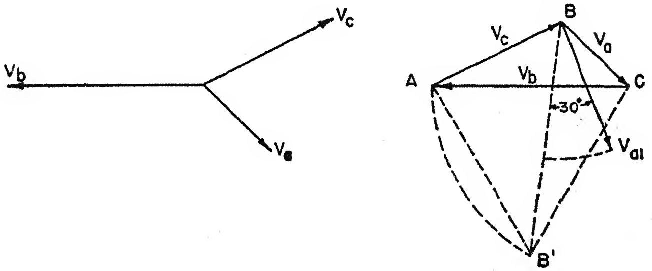

FIG. 6. Method of securing positive-sequence component V_{a1} when the vectors add to zero.

From [13], V_{a1} will be correctly represented both in magnitude and phase if (as in Fig. 6) V_b is rotated 60° in the positive direction and added to V_a, the resultant vector BB' being then divided by $\sqrt{3}$ and rotated positively 30° about B as a center.

Referring to Fig. 7 and equation [14], if V_b is rotated negatively 60° and added to V_a, the resultant vector BB' divided by $\sqrt{3}$, and the new vector thus obtained rotated negatively 30° about B as a center, V_{a2} is obtained.

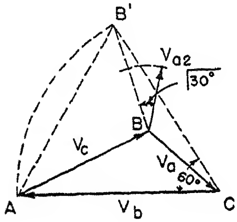

FIG. 7. Method of securing negative-sequence component V_{a2} when the vectors add to zero.

The positive- and negative-sequence components may also be obtained from the same diagram. In [15] and [16], V_{a1} and V_{a2} are given as the sum and difference of two vectors; one is $V_a/2$, the other $\frac{1}{\sqrt{3}}\left(\frac{V_a}{2} + V_b\right)\underline{/90°}$. To obtain a graphical construction, the vector $\left(\frac{V_a}{2} + V_b\right)$ must be turned through 90° and divided by $\sqrt{3}$ before it is added and subtracted from $V_a/2$ to give V_{a1} and V_{a2}, respectively.

A vector may be turned through 90° and its magnitude divided by $\sqrt{3}$ if a 30°, 60°, 90° triangle is constructed upon it with the 60° angle opposite it. In triangle ADE, Fig. 8, using scalar values, $DE = AD \tan 30° = AD/\sqrt{3}$; but vector $DE = 1/\sqrt{3}$ vector $AD\,\underline{/90°}$. Equations [15] and [16] are solved graphically in Fig. 9. AD is drawn from A to D, the midpoint of BC. Then vector $DA = (V_a/2) + V_b$. DE is drawn at D, making a positive angle of 90° with DA; AE is drawn from A making an angle of 30° with AD and closing the triangle ADE. Then

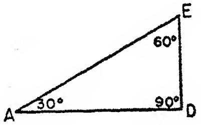

FIG. 8. Construction for dividing a vector AD by $\sqrt{3}$ and rotating it through 90°.

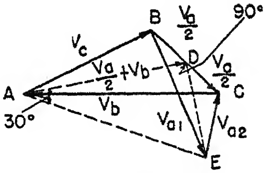

FIG. 9. Positive- and negative-sequence components obtained from the same diagram when the vectors add to zero.

$$\text{vector } DE = \frac{\text{vector } DA}{\sqrt{3}}\underline{/90°} = \frac{1}{\sqrt{3}}\left(\frac{V_a}{2} + V_b\right)\underline{/90°}$$

$$\text{vector } BE = \text{vector } BD + \text{vector } DE$$

$$= \frac{V_a}{2} + \frac{1}{\sqrt{3}}\left(\frac{V_a}{2} + V_b\right)\underline{/90°} = V_{a1} \qquad [17]$$

$$\text{vector } EC = \text{vector } DC - \text{vector } DE$$

$$= \frac{V_a}{2} - \frac{1}{\sqrt{3}}\left(\frac{V_a}{2} + V_b\right)\underline{/90^\circ} = V_{a2} \qquad [18]$$

With no zero-sequence components, V_{a1} and V_{a2} have been expressed in terms of V_a and V_b; they can also be expressed in terms of V_a and V_c, or V_b and V_c.

Voltage and Current Vectors. In the preceding developments V_a, V_b, and V_c have been used to represent a set of three vectors in a three-phase system rotating at the same rate. If voltage vectors are represented by V and current vectors by I, the basic symmetrical component equations for voltages and currents will be written:

Symmetrical Component Equations for Voltages

$$V_a = V_{a1} + V_{a2} + V_{a0} \qquad [7]$$

$$V_b = a^2 V_{a1} + a V_{a2} + V_{a0} \qquad [8]$$

$$V_c = a V_{a1} + a^2 V_{a2} + V_{a0} \qquad [9]$$

$$V_{a0} = \tfrac{1}{3}(V_a + V_b + V_c) \qquad [10]$$

$$V_{a1} = \tfrac{1}{3}(V_a + a V_b + a^2 V_c) \qquad [11]$$

$$V_{a2} = \tfrac{1}{3}(V_a + a^2 V_b + a V_c) \qquad [12]$$

Symmetrical Component Equations for Currents

$$I_a = I_{a1} + I_{a2} + I_{a0} \qquad [19]$$

$$I_b = a^2 I_{a1} + a I_{a2} + I_{a0} \qquad [20]$$

$$I_c = a I_{a1} + a^2 I_{a2} + I_{a0} \qquad [21]$$

$$I_{a0} = \tfrac{1}{3}(I_a + I_b + I_c) \qquad [22]$$

$$I_{a1} = \tfrac{1}{3}(I_a + a I_b + a^2 I_c) \qquad [23]$$

$$I_{a2} = \tfrac{1}{3}(I_a + a^2 I_b + a I_c) \qquad [24]$$

In the above equations, the voltage vectors V_a, V_b, V_c may be the voltages to ground of phases a, b, c at a specified point in the three-phase system; or they may be voltages to neutral, line-to-line voltages, generated voltages, induced voltages, in fact, any set of three voltage vectors revolving at the same rate which may exist in a three-phase system. Likewise, the three current vectors may be the three line currents, the three currents in a Δ-connected circuit, the currents flowing into a fault from the three conductors, etc. In the work which follows, V and I with appropriate subscripts will be used to indicate various voltage and current vectors, depending upon the type of problem to be solved.

Alternative Equations for Numerical Calculations. If a and a^2 in [8], [9], [11], [12], [20], [21], [23], and [24] are replaced by $\left(-\frac{1}{2}+j\frac{\sqrt{3}}{2}\right)$ and $\left(-\frac{1}{2}-j\frac{\sqrt{3}}{2}\right)$, respectively, the following equations more suitable for numerical calculations are obtained:

$$V_b = V_{a0} - \tfrac{1}{2}(V_{a1} + V_{a2}) - j\frac{\sqrt{3}}{2}(V_{a1} - V_{a2}) \qquad [25]$$

$$V_c = V_{a0} - \tfrac{1}{2}(V_{a1} + V_{a2}) + j\frac{\sqrt{3}}{2}(V_{a1} - V_{a2}) \qquad [26]$$

$$V_{a1} = \tfrac{1}{3}\left[V_a - \tfrac{1}{2}(V_b + V_c) + j\frac{\sqrt{3}}{2}(V_b - V_c)\right] \qquad [27]$$

$$V_{a2} = \tfrac{1}{3}\left[V_a - \tfrac{1}{2}(V_b + V_c) - j\frac{\sqrt{3}}{2}(V_b - V_c)\right] \qquad [28]$$

$$I_b = I_{a0} - \tfrac{1}{2}(I_{a1} + I_{a2}) - j\frac{\sqrt{3}}{2}(I_{a1} - I_{a2}) \qquad [29]$$

$$I_c = I_{a0} - \tfrac{1}{2}(I_{a1} + I_{a2}) + j\frac{\sqrt{3}}{2}(I_{a1} - I_{a2}) \qquad [30]$$

$$I_{a1} = \tfrac{1}{3}\left[I_a - \tfrac{1}{2}(I_b + I_c) + j\frac{\sqrt{3}}{2}(I_b - I_c)\right] \qquad [31]$$

$$I_{a2} = \tfrac{1}{3}\left[I_a - \tfrac{1}{2}(I_b + I_c) - j\frac{\sqrt{3}}{2}(I_b - I_c)\right] \qquad [32]$$

Instantaneous Power. The instantaneous power at any point in a three-phase system in the direction of current flow obtained by adding the instantaneous power in the three phases is

$$p = p_a + p_b + p_c = i_a v_a + i_b v_b + i_c v_c \qquad [33]$$

where p, v, and i indicate instantaneous power, voltage, and current, and subscripts a, b, and c refer to phases a, b, and c, respectively.

With sinusoidal currents and voltages of the same frequency in the three phases, instantaneous phase currents and voltages can be replaced by their instantaneous symmetrical components. Instantaneous power then becomes

$$p = (v_{a1} + v_{a2} + v_{a0})(i_{a1} + i_{a2} + i_{a0}) + (v_{b1} + v_{b2} + v_{b0})$$
$$(i_{b1} + i_{b2} + i_{b0}) + (v_{c1} + v_{c2} + v_{c0})(i_{c1} + i_{c2} + i_{c0}) \qquad [34]$$

The instantaneous zero-sequence components of voltages and currents are equal in the three phases and the sum of the instantaneous positive- or negative-sequence currents or voltages at any instant is zero. Making use of these relations, [34], when expanded, is

$$\begin{aligned} p = {} & (v_{a1}i_{a1} + v_{b1}i_{b1} + v_{c1}i_{c1}) + (v_{a2}i_{a2} + v_{b2}i_{b2} + v_{c2}i_{c2}) + 3v_{a0}i_{a0} \\ & + (v_{a1}i_{a2} + v_{b1}i_{b2} + v_{c1}i_{c2}) + (v_{a2}i_{a1} + v_{b2}i_{b1} + v_{c2}i_{c1}) \end{aligned} \tag{35}$$

Instantaneous symmetrical components of voltage and current are expressed in terms of their rms vector values (see Chapter I) by the following equations:

$$\begin{aligned}
v_{a1} &= \sqrt{2}|V_{a1}| \sin (\omega t + \alpha) \\
v_{b1} &= \sqrt{2}|V_{a1}| \sin (\omega t + \alpha - 120°) \\
v_{c1} &= \sqrt{2}|V_{a1}| \sin (\omega t + \alpha + 120°) \\
i_{a1} &= \sqrt{2}|I_{a1}| \sin (\omega t + \alpha + \theta_1) \\
i_{b1} &= \sqrt{2}|I_{a1}| \sin (\omega t + \alpha + \theta_1 - 120°) \\
i_{c1} &= \sqrt{2}|I_{a1}| \sin (\omega t + \alpha + \theta_1 + 120°) \\
v_{a2} &= \sqrt{2}|V_{a2}| \sin (\omega t + \beta) \\
v_{b2} &= \sqrt{2}|V_{a2}| \sin (\omega t + \beta + 120°) \\
v_{c2} &= \sqrt{2}|V_{a2}| \sin (\omega t + \beta - 120°) \\
i_{a2} &= \sqrt{2}|I_{a2}| \sin (\omega t + \beta + \theta_2) \\
i_{b2} &= \sqrt{2}|I_{a2}| \sin (\omega t + \beta + \theta_2 + 120°) \\
i_{c2} &= \sqrt{2}|I_{a2}| \sin (\omega t + \beta + \theta_2 - 120°) \\
v_{a0} &= v_{b0} = v_{c0} = \sqrt{2}|V_{a0}| \sin (\omega t + \gamma) \\
i_{a0} &= i_{b0} = i_{c0} = \sqrt{2}|I_{a0}| \sin (\omega t + \gamma + \theta_0)
\end{aligned} \tag{36}$$

where $|V_{a1}|$, $|V_{a2}|$, $|V_{a0}|$ and $|I_{a1}|$, $|I_{a2}|$, $|I_{a0}|$ are scalar values of the rms positive-, negative-, and zero-sequence vector voltages and currents, respectively; α, β, and γ are the phase angles by which the voltage vectors V_{a1}, V_{a2}, and V_{a0}, respectively, lead the reference vector (see Fig. 5, Chapter I). θ_1, θ_2, and θ_0 are the phase angles by which the positive-, negative-, and zero-sequence current vectors lead the corresponding voltage vectors, with θ positive for leading currents and negative for lagging currents.

Replacing instantaneous symmetrical components in [35] by their values in terms of rms vector quantities from [36], and making use of the trigonometric equations

$$\sin A \sin B = \tfrac{1}{2}[\cos (A - B) - \cos (A + B)]$$

and

$$\cos A + \cos (A + 120^\circ) + \cos (A - 120^\circ) = 0$$

the instantaneous three-phase power is

$$\begin{aligned} p = {} & 3|V_{a1}||I_{a1}| \cos \theta_1 + 3|V_{a2}||I_{a2}| \cos \theta_2 + 3|V_{a0}||I_{a0}| \cos \theta_0 \\ & - 3|V_{a0}||I_{a0}| \cos (2\omega t + 2\gamma + \theta_0) \\ & - 3|V_{a1}||I_{a2}| \cos (2\omega t + \alpha + \beta + \theta_2) \qquad [37] \\ & - 3|V_{a2}||I_{a1}| \cos (2\omega t + \alpha + \beta + \theta_1) \end{aligned}$$

The instantaneous three-phase power consists of three constant terms and three variable terms which pulsate sinusoidally at double impressed frequency. If currents and voltages are expressed in amperes and volts, respectively, power will be in watts; if currents and voltages are expressed in per unit of base phase current and phase voltage, respectively, power will be in per unit of *base power per phase*, base kilowatts being equal numerically to base kva. If instantaneous power is expressed in per unit of three-phase base kilowatts, numerically equal to three-phase base kva, all terms on the right-hand side of the equality sign in [37] are divided by three.

Average Three-Phase Power in Terms of Symmetrical Components. Since the average power of the double frequency terms in [37] is zero, the average three-phase power P is

$$P = 3|V_{a1}||I_{a1}| \cos \theta_1 + 3|V_{a2}||I_{a2}| \cos \theta_2 + 3|V_{a0}||I_{a0}| \cos \theta_0 \qquad [38]$$

where θ_1, θ_2, and θ_0 are the positive-, negative-, and zero-sequence power factor angles which are positive for leading current and negative for lagging currents. The average power given by [38] will be in watts if currents and voltages are in amperes and volts, respectively. It will be in per unit of *base power per phase* if currents and voltages are in per unit of base phase current and base phase voltage, respectively.

The average three-phase power in per unit of *three-phase base kilowatts* (numerically equal to three-phase base kva) is

$$P = |V_{a1}||I_{a1}| \cos \theta_1 + |V_{a2}||I_{a2}| \cos \theta_2 + |V_{a0}||I_{a0}| \cos \theta_0 \qquad [39]$$

Average Three-Phase Power in Terms of Phase Voltages and Currents.

$$P = |V_a||I_a| \cos \theta_a + |V_b||I_b| \cos \theta_b + |V_c||I_c| \cos \theta_c \qquad [40]$$

where θ_a, θ_b, and θ_c are the angles by which phase currents lead their respective phase voltages. Power will be in watts if currents and voltages are in amperes and volts, respectively; it will be in per unit of *base power per phase* if currents and voltages are in per unit of base phase current and base phase voltage, respectively.

The product of the magnitudes of two vectors multiplied by the cosine of the angle between them is called a dot product. Using the dot product notation, [40] may be rewritten

$$P = V_a \cdot I_a + V_b \cdot I_b + V_c \cdot I_c \tag{41}$$

When voltages and currents are expressed in rectangular coordinates, [41] may be restated in terms of the real and j-axis components. Let phase a be considered first and assume that, with respect to the reference vector,

$$V_a = A + jB$$

$$I_a = C + jD$$

It can be shown[10] that average power in phase $a = AC + BD$, where the signs of A, B, C, and D may be positive or negative. Similar relations will hold for the power in phases b and c. These relations are often convenient in making numerical calculations.

Alternate Method of Deriving Equation [38]. Expressing phase voltages and currents, in [41] in terms of their symmetrical components

$$\begin{aligned} P = {} & (V_{a0} + V_{a1} + V_{a2}) \cdot (I_{a0} + I_{a1} + I_{a2}) \\ & + (V_{a0} + a^2V_{a1} + aV_{a2}) \cdot (I_{a0} + a^2I_{a1} + aI_{a2}) \\ & + (V_{a0} + aV_{a1} + a^2V_{a2}) \cdot (I_{a0} + aI_{a1} + a^2I_{a2}) \end{aligned} \tag{42}$$

Each term of [42] may be expanded by the principle of vector analysis,[11] that the dot product of two polynomials is the sum of the dot products of the terms resulting from algebraic multiplication. For example,

$$\begin{aligned} (V_{a0} + a^2V_{a1} + aV_{a2}) \cdot (I_{a0} + a^2I_{a1} + aI_{a2}) = {} & V_{a0} \cdot I_{a0} \\ & + V_{a0} \cdot a^2I_{a1} + V_{a0} \cdot aI_{a2} + a^2V_{a1} \cdot I_{a0} \\ & + a^2V_{a1} \cdot a^2I_{a1} + a^2V_{a1} \cdot aI_{a2} + aV_{a2} \cdot I_{a0} \\ & + aV_{a2} \cdot a^2I_{a1} + aV_{a2} \cdot aI_{a2} \end{aligned}$$

In adding the individual terms of the expansion the fact that dot products are scalar quantities should be kept in mind. This means that the dot product of the vectors does not change when both vectors are rotated through the same angle. For example,

$$a^2V_{a1} \cdot a^2I_{a1} \equiv V_{a1} \cdot I_{a1}$$

$$aV_{a2} \cdot a^2I_{a1} \equiv V_{a2} \cdot aI_{a1}$$

The addition of the terms resulting from the expansion gives:

$$\begin{aligned} P = 3V_{a0}\cdot I_{a0} &+ V_{a0}\cdot I_{a1}(1 + a^2 + a) + V_{a0}\cdot I_{a2}(1 + a + a^2) \\ &+ V_{a1}\cdot I_{a0}(1 + a + a^2) + 3V_{a1}\cdot I_{a1} \\ &+ V_{a1}\cdot I_{a2}(1 + a + a^2) + V_{a2}\cdot I_{a0}(1 + a^2 + a) \\ &+ V_{a2}\cdot I_{a1}(1 + a^2 + a) + 3V_{a2}\cdot I_{a2} \end{aligned}$$

But $(a^2 + a + 1) = 0$, hence

$$P = 3(V_{a0}\cdot I_{a0} + V_{a1}\cdot I_{a1} + V_{a2}\cdot I_{a2})$$

or

$$P = 3[|V_{a1}|\,|I_{a1}| \cos\theta_1 + |V_{a2}|\,|I_{a2}| \cos\theta_2 + |V_{a0}|\,|I_{a0}| \cos\theta_0] \qquad [43]$$

Equation [43] is the same as [38].

Problem 1. The measured line-to-ground voltages in kilovolts on the high voltage side of a step-up transformer are 100, 32.4, and 37.4 on phases a, b, and c respectively. The voltage of phase a leads that of phase b by 94.8° and lags that of phase c by 176.5°. Determine analytically the symmetrical components of voltage and check results by recombining these components into the line-to-ground voltages.

Solution. With V_a, the voltage of phase a, as reference vector, the voltages may be written

$$V_a = 100\underline{/0^\circ} = 100 + j0$$

$$V_b = 32.4/\overline{94.8^\circ} = -2.7 - j32.3$$

$$V_c = 37.4\underline{/176.5^\circ} = -37.3 + j2.3$$

Substituting the above values of V_a, V_b, and V_c in [10], [11], [12],

$$\begin{aligned} V_{a0} = \tfrac{1}{3}(V_a + V_b + V_c) &= \tfrac{1}{3}[100 + j0 - 2.7 - j32.3 - 37.3 + j2.3] \\ &= \tfrac{1}{3}[60 - j30] = 20 - j10 \\ V_{a1} = \tfrac{1}{3}(V_a + aV_b + a^2V_c) &= \tfrac{1}{3}[100 + j0 + (-0.5 + j0.866)(-2.7 - j32.3) \\ &\qquad + (-0.5 - j0.866)(-37.5 + j2.3)] \\ &= \tfrac{1}{3}[150 + j45] = 50 + j15 \\ V_{a2} = \tfrac{1}{3}(V_a + a^2V_b + aV_c) &= \tfrac{1}{3}[100 + j0 + (-0.5 - j0.866)(-2.7 - j32.2) \\ &\qquad + (-0.5 + j0.866)(-37.3 + j2.3)] \\ &= \tfrac{1}{3}[90 - j15] = 30 - j5 \end{aligned}$$

V_{a1} and V_{a2} may also be determined as follows:

$$\begin{aligned} V_{a1} &= \tfrac{1}{3}(V_a + aV_b + a^2V_c) \\ &= \tfrac{1}{3}[100\underline{/0^\circ} + 1\underline{/120^\circ} \times 32.4/\overline{94.8^\circ} + 1/\overline{120^\circ} \times 37.4\underline{/176.5^\circ}] \\ &= \tfrac{1}{3}[100\underline{/0^\circ} + 32.4\underline{/25.2^\circ} + 37.4\underline{/56.5^\circ}] \\ &= \tfrac{1}{3}[100 + j0 + (29.3 + j13.8) + (20.7 + j31.2)] \\ &= \tfrac{1}{3}[150 + j45] = 50 + j15 \end{aligned}$$

$$\begin{aligned}
V_{a2} &= \tfrac{1}{3}(V_a + a^2 V_b + a V_c)\\
&= \tfrac{1}{3}[100\underline{/0^\circ} + 1\overline{/120^\circ} \times 32.4\overline{/94.8^\circ} + 1\underline{/120^\circ} \times 37.4\underline{/176.5^\circ}]\\
&= \tfrac{1}{3}[100\underline{/0^\circ} + 32.4\underline{/145.2^\circ} + 37.4\overline{/63.5^\circ}]\\
&= \tfrac{1}{3}[100 + j0 + (-26.7 + j18.5) + (16.7 - j33.5)]\\
&= \tfrac{1}{3}[90 - j15] = 30 - j5
\end{aligned}$$

Employing [27] and [28] instead of [11] and [12],

$$\begin{aligned}
V_{a1} &= \tfrac{1}{3}[V_a - 0.5(V_b + V_c) + j0.866(V_b - V_c)]\\
&= \tfrac{1}{3}[100 + j0 - 0.5(-40 - j30) + j0.866(34.6 - j34.6)]\\
&= \tfrac{1}{3}[150 + j45] = 50 + j15\\
V_{a2} &= \tfrac{1}{3} V_a - 0.5(V_b + V_c) - j0.866(V_b - V_c)]\\
&= \tfrac{1}{3}[100 + j0 - 0.5(-40 - j30) - j0.866(34.6 - j34.6)]\\
&= \tfrac{1}{3}[90 - j15] - 30 - j5
\end{aligned}$$

Knowing the components of voltage V_{a1}, V_{a2}, and V_{a0} of phase a, the component voltages of phases b and c are determined.

$$\begin{aligned}
V_{b0} &= V_{c0} = V_{a0} = 20 - j10\\
V_{b1} &= a^2 V_{a1} = (-0.5 - j0.866)(50 + j15) = -12 - j50.8\\
V_{c1} &= a V_{a1} = (-0.5 + j0.866)(50 + j15) = -38.0 + j35.8\\
V_{b2} &= a V_{a2} = (-0.5 + j0.866)(30 - j5) = -10.7 + j28.5\\
V_{c2} &= a^2 V_{a2} = (-0.5 - j0.866)(30 - j5) = -19.3 - j23.5
\end{aligned}$$

Check on Symmetrical Components

$$\begin{aligned}
V_a = V_{a0} + V_{a1} + V_{a2} &= (20 - j10) + (50 + j15) + (30 - j5)\\
&= 100 - j0 = 100\underline{/0^\circ}\\
V_b = V_{b0} + V_{b1} + V_{b2} &= (20 - j10) + (-12 - j50.8) + (-10.7 + j28.5)\\
&= -2.7 - j32.3 = 32.4\overline{/94.8^\circ}\\
V_c = V_{c0} + V_{c1} + V_{c2} &= (20 - j10) + (-38.0 + j35.8) + (-19.3 - j23.5)\\
&= -37.3 + j2.3 = 37.4\underline{/176.5^\circ}
\end{aligned}$$

Problem 2. (*a*) Solve Problem 1 graphically.

Problem 3. The line currents at the terminals of an ungrounded generator are 300, 400, and 500 amp in phases a, b, and c respectively. Determine the symmetrical components of current in magnitude and phase, taking the line current of phase a as reference vector. Draw a vector diagram of the line currents and their symmetrical components. Check results.

Problem 4 The line-to-ground voltage on one terminal of a three-phase generator is zero. Write an equation connecting the symmetrical components of voltage at this point.

Problem 5. The symmetrical components I_{a0}, I_{a1}, and I_{a2} of the line currents at a given point on a transmission system are equal in magnitude and in phase. Express the three line currents in terms of I_{a1} and reduce them to their simplest form,

Problem 6. The line-to-ground voltages of phases b and c at a specified point, P, in the transmission system are equal in magnitude and phase. What relation exists between the positive- and negative-sequence components of voltage in phase a?

Problem 7. The line currents in amperes in phases a, b, and c, respectively, are $500 + j150$, $100 - j600$, and $-300 + j600$ referred to the same reference vector. Find the symmetrical components of currents. Check these values.

Problem 8. Solve Problem 3 graphically by the method illustrated in Figs. 6–9.

Problem 9. Suggest three sets of components other than symmetrical components for the solution of unbalanced system problems.

Problem 10. Do equations [7], [25], and [26] suggest a possible answer to Problem 9?

Problem 11. With $V_a + V_b + V_c = 0$, express V_{a1} and V_{a2} in terms of V_b and V_c.

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CHAPTER III

SHORT CIRCUITS ON SYSTEMS WITH ONE POWER SOURCE

In Chapter II, currents, voltages, and power at any point in a three-phase system are expressed in terms of the symmetrical components of current and voltage at that point. In this chapter, the method of symmetrical components is used to determine fundamental-frequency currents and voltages during a short circuit at the terminals of a symmetrical three-phase unloaded synchronous generator or on an unloaded circuit in series with a generator and transformer bank. In Chapter IV, the application is extended to any symmetrical three-phase power system, normally balanced but rendered unbalanced by an unsymmetrical fault.

Sequence Impedances of Symmetrical Three-Phase Circuits. In dealing with sinusoidal currents and voltages of fundamental frequency, the impedances offered to positive-sequence currents in the three phases of a circuit will be defined as the ratios of the voltage drops in the three phases to the corresponding phase currents, with only positive-sequence currents flowing in the circuit. Likewise, the impedances offered to negative-sequence currents in the three phases will be defined as the ratios of the voltage drops in the three phases to the corresponding phase currents with only negative-sequence currents flowing in the circuit. Zero-sequence currents by definition are the same in magnitude and phase in the three phases; therefore their sum is not zero, and there must be a return path in which the sum $3I_{a0}$ can flow or the zero-sequence impedance of the circuit will be infinite. The impedance per phase met by zero-sequence currents in a symmetrical three-phase circuit with only zero-sequence currents flowing is the impedance (or equivalent impedance) offered to any one of the three equal currents flowing in the phases and their sum returning through the earth or some other conductor to which the neutral is connected. Figure 1 shows the path of zero-sequence currents and indi-

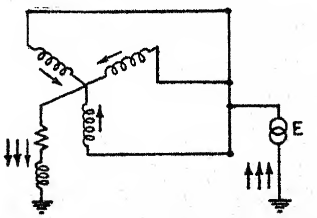


FIG. 1. Path of zero-sequence currents.

cates how zero-sequence impedance can be obtained by test. The zero-sequence impedance per phase of this circuit, which is Y-connected with neutral grounded through impedance, can be obtained by connecting the three terminals and applying a single-phase voltage to ground at the terminals. In a symmetrical circuit, the zero-sequence impedance per phase is three times the ratio of the applied voltage to the total current $3I_{a0}$.

Methods of calculating and measuring the sequence impedances of transformers, transmission lines, etc., are discussed in later chapters. It is shown in Chapter VIII that in a *symmetrical* static circuit without internal voltages the impedances to the currents of any sequence are the same in the three phases. It is further shown that currents of a given sequence produce voltage drops of like sequence only, and voltages of a given sequence produce currents of the same sequence only; consequently, there is no mutual coupling between the sequence systems. Since the impedance of a symmetrical static network to balanced three-phase currents is independent of the phase order, the positive- and negative-sequence impedances are equal; the zero-sequence impedance which includes the impedance of the return path of $3I_{a0}$, in the general case, is different from the positive- and negative-sequence impedances. In symmetrical rotating mchines, the impedances met by armature currents of a given sequence are equal in the three phases. As the impedance to currents of a given sequence depends upon their phase order relative to the direction of rotation of the rotor, positive-, negative-, and zero-sequence impedances are unequal in the general case.

For the present, let it be assumed that the sequence impedances of the symmetrical three-phase circuits discussed in this and the following chapter are known. Z_1, Z_2, and Z_0 will be used to indicate positive-, negative-, and zero-sequence impedances, respectively, of a symmetrical three-phase circuit or any portion of a symmetrical three-phase system. Let it also be assumed that there is no mutual coupling between the three sequence systems. The three sequence systems can then be considered separately, and phase currents and voltages determined by superposing their symmetrical components of current and voltage, respectively.

The division of currents and voltages into symmetrical components, with currents of each sequence meeting their own particular sequence impedances, is based on the principle of superposition (see Chapter I). Symmetrical components can be rigorously applied to electrical circuits only when the circuit impedances and admittances at the impressed frequency are constant. They can, however, be satisfactorily applied to many problems where the circuit parameters are not strictly constant, provided the resultant calculated phase voltages and currents are not of such magnitudes as to change materially the impedances and admittances *assumed* in making the calculations.

Generated or Internal Voltages. By generated or internal voltage is meant the voltage which would exist at the terminals of a machine on open circuit. It is the voltage behind the positive-sequence impedance, Z_1, of the generator (see Chapter I, Fig. 17), where Z_1 may be subtransient, transient, synchronous or equivalent steady-state impedance, depending upon the nature of the problem. The letter E will be used to designate generated voltages of machines, in order to distinguish them from voltages at the terminals or other points in the system, designated by the letter V. If the generated voltages in the three phases are E_a, E_b, and E_c, they can be resolved into their positive-, negative-, and zero-sequence components of generated voltages by [10]–[12], Chapter II, giving

$$\begin{aligned} E_{a0} &= \tfrac{1}{3}(E_a + E_b + E_c) \\ E_{a1} &= \tfrac{1}{3}(E_a + aE_b + a^2E_c) \\ E_{a2} &= \tfrac{1}{3}(E_a + a^2E_b + aE_c) \end{aligned} \qquad [1]$$

Since alternators are designed to generate balanced voltages, E_{a0} and E_{a2} in general will be zero. With balanced generated voltages,

$$\begin{aligned} E_a &= E_a \\ E_b &= a^2E_a = E_a\overline{\big/120^\circ} = \left(-\tfrac{1}{2} - j\frac{\sqrt{3}}{2}\right)E_a \\ E_c &= aE_a = E_a\underline{\big/120^\circ} = \left(-\tfrac{1}{2} + j\frac{\sqrt{3}}{2}\right)E_a \end{aligned} \qquad [2]$$

Voltage of the Neutral. In ungrounded circuits or circuits grounded through impedances, the neutrals of the circuits may or may not be at ground potential. Figure 2 shows a circuit with neutral N grounded through an impedance Z_n. If current flows in Z_n, the neutral N will not be at ground potential. That the voltage of the neutral referred to the ground is a zero-sequence voltage may be shown by applying equations [10]–[12], Chapter II. Since N is common to all three phases, the three voltages to ground

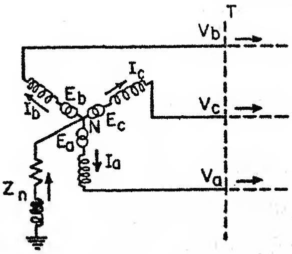


FIG. 2. Synchronous machine with neutral grounded through impedance.

as this point is approached become equal in the limit, thus: $V_a = V_b = V_c = V_n$. Hence

$$V_{a0} = \tfrac{1}{3}(V_n + V_n + V_n) = \frac{3V_n}{3} = V_n \qquad [3]$$

$$V_{a1} = \tfrac{1}{3}(V_n + aV_n + a^2V_n) = (1 + a + a^2)V_n = 0 \qquad [4]$$

$$V_{a2} = \tfrac{1}{3}(V_n + a^2V_n + aV_n) = (1 + a^2 + a)V_n = 0 \qquad [5]$$

From [4] and [5], it is evident that both the positive- and negative-sequence voltages from the neutral to ground are zero; and from [3], that the voltage of the neutral to ground is a zero-sequence voltage. For the positive- and negative-sequence systems, therefore, the expressions *voltage to neutral* and *voltage to ground* may be used interchangeably, but for the zero-sequence system it is important to distinguish between the two terms.

Reference for Voltages. The phase voltages at any point in a grounded system and their zero-sequence components of voltage will be referred to the ground at that point. The positive- and negative-sequence components of voltage are referred to neutral.

Convention for Direction of Current. For any given problem, the direction assumed as positive for current flow will be stated or indicated by arrows. (See Chapter I for a discussion of direction of current flow.)

Neutral or Ground Current. In Fig. 2, let I_n represent the neutral or ground current and I_a, I_b, I_c the three line currents, positive direction for neutral current being from the ground towards the neutral and for the line currents from the neutral towards the terminals. Applying Kirchhoff's law, that the sum of the currents flowing into a point is zero,

$$I_n = I_a + I_b + I_c$$

Replacing I_a, I_b, and I_c by their symmetrical components given by [19]–[21], Chapter II, and $(1 + a + a^2)$ by zero,

$$\begin{aligned} I_n &= (I_{a0} + I_{a1} + I_{a2}) + (I_{a0} + a^2I_{a1} + aI_{a2}) + (I_{a0} + aI_{a1} + a^2I_{a2}) \\ &= 3I_{a0} + I_{a1}(1 + a + a^2) + I_{a2}(1 + a^2 + a) = 3I_{a0} \qquad [6] \end{aligned}$$

From [6], the current from the ground flowing into the neutral of a circuit has no positive- or negative-sequence components, but is equal to three times the zero-sequence line current in the circuit. It follows therefore that, if Z_n = neutral grounding impedance,

$$V_n = -I_nZ_n = -3I_{a0}Z_n = -I_{a0}(3Z_n) \qquad [7]$$

Equation [7] states that *the zero sequence current flowing through three times the neutral grounding impedance produces the same voltage at the neutral as the neutral current flowing through the neutral grounding impedance.* This is a convenient relation which makes it possible to obtain the equivalent impedance offered to zero-sequence currents per phase by adding three times the impedance of the return path to the zero-sequence impedance of the symmetrical part of the system.

Symmetrical Components of Phase Voltages to Ground at the Terminals of a Symmetrical Machine in Terms of the Sequence Impedances and Symmetrical Components of Current. Figure 2 represents a symmetrical three-phase synchronous machine, with neutral grounded through an impedance Z_n. Positive direction of phase currents and their symmetrical components is taken from the neutral towards the terminals; that of the neutral current, from ground towards the neutral.

The positive-sequence component of the voltage of phase a at terminal T is equal to the generated positive-sequence component of voltage of phase a, E_{a1}, minus the voltage drop due to the positive-sequence current of phase a flowing through the positive-sequence impedance between the generator neutral and T. The negative-sequence component of the voltage of phase a at T is the generated negative-sequence component of voltage of phase a, E_{a2}, minus the voltage drop due to the negative-sequence current of phase a flowing through the negative-sequence impedance between N and T. The zero-sequence component of voltage at T is V_n (the voltage of the neutral) plus E_{a0} (the generated zero-sequence component of voltage of phase a) minus the voltage drop caused by zero-sequence current flowing through the zero-sequence impedance between N and T, where V_n is given by [7]. With balanced generated voltages, $E_{a2} = E_{a0} = 0$ and $E_{a1} = E_a$.

The equations for the symmetrical components of line-to-ground voltage of phase a at the terminals of a symmetrical three-phase synchronous machine with balanced generated voltages are

$$V_{a1} = E_a - I_{a1}Z_1 \qquad [8]$$

$$V_{a2} = -I_{a2}Z_2 \qquad [9]$$

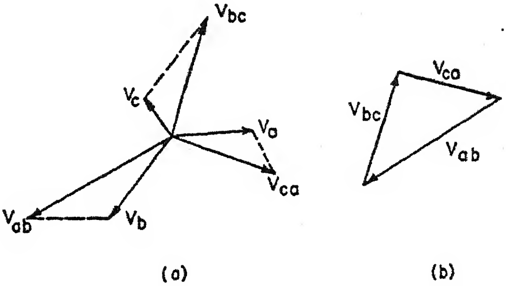
$$V_{a0} = V_n - I_{a0}Z_0' = -I_{a0}(3Z_n + Z_0') = -I_{a0}Z_0 \qquad [10]$$

where Z_1, Z_2, and Z_0' are the positive-, negative-, and zero-sequence impedances, respectively, between machine neutral and terminals. $Z_0 = 3Z_n + Z_0'$ is the zero-sequence impedance between ground and T. Z_0' in a Y-connected machine is a finite impedance. When the generator neutral is grounded, Z_n will also be finite; but when ungrounded, Z_n will be infinite.

When the symmetrical components of voltages are known, they can be substituted in [7]–[9] of Chapter II, and the line-to-ground voltages of the three phases obtained.

Note. V_{a0}, V_{a1}, and V_{a2} in [3]–[5] designate the symmetrical components of the neutral point N, while the same symbols in [8]–[10] refer to the terminal point T. Each point of a system will have its own symmetrical components of current and voltage, which, in general, will differ from those at other points.

Line-to-Line Voltages. The line-to-line voltages at any point T in a three-phase system will be the respective differences of the line-to-ground voltages at T, as shown in Fig. 3(*a*). When the differences of the equations [7], [8], and [9] of Chapter II for the line-to-ground voltages in terms of their symmetrical components are taken, it is found that the zero-sequence components disappear. The line-to-line voltages are



(a) (b)

FIG. 3. (*a*) Line-to-ground and line-to-line voltages. (*b*) Sum of line-to-line voltages.

$$\begin{aligned} V_{ab} &= V_b - V_a = (a^2 - 1)V_{a1} + (a - 1)V_{a2} \\ &= \sqrt{3}(V_{a1}/\overline{150^\circ} + V_{a2}/\underline{150^\circ}) \\ V_{bc} &= V_c - V_b = (a - a^2)V_{a1} + (a^2 - a)V_{a2} \\ &= \sqrt{3}(V_{a1}/\underline{90^\circ} + V_{a2}/\overline{90^\circ}) \\ V_{ca} &= V_a - V_c = (1 - a)V_{a1} + (1 - a^2)V_{a2} \\ &= \sqrt{3}(V_{a1}/\overline{30^\circ} + V_{a2}/\underline{30^\circ}) \end{aligned} \qquad [11]$$

Equations [11] express the line-to-line voltages in terms of the positive and negative sequence components of line-to-ground voltages. If V_{a1} and V_{a2} are expressed in volts, V_{ab}, V_{bc}, and V_{ca} will be in volts. If V_{a1} and V_{a2} are in per unit of base line-to-neutral voltage, V_{ab}, V_{bc}, and V_{ca} in [11] will also be in per unit of base line-to-neutral voltage.

The sum of the three line-to-line voltages shown in Fig. 3(*b*) is zero.

SHORT CIRCUITS

The simplest short-circuit problem, that presented by a fault at the terminals of a symmetrical Y-connected synchronous generator operated at no load with balanced generated voltages, will be treated first. Formulas will be developed for determining the three line currents and the line-to-ground and line-to-line voltages at the terminals of the generator for the following types of short circuits:

1. Three-phase fault.
2. Line-to-line fault.
3. Line-to-ground fault.
4. Double line-to-ground fault.

In applying the method of symmetrical components to the solution of problems involving a symmetrical three-phase system with but one point of dissymmetry, the procedure is to replace the phase currents and voltages at the point of dissymmetry by their symmetrical components of current and voltage. The phase currents and voltages may be any three currents and any three voltages associated with the three phases. These three currents and three voltages are the six unknowns to be determined. If the problem is to determine the line currents and the voltages to ground at the terminals of an unloaded generator with a fault at its terminals, the unknown currents and voltages are I_a, I_b, I_c, V_a, V_b, V_c, where I and V represent line currents and phase voltages to ground, respectively, and the subscripts refer to the phases. In any given problem, certain conditions are known about the unknown phase currents and voltages which can be expressed in equations. For example, if conductor a is faulted to ground at point P, the voltage to ground of phase a at P is zero, and the equation expressing this condition is $V_a = 0$. In a three-phase system, three equations can be written in terms of the three unknown phase currents and voltages at the point of dissymmetry. Three more equations are needed for a solution of the six unknowns. Equations [7]–[9] and [19]–[21] of Chapter II express the unknown phase voltages and currents, respectively, in terms of their symmetrical components, but they merely replace the six unknowns by six other unknowns: V_{a1}, V_{a2}, V_{a0}, I_{a1}, I_{a2}, I_{a0}. The advantage in using the six unknown components instead of the six unknown phase quantities is that the impedances met by the sequence currents can be determined either by calculation or test. This is not usually the case with phase impedances. However, if the phase impedances can also be readily obtained, there may be no advantage in introducing components; in fact, the use of phase quantities may give a simpler solution. If the generated voltages and the sequence impedances associated with each of the symmetrical components of current are known, three equations can be written expressing the components of voltage of each sequence at the fault in terms of the corresponding sequence current and the impedance associated with it. In the case of an unloaded symmetrical generator with a fault at its terminals, these three equations are given by [8]–[10]. Simultaneous solution of these three equations, together with the three equations determined by the boundary conditions, in which phase currents and voltages have been replaced by their symmetrical components of current and voltage, will give the six unknown symmetrical components of current and voltage. If more convenient, equations [10]–[12] and [22]–[24] of Chapter II can be used instead of [7]–[9] and [19]–[21]. The two sets of equations are not independent. One set expresses the phase quantities in terms of their symmetrical components; the other set, the symmetrical components in terms of the phase quantities. Either set may be used, or some equations from one set and some from the other, depending upon the problem. When the symmetrical components have been determined, the unknown phase currents and voltages are then calculated, using equations [7]–[9] and [19]–[21] of Chapter II.

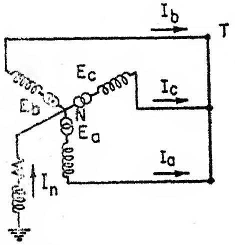


FIG. 4. Y-connected generator with three-phase short circuit at its terminals.

Three-Phase Fault. Since a symmetrical three-phase short circuit on a balanced system does not unbalance the system, it is evident that there will be only positive-sequence currents and voltages in the system. However, this case will be solved by the method of symmetrical components to illustrate the procedure with a balanced system.

(*a*) *Neutral Grounded.* Figure 4 shows a Y-connected generator with grounded neutral, and the three terminals short-circuited at T. The three equations expressing conditions at the fault are

$$V_b = V_a; \quad V_c = V_a; \quad I_a + I_b + I_c = 0$$

Equations [8]–[10] provide the other three simultaneous equations needed. Substituting V_a for V_b and V_c in [10]–[12] of Chapter II, and remembering that $1 + a + a^2 = 0$,

$$V_{a0} = V_a \qquad V_{a1} = 0 \qquad V_{a2} = 0$$

Substituting 0 for $I_a + I_b + I_c$ in [22] of Chapter II,

$$I_{a0} = 0$$

Substituting $V_{a2} = 0$ in [9],

$$I_{a2} = 0$$

Substituting $V_{a1} = 0$ in [8],

$$V_{a1} = 0 = E_a - I_{a1}Z_1$$

Therefore

$$I_{a1} = \frac{E_a}{Z_1} \tag{12}$$

Substituting $I_{a0} = 0$, $I_{a2} = 0$, and I_{a1} from [12] in [19]–[21] of Chapter II, the line currents with a three-phase fault are

$$I_a = I_{a1} = \frac{E_a}{Z_1} \tag{13}$$

$$I_b = a^2 I_{a1} = \frac{a^2 E_a}{Z_1} \tag{14}$$

$$I_c = aI_{a1} = \frac{aE_a}{Z_1} \tag{15}$$

Substituting $I_{a0} = 0$ in [10],

$$V_{a0} = -0 \cdot Z_0 \tag{16}$$

When Z_0 is finite, $V_{a0} = 0$. When Z_0 is infinite, V_{a0} in [16] is indeterminate. It may be evaluated, however, from the following considerations:

The point of fault T is common to the three phases; therefore the voltage from the fault to ground V_T, just like the voltage of the neutral to ground V_n, can have neither positive- nor negative-sequence components of voltage. (See [4] and [5].) The zero-sequence impedance between N and T is finite; therefore, with no zero-sequence current, there will be no zero-sequence voltage* drop, and T and N will be at the same potential above ground. In Fig. 4, T will be at ground potential, since the neutral point N is grounded.

(*b*) *Isolated Neutral.* With the neutral isolated but the fault point grounded, V_{a0} at T will be zero, and the voltage to ground at N, being the same as that at T, will be zero.

With the neutral and fault both isolated from ground, V_{a0} in [16] is mathematically indeterminate. However, in an ungrounded system in which capacitance and leakance are neglected, the term *voltage-to-ground* has no significance unless there should be an accidental ground on the system. In actual systems, leakance and capacitance to ground are always present, so that zero-sequence impedances to ground, although large, are not infinite. Since equal finite zero-sequence impedances to ground exist in the three phases of a symmetrical circuit with constant impedances, if there is no zero-sequence current, $V_{a0} = 0$ and $V_T = V_n = 0$.

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* It will be shown in Chapter VIII that in an *unsymmetrical* circuit there may be zero-sequence voltages present with no zero-sequence currents flowing.

The d-c voltage due to a static charge on an isolated system is to be distinguished from zero-sequence voltage, which is a single-phase voltage of fundamental frequency.

Problem 1. A generator whose subtransient, transient, and synchronous reactances are 12, 25, and 110%, respectively, and its resistance 0.6%, based on the rating of the generator, is operated at rated terminal voltage on open circuit. If a three-phase fault occurs, what are the initial symmetrical, transient, and sustained rms line currents expressed in per unit of rated current?

Solution. In a three-phase fault the rms values of I_b and I_c are equal to I_a in magnitude; therefore I_a only will be determined. Since $I_a = I_{a1}$, the first step will be to solve for I_{a1} by [13]. In this problem the resistance component of Z_1 will be neglected, since it is small relative to the reactance and its inclusion would not appreciably affect the magnitudes of the short-circuit currents.

The internal voltage of phase a, E_a, will be used as reference vector. Expressed in per unit, E_a will be 1.0 for all cases, since the generator is operated at rated terminal voltage on open circuit.

$$I_a = I_{a1} \text{ initial symmetrical rms} = \frac{1.0 + j0}{0 + j0.12} = -j8.33 \text{ per unit of rated current}$$

$$I_a = I_{a1} \text{ rms transient} = \frac{1 + j0}{0 + j0.25} = -j4.0 \text{ per unit of rated current}$$

$$I_a = I_{a1} \text{ rms sustained} = \frac{1 + j0}{0 + j1.10} = -j0.91 \text{ per unit of rated current}$$

Line-to-Line Fault. Figure 5 shows an unloaded Y-connected generator with grounded neutral and a line-to-line fault through zero fault impedance between terminals b and c. From the conditions of the problem it is evident that there can be no current in phase a, and that the currents in phases b and c are equal in magnitude and opposite in phase. The line-to-ground voltages at b and c must be the same, since there is no impedance between them.

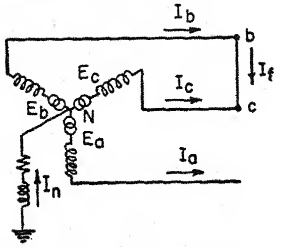


FIG. 5. Unloaded Y-connected generator with line-to-line short circuit at its terminals.

Expressing the conditions at the fault in equations,

$$I_a = 0 \qquad I_c = -I_b \qquad V_c = V_b$$

Substituting $I_a = 0$ and $I_c = -I_b$ in [22]–[24] of Chapter II,

$$I_{a0} = \tfrac{1}{3}(0 + I_b - I_b) = 0$$

$$I_{a1} = \tfrac{1}{3}(0 + aI_b - a^2 I_b) = \frac{a - a^2}{3} I_b = j\frac{I_b}{\sqrt{3}} \qquad [17]$$

$$I_{a2} = \tfrac{1}{3}(0 + a^2 I_b - aI_b) = \frac{a^2 - a}{3} I_b = -j\frac{I_b}{\sqrt{3}}$$

Therefore

$$I_{a2} = -I_{a1} \qquad [18]$$

Subtracting [9] from [8] in Chapter II,

$$V_b - V_c = (a^2 - a)V_{a1} - (a^2 - a)V_{a2}$$

Since $V_b = V_c$,

$$V_{a1} = V_{a2} \qquad [19]$$

Replacing V_{a1} and V_{a2} in [19] by their values from [8] and [9], and substituting $-I_{a1}$ for I_{a2},

$$V_{a1} = E_a - I_{a1}Z_1 = V_{a2} = -I_{a2}Z_2 = I_{a1}Z_2 \qquad [20]$$

Therefore

$$I_{a1} = \frac{E_a}{Z_1 + Z_2}; \quad \text{and} \quad I_{a2} = -I_{a1} = -\frac{E_a}{Z_1 + Z_2} \qquad [21]$$

The current in the fault is

$$I_f = I_b = -I_c = a^2 I_{a1} + aI_{a2} + I_{a0} = (a^2 - a)I_{a1} = \frac{-j\sqrt{3}E_a}{Z_1 + Z_2} \qquad [22]$$

Substituting [21] in [20],

$$V_{a1} = V_{a2} = I_{a1}Z_2 = E_a \frac{Z_2}{Z_1 + Z_2} \qquad [23]$$

If the neutral is solidly grounded or grounded through an impedance, Z_0 is finite. Substituting $I_{a0} = 0$ in [10],

$$V_{a0} = -0 \cdot Z_0 = 0 \qquad [24]$$

When the generator neutral is ungrounded, Z_0 is infinite, and V_{a0} is indeterminate. As previously stated, in a system without an intentional or accidental ground when capacitance and leakance are neglected, V_{a0} is indeterminate; but, since capacitance and leakance to ground are always present, V_{a0} will be zero if the system is symmetrical and $I_{a0} = 0$.

Equations [17] and [24] give $I_{a0} = 0$, $V_{a0} = 0$. I_{a1}, I_{a2}, V_{a1}, and V_{a2} in terms of E_a and Z_1 and Z_2 are given by [21] and [23]. When numerical values of the symmetrical components have been calculated, the line currents and the line-to-ground voltages may be obtained by substituting them in [7]–[9] and [19]–[21] of Chapter II.

Problem 2. Find the line currents and the line-to-ground and line-to-line voltages in per unit of rated current and voltage, respectively, when a line-to-line fault through zero fault impedance occurs at the terminals of an unloaded generator with solidly grounded neutral. The field current is such as to produce rated voltage on open circuit. The generator is rated 15,000 kva, 13,800 volts, and the positive-, negative-, and zero-sequence impedances in per unit are $0.007 + j0.35$, $0.05 + j0.45$, and $0.007 + j0.06$, respectively. Draw the vector diagram giving the symmetrical components of current and voltage, line currents, and line-to-ground voltages.

Solution. Assume the fault to occur between terminals b and c. E_a, the generated voltage of phase a, is $13{,}800/\sqrt{3}$ in volts and 1.0 in per unit of rated line-to-neutral voltage. E_a will be taken as reference vector, and calculations will be made in per unit based on the generator rating.

From [17] and [24],

$$I_{a0} = 0; \quad V_{a0} = 0$$

Substituting $E_a = 1$ and the given per unit values of Z_1 and Z_2 in [21] and [23],

$$I_{a1} = -I_{a2} = \frac{E_a}{Z_1 + Z_2} = \frac{1.0}{0.057 + j0.80} = 1.247/\overline{85.9^\circ} = 0.089 - j1.244$$

$$V_{a1} = V_{a2} = I_{a1}Z_2 = (0.089 - j1.244)(0.05 + j0.45) = 0.564 - j0.022$$
$$= 0.565/\overline{2.3^\circ}$$

Substituting the symmetrical components in [19]–[21] and [7]–[9], of Chapter II,

$$I_a = 0 + I_{a1} - I_{a1} = 0$$

$$I_b = -I_c = 0 + a^2 I_{a1} - aI_{a1} = (a^2 - a)I_{a1} = -j\sqrt{3}I_{a1} = -2.155 - j0.154$$
$$= 2.160/\overline{175.9^\circ}$$

$$V_a = 0 + V_{a1} + V_{a1} = 2V_{a1} = 1.128 - j0.044 = 1.129/\overline{2.3^\circ}$$

$$V_b = V_c = 0 + a^2 V_{a1} + aV_{a1} = -V_{a1} = -0.564 + j0.022 = 0.565/\underline{177.7^\circ}$$

From [11],

$$V_{ab} = V_b - V_a = -3V_{a1} = -1.692 + j0.066$$

$$V_{bc} = V_c - V_b = 0$$

$$V_{ca} = V_a - V_c = 3V_{a1} = 1.692 - j0.066$$

The symmetrical components of current and voltage, the line currents, and the line-to-ground voltages for this problem are given in the vector diagram of Fig. 6 in per unit of rated line current and line-to-neutral voltage with E_a as reference vector. If the vector length taken to represent unit voltage be assigned a scale value of $13{,}800/\sqrt{3} = 7970$ volts, and the length corresponding to unit current be considered as $15{,}000/(\sqrt{3}\ 13.8) = 628$ amp, the vector diagrams can be read in actual volts and amperes.

Note. It is suggested that the student close his book at this point and determine for himself the line currents and phase voltages to ground for the other types of short circuits. The following development, in that case, will serve as a check for his work.

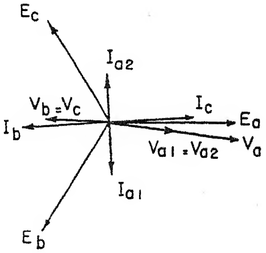


FIG. 6. Per unit vector diagram for Problem 2 with E_a the generated voltage of phase a as reference vector. Unit current scale one-third unit voltage scale.

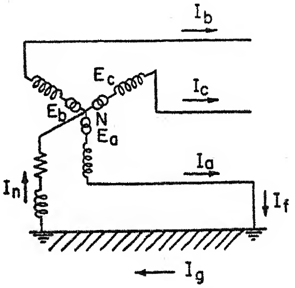


FIG. 7. Unloaded Y-connected generator with grounded neutral and line-to-ground fault on phase a.

Line-to-Ground Fault. (*a*) *Neutral of Generator Grounded.* Figure 7 shows a generator with grounded neutral, and a line-to-ground fault through zero fault impedance on phase a. The conditions at the fault are given by the following equations:

$$I_b = 0 \qquad I_c = 0 \qquad V_a = 0$$

Substituting $I_b = I_c = 0$ in [22]–[24] of Chapter II,

$$I_{a0} = \tfrac{1}{3}(I_a + 0 + 0) = \frac{I_a}{3}$$

$$I_{a1} = \tfrac{1}{3}(I_a + 0 + 0) = \frac{I_a}{3}$$

$$I_{a2} = \tfrac{1}{3}(I_a + 0 + 0) = \frac{I_a}{3}$$

Therefore

$$I_{a0} = I_{a1} = I_{a2} \tag{25}$$

Substituting $V_a = 0$ in [7] of Chapter II,

$$V_{a1} = -(V_{a0} + V_{a2}) \tag{26}$$

Substituting I_{a1} for I_{a0} and I_{a2} in [9] and [10],

$$V_{a2} = -I_{a2}Z_2 = -I_{a1}Z_2 \tag{27}$$

$$V_{a0} = -I_{a0}Z_0 = -I_{a1}Z_0 \tag{28}$$

Substituting [27] and [28] in [26],

$$V_{a1} = I_{a1}(Z_0 + Z_2) \tag{29}$$

Substituting [29] in [8] and solving for I_{a1},

$$I_{a1} = I_{a2} = I_{a0} = \frac{E_a}{Z_1 + Z_2 + Z_0} \tag{30}$$

The current in the fault I_f flows through ground and returns to the generator through the grounded neutral. From [6], neutral or ground current is a zero-sequence current equal to three times the zero-sequence line current flowing from the neutral. Therefore

$$I_f = I_g = I_n = 3I_{a0}$$

Also from Fig. 7, and equation [30],

$$I_f = I_g = I_n = I_a = I_{a1} + I_{a2} + I_{a0} = 3I_{a0} = \frac{3E_a}{Z_1 + Z_2 + Z_0} \tag{31}$$

The fault current and the symmetrical components of current and voltage can be determined when I_{a1} has been evaluated. The line currents and voltages to ground at the generator terminals may then be obtained by substituting the values of the components in [7]–[9] and [19]–[21] of Chapter II.

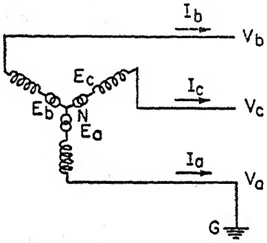


FIG. 8. Unloaded Y-connected generator with isolated neutral and line-to-ground fault on phase a.

(*b*) *Neutral of Generator Isolated.* Figure 8 shows a Y-connected generator with isolated neutral and phase a grounded through zero impedance. Since there is only one ground, neglecting capacitance and leakance, Z_0 will be infinite.

Substituting $Z_0 = \infty$ in [30],

$$I_{a1} = I_{a2} = I_{a0} = \frac{E}{Z_1 + Z_2 + \infty} = 0 \tag{32}$$

Substituting 0 for I_{a1} and I_{a2} in [8] and [9],

$$V_{a1} = E_a \qquad V_{a2} = 0$$

Substituting $V_{a1} = E_a$ and $V_{a2} = 0$ in [26],

$$V_{a0} = -E_a$$

The zero-sequence voltage at the fault is $-E_a$. Since there is no zero-sequence current, there can be no zero-sequence voltage drop and the zero-sequence voltage of the neutral will be the same as that at the fault. Therefore

$$V_n = -E_a$$

The line-to-ground voltages are

$$V_a = V_{a1} + V_{a2} + V_{a0} = E_a + 0 - E_a = 0$$

$$V_b = a^2V_{a1} + aV_{a2} + V_{a0} = a^2E_a + 0 - E_a = (a^2 - 1)E_a = E_{ab}$$

$$V_c = aV_{a1} + a^2V_{a2} + V_{a0} = aE_a + 0 - E_a = (a - 1)E_a = E_{ac}$$

The line-to-line voltages are

$$V_{ab} = V_b - V_a = (a^2 - 1)E_a = E_{ab}$$

$$V_{bc} = V_c - V_b = (a - a^2)E_a = E_{bc}$$

$$V_{ca} = V_a - V_c = (1 - a)E_a = E_{ca}$$

The line-to-line and line-to-neutral voltages will be unaffected by the fault, but the line-to-ground voltages will be unbalanced, one being zero and the other two equal to line-to-line voltage in magnitude. The voltage vector diagram is given in Fig. 9. E_a, E_b, and E_c are the generated voltages and also the terminal voltages to neutral or to ground before the fault, the neutral being at ground potential before the fault. V_a, V_b, and V_c are the phase voltages to ground and V_n the voltage of the neutral to ground after the fault. The fault causes the neutral to *shift* from its position of zero voltage before the fault to $-E_a$ after the fault.

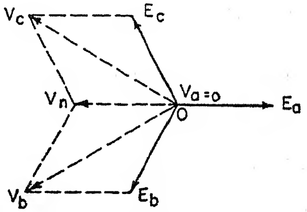


FIG. 9. Line-to-ground voltage vector diagram of isolated neutral generator with phase *a* grounded.

Problem 3. Find the per unit values of the symmetrical components of current and voltage, the line currents and the line-to-ground and line-to-line voltages for a fault of zero fault impedance on phase *a* of the generator described in Problem 2. Draw the vector diagram.

Solution. The given conditions are

$$E_a = 1.0; \quad Z_1 = 0.007 + j0.35; \quad Z_2 = 0.05 + j0.45; \quad Z_0 = 0.007 + j0.06$$

Substituting these values in [30], $I_{a1} = I_{a2} = I_{a0}$ is obtained. From [27]–[29], the three symmetrical components of voltage are obtained. Thus,

$$I_{a0} = I_{a2} = I_{a1} = \frac{1.0}{0.064 + j0.86} = 1.160\overline{\big/85.7^\circ} = 0.086 - j1.156$$

$$V_{a0} = -I_{a0}Z_0 = -(0.086 - j1.156)(0.007 + j0.06) = -0.070 + j0.003$$

$$V_{a2} = -I_{a2}Z_2 = -(0.086 - j1.156)(0.05 + j0.45) = -0.524 + j0.019$$

$$V_{a1} = I_{a1}(Z_0 + Z_2) = -(V_{a0} + V_{a2}) = 0.594 - j0.022$$

The line currents, the fault current, and the line-to-ground voltages obtained by substituting their symmetrical components in [19]–[21] and [7]–[9] of Chapter II are

$$I_a = I_f = I_g = I_n = 3I_{a1} = 0.258 - j3.468$$

$$I_b = I_c = 0$$

$$V_a = 0$$

$$\begin{aligned} V_b &= (-0.070 + j0.003) + a^2(0.594 - j0.022) + a(-0.524 + j0.019) \\ &= (-0.070 + j0.003) - \tfrac{1}{2}(0.070 - j0.003) - j\frac{\sqrt{3}}{2}(1.118 - j0.041) \\ &= -0.140 - j0.964 \\ V_c &= (-0.070 + j0.003) + a(0.594 - j0.022) + a^2(-0.524 + j0.019) \\ &= (-0.070 + j0.003) - \tfrac{1}{2}(0.070 - j0.003) + j\frac{\sqrt{3}}{2}(1.118 - j0.041) \\ &= -0.070 + j0.973 \end{aligned}$$

The line-to-line voltages, obtained from the line-to-ground voltages, are

$$V_{ab} = V_b - V_a = V_b = -0.140 - j0.964$$

$$V_{bc} = V_c - V_b = 0.070 + j1.937$$

$$V_{ca} = V_a - V_c = -V_c = 0.070 - j0.973$$

In the vector diagram of Fig. 10, the symmetrical components of current and voltage, the line currents, and the line-to-ground voltages are given in per unit of rated line current and line-to-neutral voltage, with $E_a = 1$ as reference vector.

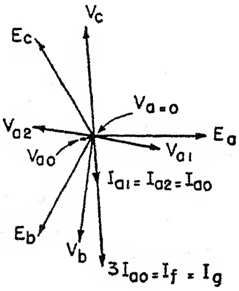


FIG. 10. Per unit vector diagram for Problem 3. Unit current scale one-third unit voltage scale.

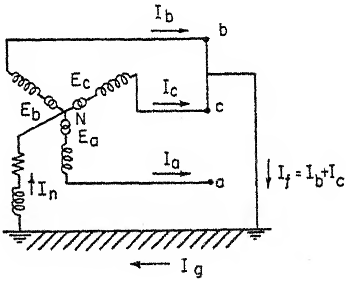


FIG. 11. Unloaded generator with grounded neutral and double line-to-ground fault at its terminals.

Double Line-to-Ground Fault. (*a*) *Neutral of Generator Grounded.* Figure 11 shows an unloaded generator with grounded neutral and terminals *b* and *c* grounded through zero impedances. The conditions at the fault are given by the following equations:

$$V_b = 0 \qquad V_c = 0 \qquad I_a = 0$$

Replacing V_b and V_c by zero in [10]–[12], Chapter II,

$$V_{a1} = V_{a2} = V_{a0} = \frac{V_a}{3} \qquad [33]$$

Replacing I_a by zero in [19] of Chapter II and solving for I_{a1},

$$I_{a1} = -(I_{a0} + I_{a2}) \qquad [34]$$

Substituting for V_{a2} and V_{a0} in [33] their values from [9] and [10],

$$V_{a1} = -I_{a2}Z_2 = -I_{a0}Z_0 \qquad [35]$$

Therefore

$$I_{a2} = -\frac{V_{a1}}{Z_2} \quad \text{and} \quad I_{a0} = -\frac{V_{a1}}{Z_0} \qquad [36]–[37]$$

Substituting [36] and [37] in [34],

$$I_{a1} = -(I_{a2} + I_{a0}) = V_{a1}\left(\frac{1}{Z_2} + \frac{1}{Z_0}\right) = V_{a1}\frac{Z_2 + Z_0}{Z_2Z_0} \qquad [38]$$

Therefore

$$V_{a1} = V_{a2} = V_{a0} = I_{a1}\frac{Z_2Z_0}{Z_2 + Z_0} \qquad [39]$$

From [36], [37], and [39], I_{a2} and I_{a0} can be expressed in terms of I_{a1}. Thus

$$I_{a2} = -I_{a1}\frac{Z_0}{Z_0 + Z_2} \qquad [40]$$

$$I_{a0} = -I_{a1}\frac{Z_2}{Z_0 + Z_2} \qquad [41]$$

Replacing V_{a1} in [8] by its value from [39] in terms of I_{a1} and solving for I_{a1},

$$I_{a1} = \frac{E_a}{Z_1 + \dfrac{Z_0Z_2}{Z_0 + Z_2}} = \frac{E_a(Z_0 + Z_2)}{Z_0Z_1 + Z_0Z_2 + Z_1Z_2} \qquad [42]$$

At this point it is interesting to note that the value of I_{a1} was determined in the line-to-line fault by $Z_1 + Z_2$, in the single line-to-ground fault by $Z_1 + Z_0 + Z_2$, but in the present case by the sum of Z_1 and the parallel value of Z_0 and Z_2.

After the numerical value of I_{a1} has been calculated from [42] the other symmetrical components can be obtained, and from the symmetrical components the line currents and the line-to-ground and line-to-line voltages determined.

(*b*) *Neutral of Generator Isolated.* Figure 12 shows an unloaded generator with isolated neutral and phases *b* and *c* grounded through zero impedance. From Fig. 12 the conditions of the problem are

$$I_a = 0 \qquad I_b + I_c = 0 \qquad V_b = 0 \qquad V_c = 0$$

Substituting 0 for I_a and $-I_b$ for I_c in [22]–[24] of Chapter II and 0 for V_b and V_c in [10]–[12] of Chapter II,

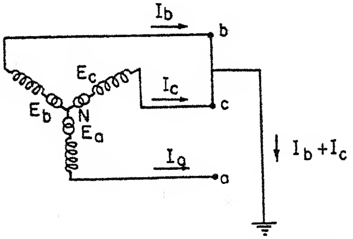


FIG. 12. Unloaded generator with isolated neutral and double line-to-ground fault at its terminals.

$$I_{a0} = 0 \tag{43}$$

$$I_{a2} = -I_{a1} \tag{44}$$

$$V_{a0} = V_{a1} = V_{a2} = \frac{V_a}{3} \tag{45}$$

Replacing I_{a2} by $-I_{a1}$ in [9],

$$V_{a2} = -I_{a2}Z_2 = I_{a1}Z_2 \tag{46}$$

Substituting [46] in [45],

$$V_{a0} = V_{a1} = V_{a2} = I_{a1}Z_2 \tag{47}$$

Substituting $V_{a1} = I_{a1}Z_2$ from [47] in [8] and solving for I_{a1},

$$I_{a1} = \frac{E_a}{Z_1 + Z_2} \tag{48}$$

Substituting [48] in [47],

$$V_{a1} = V_{a2} = V_{a0} = E_a \frac{Z_2}{Z_1 + Z_2} \tag{49}$$

Comparing the symmetrical components of currents for the double line-to-ground fault with isolated neutral with those for a line-to-line fault, it can be concluded that when a double line-to-ground fault occurs at the terminals of a generator with isolated neutral the line currents are the same whether the point of fault is isolated or grounded. Also, if the line-to-line voltages for the two cases are compared, they will be found to be identical. For the line-to-line fault with isolated neutral, the zero-sequence voltage is mathematically indeterminate, but practically it is zero. For the double line-to-ground fault with isolated neutral, it has been shown that

$$V_{a0} = V_{a1} = V_{a2} = I_{a1}Z_2 = \frac{E_aZ_2}{Z_1 + Z_2}$$

The grounding of the fault point causes the neutral to shift from its initial position to $\dfrac{E_aZ_2}{Z_1 + Z_2}$.

Problem 4. Using per unit quantities, find the line currents, the fault current, and the line-to-ground and line-to-line voltages for a double line-to-ground fault of zero fault impedance on terminals b and c of a solidly grounded generator rated 10,000 kva, 13.8 kv, with positive-, negative-, and zero-sequence reactances of 30, 40, and 5%, respectively. The generator was operated at normal terminal voltage on open circuit before the fault.

Solution. From the given conditions

$$E_a = 1.0; \quad Z_0 = 0 + j0.05; \quad Z_1 = 0 + j0.30; \quad Z_2 = 0 + j0.40$$

Selecting E_a as reference vector, the value of I_{a1} calculated from [42] is

$$I_{a1} = \frac{1 + j0}{0 + j0.30 + \dfrac{(0 + j0.05)(0 + j0.40)}{0 + j0.45}} = \frac{1}{j0.30 + \dfrac{-0.02}{j0.45}}$$

$$= \frac{1}{j0.30 + j0.044} = \frac{1}{j0.344} = -j2.90$$

From [39]–[41] the other symmetrical components are

$$V_{a0} = V_{a1} = V_{a2} = (-j2.90)\frac{(0 + j0.05)(0 + j0.40)}{0 + j0.45} = (-j2.90)(j0.044) = 0.13$$

$$I_{a2} = -(-j2.90)\frac{0 + j0.05}{0 + j0.45} = j0.32$$

$$I_{a0} = -(-j2.90)\frac{0 + j0.40}{0 + j0.45} = j2.58$$

Substituting the symmetrical components in [19]–[21] and [7]–[9] of Chapter II (to shorten numerical work use [29], [30], [25], and [26] of Chapter II),

$$I_a = j2.58 - j2.90 + j0.32 = 0$$
$$I_b = j2.58 - 0.5(-j2.58) - j0.866(-j3.22) = -2.79 + j3.87$$
$$I_c = j2.58 - 0.5(-j2.58) + j0.866(-j3.22) = 2.79 + j3.87$$
$$I_f = I_b + I_c = I_g = I_n = 3I_{a0} = 3 \times j2.58 = j7.74$$
$$V_a = 3(0.13) = 0.39$$
$$V_b = (1 + a^2 + a)V_{a1} = 0$$
$$V_c = (1 + a + a^2)V_{a1} = 0$$
$$V_{ab} = V_b - V_a = -0.39$$
$$V_{bc} = V_c - V_b = 0$$
$$V_{ca} = V_a - V_c = 0.39$$

Windings of the Generator Connected in Δ

If the windings of the generator are connected in Δ, there is no path for zero-sequence currents and the zero-sequence impedance of the generator is infinite. For calculating conditions at its terminals, the Δ-connected generator can be replaced by an equivalent Y-connected generator with isolated neutral. The positive- and negative-sequence impedances of the equivalent Y-connected generator, if expressed in ohms, will be one-third those of the given Δ-connected generator; if expressed in per unit, they will be the same as those of the given Δ-connected generator. (See Chapter I.) For faults at the terminals of a Δ-connected generator, the line currents and the line-to-ground and line-to-line voltages at the fault will be the same as for a Y-connected ungrounded generator having the same per unit positive- and negative-sequence impedances based on the rating of the generator.

Voltages of Δ-Connected Windings. Voltages across windings connected in Δ are line-to-line voltages. Line-to-line voltages are expressed in terms of the symmetrical components of the line-to-ground voltages of phase *a* by [11]. When the line-to-ground voltages have only positive-sequence components of voltage, V_{a2} in [11] is zero and the Δ voltages have only positive-sequence components of voltage; these equations then become

$$\begin{aligned} V_{ab1} &= V_{b1} - V_{a1} = (a^2 - 1)V_{a1} = \sqrt{3}V_{a1}/\overline{150^\circ} \\ V_{bc1} &= V_{c1} - V_{b1} = (a - a^2)V_{a1} = \sqrt{3}V_{a1}\underline{/90^\circ} = j\sqrt{3}V_{a1} \qquad [50] \\ V_{ca1} &= V_{a1} - V_{c1} = (1 - a)V_{a1} = \sqrt{3}V_{a1}/\overline{30^\circ} \end{aligned}$$

When the line-to-ground voltages have only negative-sequence components of voltages, V_{a1} in [11] is zero and the Δ voltages have only negative-sequence components of voltage; these equations then become

$$\begin{aligned} V_{ab2} &= V_{b2} - V_{a2} = (a - 1)V_{a2} = \sqrt{3}V_{a2}\underline{/150^\circ} \\ V_{bc2} &= V_{c2} - V_{b2} = (a^2 - a)V_{a2} = \sqrt{3}V_{a2}/\overline{90^\circ} = -j\sqrt{3}V_{a2} \qquad [51] \\ V_{ca2} &= V_{a2} - V_{c2} = (1 - a^2)V_{a2} = \sqrt{3}V_{a2}\underline{/30^\circ} \end{aligned}$$

Equations [50] and [51] express positive- and negative-sequence components of line-to-line voltages in terms of the positive- and negative-sequence components, respectively, of V_a. Figures 13(*a*) and (*b*) show these relations graphically.

As the line-to-line voltages themselves constitute a set of three-phase vectors, they can be resolved into positive- and negative-sequence components; but they will have no zero-sequence components since their sum is zero. (See Fig. 3(*b*).) The choice of the line-to-line voltage to be selected as reference is arbitrary. The simplest relations between the symmetrical components of I_a and the symmetrical components of the line-to-line voltages given by [50] and [51] are those for the components of V_{bc} which involve phase differences of ±90°. Those for V_{ab} or V_{ca} involve ±150° or ±30°, respectively. The components of

V_{bc} ($= V_c - V_b$) will therefore be selected as reference components, where the symmetrical components of V_{bc} in terms of the symmetrical components of V_a are

$$\begin{aligned} V_{bc0} &= 0 \\ V_{bc1} &= (a - a^2)V_{a1} = \sqrt{3}V_{a1}\underline{/90^\circ} = j\sqrt{3}V_{a1} \\ V_{bc2} &= (a^2 - a)V_{a2} = \sqrt{3}V_{a2}\overline{/90^\circ} = -j\sqrt{3}V_{a2} \end{aligned} \qquad [52]$$

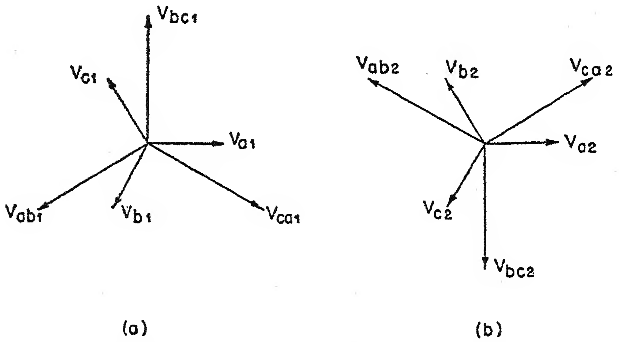


FIG. 13. Voltage vector diagrams of components of line-to-line and line-to-neutral voltages. (*a*) Positive sequence. (*b*) Negative sequence.

The symmetrical components of V_a in terms of the symmetrical components of V_{bc} from [52] are

$$\begin{aligned} &V_{a0} \text{ is indeterminate} \\ &V_{a1} = \frac{V_{bc1}}{\sqrt{3}}\overline{/90^\circ} = -j\frac{V_{bc1}}{\sqrt{3}} \\ &V_{a2} = \frac{V_{bc2}}{\sqrt{3}}\underline{/90^\circ} = j\frac{V_{bc2}}{\sqrt{3}} \end{aligned} \qquad [53]$$

The line-to-line voltages will be expressed in terms of the symmetrical components of V_{bc} by equations similar to those used to express V_a, V_b, and V_c in terms of the symmetrical components of V_a, except that there are no zero-sequence components in line-to-line voltages. See equations [7]–[9] of Chapter II. From Fig. 13(*a*), $V_{ca1} = a^2V_{bc1}$, $V_{ab1} = aV_{bc1}$; from Fig. 13(*b*), $V_{ca2} = aV_{bc2}$, $V_{ab2} = a^2V_{bc2}$. Therefore

$$\begin{aligned} V_{bc} &= V_{bc1} + V_{bc2} \\ V_{ca} &= V_{ca1} + V_{ca2} = a^2V_{bc1} + aV_{bc2} \\ V_{ab} &= V_{ab1} + V_{ab2} = aV_{bc1} + a^2V_{bc2} \end{aligned} \qquad [54]$$

Equations [52] and [53] express the relations between the symmetrical components of the line-to-line voltage V_{bc} and the line-to-ground voltage V_a when both are expressed in volts, or in per unit on a *common voltage base.*

If the components of line-to-line voltages are expressed on *base line-to-line voltage*, and the components of line-to-ground voltages on *base line-to-neutral voltage*, the $\sqrt{3}$ disappears, and [52] and [53] become

$$\begin{aligned} V_{bc0} &= 0 \\ V_{bc1} &= V_{a1}\underline{/90^\circ} = jV_{a1} \\ V_{bc2} &= V_{a2}\overline{/90^\circ} = -jV_{a2} \end{aligned} \qquad [55]$$

and

$$\begin{aligned} &V_{a0} \text{ is indeterminate} \\ V_{a1} &= V_{bc1}\overline{/90^\circ} = -jV_{bc1} \\ V_{a2} &= V_{bc2}\underline{/90^\circ} = jV_{bc2} \end{aligned} \qquad [56]$$

Currents in Δ-Connected Windings. With $V_{bc} = V_c - V_b$ selected as reference phase for line-to-line voltages, where V_{bc} represents the rise in voltage in going from b to c, currents in Δ-connected windings will be expressed in terms of the symmetrical components of the current in the winding bc. The choice, however, of positive direction for current flow in the winding bc is arbitrary. (See Chapter I for a discussion of voltage rise, voltage drop, and positive direction of current flow.) Consider, for example, a Y-connected generator supplying a pure resistance Y-connected load at its terminals as in Fig. 14(a). Arrows are used to indicate positive direction of current flow in phase a. The current I_a flowing out of the generator and into the load flows in the direction of voltage rise through the generator but in the direction of voltage drop through the load. By analogy, positive direction for current flow in the Δ windings will be taken in the direction of voltage rise (i.e., from b to c) when positive direction for line currents is away from the Δ terminals; and in the direction of voltage drop (i.e., from c to b) when positive direction for line currents is towards the Δ terminals. This convention is indicated in Figs. 14(b) and (c), where I with two subscripts represents current flowing from the point indicated by the first subscript towards the point indicated by the second.

It should be noted that arrows in Figs. 14(a), (b), and (c) are used to indicate the direction of current flow and should not be confused with the phase of the current. For example, in Fig. 14(a), with a unity power factor load delivered by the generator, I_a in the generator flowing from N to T is in phase with the terminal voltage indicated by the vector V_a. For this case, the directional arrow accompanying I_a in the generator might also represent its phase, but the directional arrow accompanying I_a flowing into the load from T to N' could not represent its phase (which is the same as that of I_a flowing from the generator) on the same current vector diagram. Likewise in the Δ circuits of Figs. 14(b) and (c), for a unity power factor load delivered by the Δ of Fig. 14(b) and received by the Δ of Fig. 14(c), I_{bc} in Fig. 14(b) is in phase with V_{bc} and I_{cb} in Fig. 14(c) is also in phase with V_{bc}. The arrow accompanying I_{bc} and representing its direction might also represent its phase, but the arrow accompanying I_{cb} could not repre-

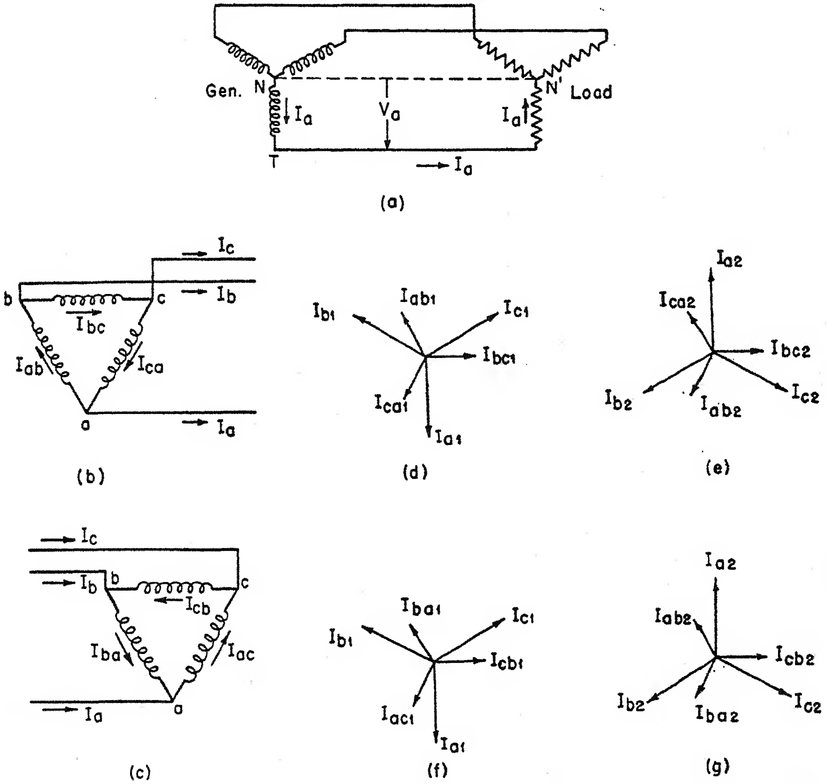

Gen.
N
Ia
Va
Ia
N'
Load
T
Ia
(a)
Ic
b
c
Ib
Ibc
Iab
Ica
a
Ia
(b)
Iab1
Ib1
Ic1
Ibc1
Ica1
Ia1
(d)
Ia2
Ica2
Ibc2
Ib2
Iab2
Ic2
(e)
Ic
Ib
b
c
Icb
Iba
Iac
Ia
a
(c)
Iba1
Ib1
Ic1
Icb1
Iac1
Ia1
(f)
Ia2
Iab2
Icb2
Ib2
Iba2
Ic2
(g)

FIG. 14. (a) Y-connected generator supplying Y-connected resistance load. (b) and (c) Currents in Δ-connected windings and line currents flowing *from* and *toward* the Δ, respectively. (d) and (e) Positive- and negative-sequence vector diagrams, respectively, of currents indicated by arrows in (b) with I_{bc1} and I_{bc2} as reference vectors. (f) and (g) Positive- and negative-sequence vector diagrams, respectively, of currents indicated by arrows in (c) with I_{cb1} and I_{cb2} as reference vectors.

sent its phase (which is the same as that of I_{bc}) on the same vector diagram.

In Fig. 14(*b*), applying Kirchhoff's law,

$$\begin{aligned} I_a &= I_{ca} - I_{ab} \\ I_b &= I_{ab} - I_{bc} \\ I_c &= I_{bc} - I_{ca} \end{aligned} \qquad [57]$$

The currents I_{bc}, I_{ca}, and I_{ab} are expressed in terms of the symmetrical components of I_{bc} by the following equations, analogous to those of [54] with the addition of zero-sequence currents which may be present in the Δ,

$$\begin{aligned} I_{bc} &= I_{bc1} + I_{bc2} + I_{bc0} \\ I_{ca} &= a^2 I_{bc1} + a I_{bc2} + I_{bc0} \\ I_{ab} &= a I_{bc1} + a^2 I_{bc2} + I_{bc0} \end{aligned} \qquad [58]$$

Replacing I_a, I_b, and I_c in [57] by their values in terms of the symmetrical components of I_a given by [19]–[21] of Chapter II, and I_{bc}, I_{ca}, and I_{ab} by their values in terms of the symmetrical components of I_{bc} given by [58], the following relations are obtained:

$$\begin{aligned} I_{a0} &= 0 \\ I_{a1} &= -j\sqrt{3}I_{bc1} = \sqrt{3}I_{bc1}\overline{/90^\circ} \\ I_{a2} &= j\sqrt{3}I_{bc2} = \sqrt{3}I_{bc2}\underline{/90^\circ} \end{aligned} \qquad [59a]$$

and

$$I_{bc0} \text{ is indeterminate}$$

$$\begin{aligned} I_{bc1} &= \frac{j}{\sqrt{3}} I_{a1} = \frac{I_{a1}}{\sqrt{3}}\underline{/90^\circ} \\ I_{bc2} &= \frac{-j}{\sqrt{3}} I_{a2} = \frac{I_{a2}}{\sqrt{3}}\overline{/90^\circ} \end{aligned} \qquad [59b]$$

Equations [59] give the relations between the symmetrical components of Δ current and line current flowing from the Δ when both are expressed in amperes or in per unit on a common current base.

The corresponding relations between the symmetrical components of the Δ current I_{cb} and the line current I_a flowing towards the Δ, obtained in a similar manner, are

$$\begin{aligned} I_{a0} &= 0 \\ I_{a1} &= -j\sqrt{3}I_{cb1} = \sqrt{3}I_{cb1}\overline{/90^\circ} \\ I_{a2} &= j\sqrt{3}I_{cb2} = \sqrt{3}I_{cb2}\underline{/90^\circ} \end{aligned} \qquad [60a]$$

and

$$I_{cb0} \text{ is indeterminate}$$

$$I_{cb1} = j\frac{I_{a1}}{\sqrt{3}} = \frac{I_{a1}}{\sqrt{3}}\underline{/90^\circ} \qquad [60b]$$

$$I_{cb2} = -j\frac{I_{a2}}{\sqrt{3}} = \frac{I_{a2}}{\sqrt{3}}\overline{\diagdown 90^\circ}$$

The positive- and negative-sequence current vector diagrams for Δ currents I_{bc}, I_{ca}, and I_{ab} and line currents flowing from the Δ are shown in Figs. 14(*d*) and (*e*); and for the Δ currents I_{cb}, I_{ba}, and I_{ac} and the line currents flowing towards the Δ in Figs. 14(*f*) and (*g*). In these diagrams, currents are in amperes or in per unit on a common current base.

When line and Δ currents are expressed in per unit, each on its own current base, the $\sqrt{3}$ in [59]–[60] disappears.

Summary of Relations between Components of Current and Voltage in the Δ and Components of Line Current and Voltage to Ground at the Δ Terminals. When currents flow from a Δ into the line, or from the line into a Δ, there can be no zero-sequence components of current in the line. Zero-sequence current may appear in the delta as a circulating current, but its magnitude cannot be determined from the line currents. With line and Δ currents expressed in per unit on their respective base currents, and line-to-ground voltages and Δ voltage in per unit of base line-to-neutral voltage and base line-to-line voltage, respectively, the following relations obtain:

$$\begin{aligned} &V_{bc1} \text{ leads } V_{a1} \text{ by } 90^\circ: \quad V_{bc1} = jV_{a1} \\ &V_{bc2} \text{ lags } V_{a2} \text{ by } 90^\circ: \quad V_{bc2} = -jV_{a2} \qquad [61] \\ &V_{bc0} = 0; \quad V_{a0} \text{ is indeterminate} \end{aligned}$$

When positive direction of current flow is away from the Δ,

$$\begin{aligned} &I_{bc1} \text{ leads } I_{a1} \text{ by } 90^\circ: \quad I_{bc1} = jI_{a1} \\ &I_{bc2} \text{ lags } I_{a2} \text{ by } 90^\circ: \quad I_{bc2} = -jI_{a2} \qquad [62a] \end{aligned}$$

When positive direction of current flow is towards the Δ,

$$\begin{aligned} &I_{cb1} \text{ leads } I_{a1} \text{ by } 90^\circ: \quad I_{cb1} = jI_{a1} \\ &I_{cb2} \text{ lags } I_{a2} \text{ by } 90^\circ: \quad I_{cb2} = -jI_{a2} \qquad [62b] \end{aligned}$$

It should be noted that $V_{cb1} = -V_{bc1}$; $V_{cb2} = -V_{bc2}$; $I_{cb1} = -I_{bc1}$; $I_{cb2} = -I_{bc2}$.

Problem 5. In Problem 2, assume the generator Δ-connected and determine the currents in the generator windings and the voltages across them from the positive- and negative-sequence components of line currents and line-to-ground voltages at the generator terminals.

Solution. From the solution of Problem 2 with E_a as reference vector,

$$I_{a1} = -I_{a2} = 0.089 - j1.244 \text{ per unit of rated line current}$$

$$V_{a1} = V_{a2} = 0.564 - j0.022 \text{ per unit of rated line-to-neutral voltage}$$

With $I_{a1} = -I_{a2}$, from [62a],

$$I_{bc1} = I_{bc2} = jI_{a1} = 1.244 + j0.089 \text{ per unit of rated } \Delta \text{ current}$$

With $V_{a1} = V_{a2}$, from [61],

$$V_{bc1} = -V_{bc2} = jV_{a1} = 0.022 + j0.564 = \text{per unit of rated line-to-line voltage}$$

Base voltage in the Δ windings is 13,800 volts. Base current is 15,000/(3 × 13.8) = 362 amp.

Substituting I_{bc1} and I_{bc2} in [58] with $I_{bc0} = 0$,

$$I_{bc} = 2I_{bc1} = 2.488 + j0.178 \text{ per unit of rated } \Delta \text{ current} = 900 + j65 \text{ amp.}$$

$$I_{ca} = -I_{bc1} = -1.244 - j0.089 \text{ per unit of rated } \Delta \text{ current} = -450 - j32 \text{ amp.}$$

$$I_{ab} = -I_{bc1} = -1.244 - j0.089 \text{ per unit of rated } \Delta \text{ current} = -450 - j32 \text{ amp.}$$

Substituting V_{bc1} and V_{bc2} in [54],

$$V_{bc} = 0$$

$$V_{ca} = (a^2 - a)V_{bc1} = -j\sqrt{3}V_{bc1} = 0.977 - j0.038 \text{ per unit of rated line-to-line voltage} = 13{,}500 - j525 \text{ volts}$$

$$V_{ab} = (a - a^2)V_{bc1} = +j\sqrt{3}V_{bc1} = -0.974 + j0.038 \text{ per unit of rated line-to-line voltage} = -13{,}500 + j525 \text{ volts}$$

Faults on Circuits in Series with an Unloaded Generator

Equations for the symmetrical components of currents and voltages at the fault, with a short circuit at the terminals of an unloaded generator, can be extended by analogy to unloaded circuits in series with the generator. When there is no intervening transformer bank, the positive-, negative-, and zero-sequence impedances of the series circuits are added directly to the positive-, negative-, and zero-sequence impedances, respectively, of the generator, giving total impedances Z_1, Z_2, and Z_0 viewed from the fault. These impedances replace the sequence impedances of the generator in the formulas developed for faults at the generator terminals.

When there is a transformer bank between the generator and the fault, and currents and voltages are to be determined on both sides of the bank, one-line impedance diagrams for each of the sequence systems are useful.

Sequence Networks. A symmetrical three-phase system rendered unbalanced by a fault can be resolved into positive-, negative-, and zero-sequence systems. In each of these systems the currents and voltages are symmetrical. Their three-phase impedance networks have equal impedances in the three phases and can therefore be replaced for purposes of calculation by an equivalent single-phase network and represented by a one-line impedance diagram. In the one-line sequence impedance diagrams of a three-phase network each piece of apparatus and each transmission circuit is replaced by its equivalent circuits. An equivalent circuit at its terminals should represent the apparatus or circuit which it replaces to the degree of precision required in the problem. See Chapter I for a general discussion of equivalent circuits and the development of equivalent circuits for use in positive-sequence one-line impedance diagrams.

In a symmetrical system, the one-line impedance diagram is the same, regardless of which phase is used as reference phase, but with the reference phase specified the currents and voltages in the single-phase network are those of the reference phase. The terms *positive-*, *negative-*, and *zero-sequence networks* will be used to indicate single-phase networks in which currents and voltages of positive-, negative-, and zero-sequence, respectively, are those of the reference phase. In each sequence network, the sequence voltages are referred to the zero-potential bus for the network.

Positive-Sequence Network. The positive-sequence one-line impedance diagram of a symmetrical three-phase system is discussed in Chapter I, and equivalent circuits are developed for use in this impedance diagram. All neutral points are at zero potential in the positive-sequence system and are therefore connected to a common point which is zero potential for the network. Generated voltages of the reference phase are represented in the positive-sequence network as applied between the zero-potential bus and the impedance of the generator. See Chapter I, Fig. 17.

Negative-Sequence Network. The negative-sequence, as well as the positive-sequence, voltage of all neutral points is zero. (See [4] and [5].) In the one-line impedance diagram of the negative-sequence system, all neutrals are therefore connected to a common point which is zero potential for the network. The negative-sequence network is similar to the positive, except that there are no generated negative-sequence voltages in a system balanced before the fault occurred. In a symmetrical three-phase static circuit, the positive- and negative-sequence impedances are equal; the equivalent circuits developed in Chapter I for use in the positive-sequence one-line impedance diagram can be used also in the negative-sequence network if the given circuit is a symmetrical three-phase static circuit, with or without mutual impedances between phases. As previously stated, the negative-sequence impedances of rotating machines are, in general, different from positive-sequence impedances.

Zero-Sequence Network. The zero-sequence system is not a three-phase system, since the phase currents and voltages are equal in magnitude and in phase. It is a single-phase system, with equal currents and equal voltages in the three phases at all points of the given three-phase system. The currents and voltages in the zero-sequence network are the same, regardless of which phase is selected as reference phase. The reference for zero-sequence voltages at any point in a grounded system is the ground at that particular point.

The reference for zero-sequence voltages is of a different character from that for positive- and negative-sequence voltages. In positive- or negative-sequence systems all neutral points are at the *same potential*. Neutral points in either system can therefore be connected to a common point. On the other hand, the ground is not necessarily at the same potential at all points. Therefore, in the one-line diagram of the zero-sequence system, the individual equivalent circuits for the various circuits and apparatus of the system must be so constructed that the zero-sequence voltages to ground at the terminals of these circuits are correctly given when referred to the zero-potential bus of the network. The zero-potential bus for the zero-sequence network does not represent the potential of the ground at any particular point, but is the reference ground for zero-sequence voltages at all points of the system. In Chapter XI a further discussion of the reference for zero-sequence voltages is given.

Equivalent circuits for the zero-sequence network depend upon the impedance met by the zero-sequence currents flowing in the three phases and their sum, $3I_{a0}$, flowing through neutral impedance and returning through the ground or a neutral conductor. If there is no complete path for zero-sequence currents in a circuit, the zero-sequence impedance is infinite. In drawing the zero-sequence impedance diagram, an infinite impedance is represented as an open circuit. Thus a Y-connected circuit with ungrounded neutral has infinite impedance to zero-sequence currents. As the equivalent circuit is used to determine voltages as well as currents, the opening is placed at the point where the impedance becomes infinite. For a Y-connected circuit of three equal self-impedances Z, with neutral N ungrounded and terminals at T, as in Fig. 15(a), a finite impedance Z is indicated in the equivalent circuit of Fig. 15(b) between N and T, and an open circuit between N and the zero-potential bus. Point T is to be connected into the zero-sequence diagram of the system. With finite impedance between T and N, no current will flow in this impedance; and, if there is no induced voltage, T and N are at the same zero-sequence potential.

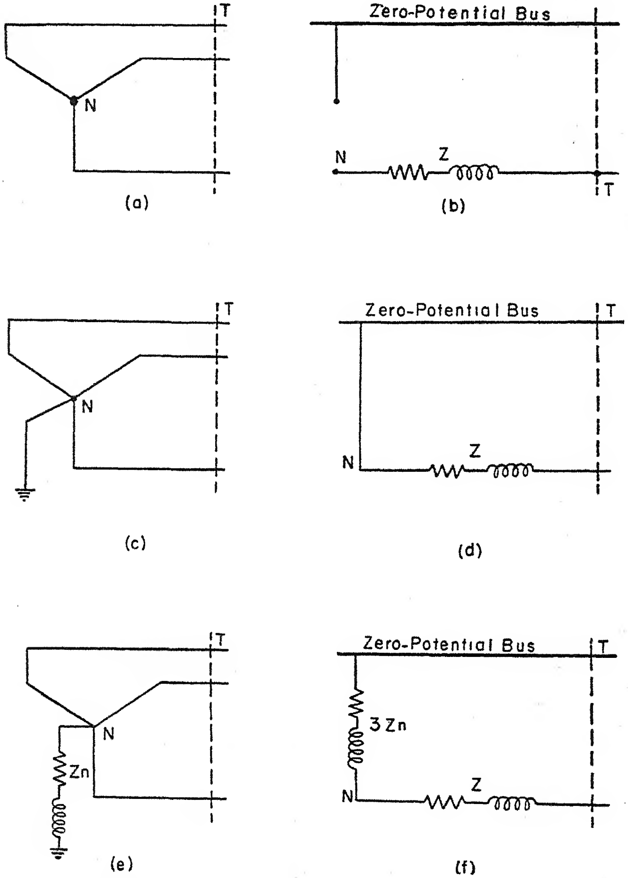

T
N
(a)
Zero-Potential Bus
N
Z
T
(b)
T
N
(c)
Zero-Potential Bus
T
N
Z
(d)
T
N
Zn
(e)
Zero-Potential Bus
T
3 Zn
N
Z
(f)

FIG. 15. Symmetrical Y-connected circuits and their zero-sequence equivalent circuits.

With the neutral solidly grounded, as in Fig. 15(*c*), N in the equivalent circuit of Fig. 15(*d*) is connected directly to the zero-potential bus. With the neutral grounded through an impedance Z_n, as in Fig. 15(*e*), N in the equivalent circuit shown in Fig. 15(*f*) is connected through $3Z_n$ to the zero-potential bus.

A Δ-connected circuit provides no path for zero-sequence currents flowing in the line. Viewed from its terminals, its zero-sequence impedance is infinite. The Δ voltages are line-to-line voltages and, since their sum is zero, they can have no zero-sequence components. The equivalent zero-sequence circuit for the symmetrical Δ-connected circuit of Fig. 16(*a*) with terminals at *T* is shown in Fig. 16(*b*). In this equivalent circuit there is an open circuit at *T* on the Δ side, indicating infinite impedance viewed from *T* towards the Δ; but beyond the opening there is a connection to the zero-potential bus, indicating that (1) there may be zero-sequence voltages at *T* but there can be none across the phases of the Δ and (2) there may be zero-sequence currents in the Δ but there can be none in the line at *T*.

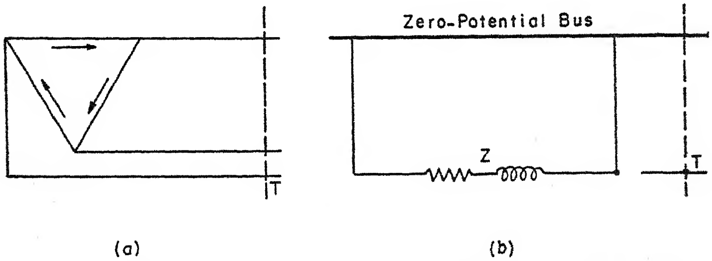


FIG. 16. Symmetrical Δ-connected circuit and its zero-sequence equivalent circuit.

Equivalent Circuits for Transformer Bank of Three Identical Single-Phase Units. Equivalent circuits for two-winding transformer banks to be used in the positive-sequence one-line impedance diagram are shown in Chapter I, Figs. 14(*e*) and (*f*): in Fig. 14(*e*) the magnetizing current is included; in Fig. 14(*f*) it is neglected. Figure 16(*b*) of Chapter I gives the equivalent circuit for a three-winding transformer bank with magnetizing current neglected. These equivalent circuits, developed for the positive-sequence one-line impedance diagram, are independent of the phase order of the balanced currents, and therefore are also the equivalent circuits to be used in the negative-sequence one-line impedance diagram.

The line-to-neutral positive- or negative-sequence leakage impedance of the two-winding bank (or equivalent line-to-neutral impedance when one or both sets of windings are Δ-connected), if expressed in per cent or per unit based on its rating with exciting currents neglected, is the same referred to either side of the bank (see Chapter I). If expressed in ohms, the impedances of the bank referred to the two sides will be proportional to the squares of the equivalent line-to-neutral turns. In problems involving the determination of currents and voltages on both sides of a transformer bank, solutions are simplified if currents, voltages, and impedances are expressed in per cent or per unit on a common kva base with base line-to-neutral voltages on the two sides of the bank directly proportional to the equivalent line-to-neutral turns.

Transformer banks made up of three identical single-phase units offer the same impedances to zero-sequence currents as to positive-sequence currents, provided there is a path for zero-sequence currents. Figure 17(*a*) shows the path of per unit zero-sequence currents in a Y–Δ transformer bank between P and Q, with the neutral of the Y solidly grounded and magnetizing current neglected. Figure 17(*b*) gives the zero-sequence equivalent circuit. Viewed from Q the impedance is infinite; viewed from P it is Z_t, the transformer leakage impedance. If the neutral of the Y is grounded through Z_n, the impedance Z_t in Fig. 17(*b*) is replaced by $(Z_t + 3Z_n)$.

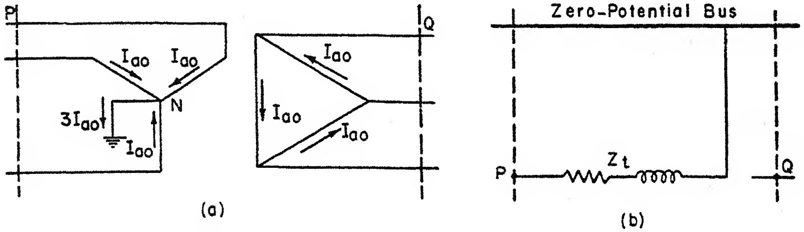


FIG. 17. (*a*) Path of zero-sequence currents in Y–Δ transformer bank with grounded neutral and negligible exciting current. (*b*) Zero-sequence equivalent circuit for (*a*) between P and Q, where Z_t is the per unit leakage impedance of one unit. If the neutral of the Y in (*a*) is grounded through Z_n, Z_t in (*b*) is replaced by $Z_t + 3Z_n$.

Figure 18 gives additional equivalent circuits for transformer banks made up of three identical single-phase units, with exciting currents neglected. The two-winding banks are connected Y–Y, Y–Δ, and Δ–Δ, with the neutrals of the Y's grounded or ungrounded. The three-winding banks are connected Y–Y–Y, Y–Y–Δ, Y–Δ–Δ, and Δ–Δ–Δ, with the Y's grounded or ungrounded. The equivalent series impedance of the two-winding transformer between primary and secondary terminals, indicated by P and S, respectively, is Z_{ps}. The impedances of the three-winding transformer between primary, secondary, and tertiary terminals, indicated by P, S, and T, respectively, taken two at a time with the other winding open, are Z_{ps}, Z_{pt}, and Z_{st}, the subscripts indicating the terminals between which the impedances are measured. (See Chapter I.) It is suggested that the student draw these equivalent circuits for himself before looking at Fig. 18.

Shift in Phase of Positive- and Negative-Sequence Line-to-Neutral Voltages and Line Currents in Passing through a Y-Δ or Δ-Y Transformer Bank of Three Identical Single-Phase Units. The two possible ways of connecting transformers Y–Δ are shown in Figs. 19(*a*) and (*b*). Capital letters refer to the Δ side of the bank and small letters to the Y side; phase *A* on the Δ side is the phase connected to the transformer phases corresponding to phases *b* and *c* on the Y side,

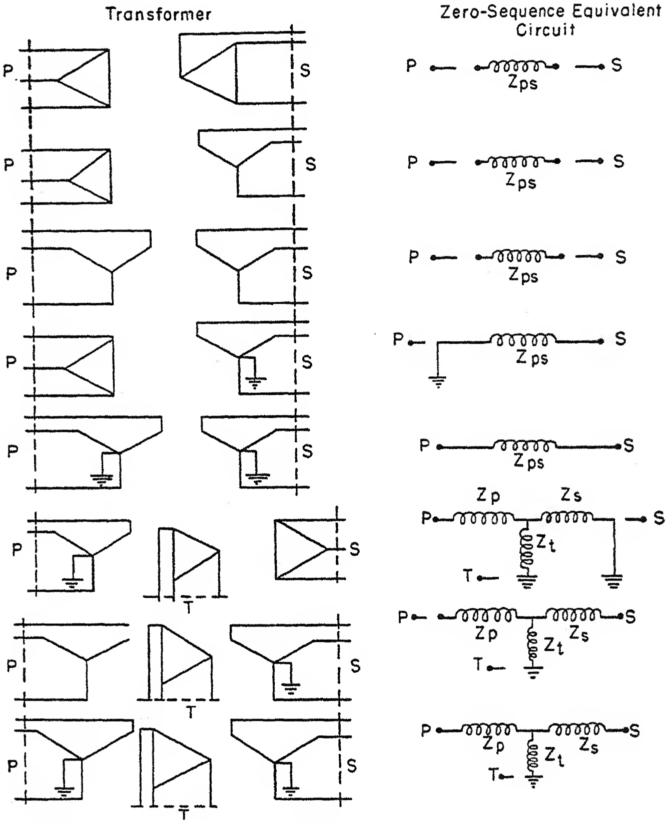


FIG. 18. Zero-sequence equivalent circuits for transformer banks of three identical single-phase units — exciting currents neglected.

$Z_p = \frac{1}{2}(Z_{ps} + Z_{pt} - Z_{st})$; $Z_s = \frac{1}{2}(Z_{ps} + Z_{st} - Z_{pt})$; $Z_t = \frac{1}{2}(Z_{pt} + Z_{st} - Z_{ps})$

as shown. Figures 19(*c*) and (*d*) give the positive-sequence voltage vector diagrams of connection diagrams (*a*) and (*b*), respectively, neglecting the voltage drop through the bank. Expressed in per unit, each on its own voltage base, V_{CB1} and V_{a1} in Fig. 19(*c*) are equal, as they are generated by the same per unit flux; and therefore

$$V_{A1} = jV_{CB1} = jV_{a1} \qquad [63]$$

In the positive-sequence per unit voltage vector diagram of Fig. 19(*d*) for the connection diagram of Fig. 19(*b*), $V_{BC1} = V_{a1}$; and

$$V_{A1} = -jV_{BC1} = -jV_{a1} \qquad [64]$$

For the connection diagram of Fig. 19(*a*), the line-to-neutral voltage V_{A1} on the Δ side of the bank leads by 90° the positive-sequence line-to-neutral voltage V_{a1} on the Y side of the bank; but for the connection

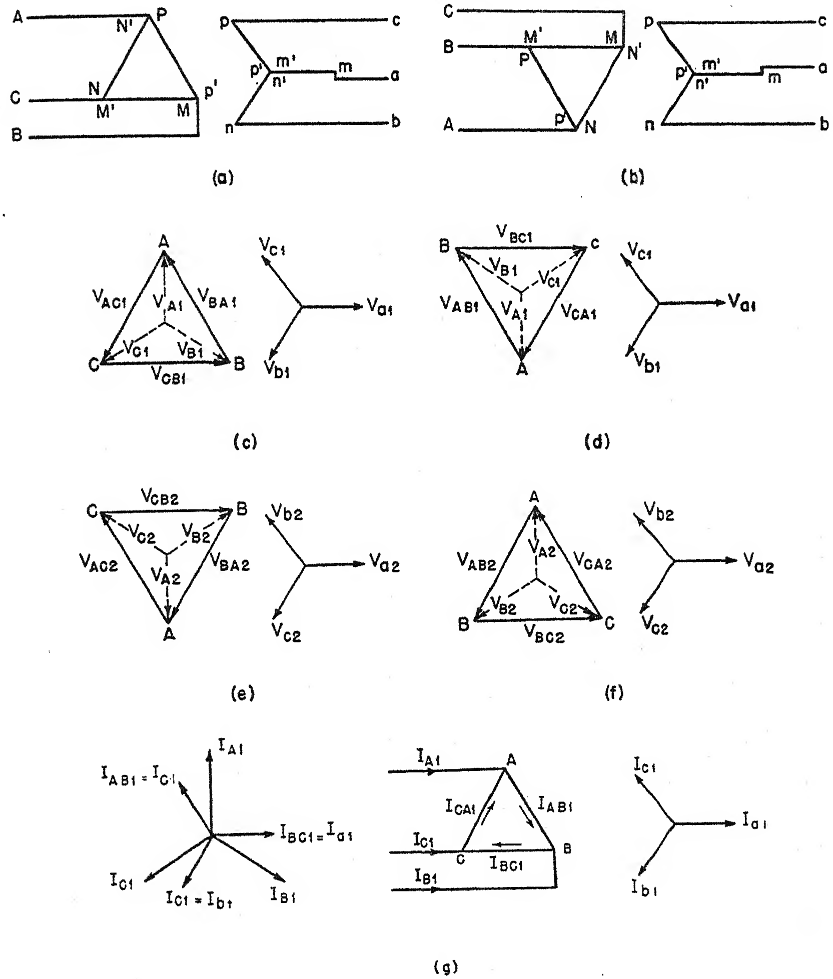

A
B
C
P
N'
N
M'
M
p'
p
m'
m
n'
n
a
b
c
(a)
(b)
V_BC1
V_AB1
V_CA1
V_AC1
V_BA1
V_CB1
V_A1
V_B1
V_C1
V_a1
V_b1
V_c1
(c)
(d)
V_CB2
V_AC2
V_BA2
V_AB2
V_CA2
V_BC2
V_A2
V_B2
V_C2
V_a2
V_b2
V_c2
(e)
(f)
I_A1
I_AB1 = I_c1
I_BC1 = I_a1
I_C1
I_C1 = I_b1
I_B1
I_CA1
I_AB1
I_BC1
I_a1
I_b1
I_c1
(g)

FIG. 19. (*a*) and (*b*) Possible connections of Δ-Y transformer banks. (*c*) and (*d*) Positive-sequence voltage vector diagrams for connection diagrams (*a*) and (*b*), respectively. (*e*) and (*f*) Negative-sequence voltage vector diagrams for connection diagrams (*a*) and (*b*), respectively. (*g*) Vector diagrams for positive-sequence currents in connection diagram (*a*).

of Fig. 19(*b*), V_{A1} lags V_{a1} by 90°. In Fig. 19, comparing (*a*) with (*c*) and (*b*) with (*d*), it may be seen that the positive-sequence voltage vector diagram in either case is the same as the transformer connection diagram; therefore, when the connection diagram is given, the phase relation between V_{A1} and V_{a1} can be determined by inspection.

Figure 19, parts (*e*) and (*f*), give the per unit negative-sequence voltage vector diagrams of connection diagrams (*a*) and (*b*), respectively, neglecting the voltage drop through the transformer bank. Making use of the relation $V_{A2} = jV_{BC2} = -jV_{CB2}$ of [56], in Fig. 19(*e*),

$$V_{A2} = -jV_{CB2} = -jV_{a2} \tag{65}$$

In Fig. 19(*f*),

$$V_{A2} = jV_{BC2} = jV_{a2} \tag{66}$$

Comparing [63] and [65], and [66] and [64], the positive-sequence line-to-neutral voltage is shifted 90° either forward or backward, depending upon the connection diagram, while the negative-sequence line-to-neutral voltage is shifted 90° in the direction opposite to the shift in phase of the positive-sequence voltage for the same connection diagram.

The shift in phase of positive- and negative-sequence line currents in passing through a Y–Δ or Δ–Y transformer bank, with transformer exciting currents neglected, must correspond exactly to the shift in phase of line-to-neutral voltages, with the voltage drop through the impedance of the bank neglected. If this were not the case, the kva and power on the two sides of the bank would not be equal with exciting current, resistance, and voltage drop through the bank neglected.

The shift in phase of positive-sequence line currents in passing through the Δ–Y connection given by Fig. 19(*a*) is shown in Fig. 19(*g*). With I_{a1} on the Y side as reference vector, and positive direction of current flow away from the neutral of the Y on the Y side and towards the Δ on the Δ side, the current in the Δ winding marked $M'M$ will flow from B to C, as indicated by the arrow. With exciting current neglected, I_{BC1} is in phase with I_{a1}. Arrows on the Δ side are used in the connection diagram to indicate direction of current flow but *do not indicate phase* referred to I_{a1}. The positive-sequence line current I_{A1} flowing towards the Δ in per unit from [62*b*] is

$$I_{A1} = -jI_{CB1} = jI_{BC1} = jI_{a1}$$

The current vector diagram is given to the left of the connection diagram in Fig. 19(*g*).

For the same connection diagram, Fig. 19(*a*), it can be shown that

$$I_{A2} = -jI_{a2}$$

For the connection diagram of Fig. 19(*b*),

$$I_{A1} = -jI_{a1}$$
$$I_{A2} = jI_{a2}$$

It can be concluded that positive-sequence line-to-neutral voltages and positive-sequence line currents are shifted 90° in phase in the same direction in passing through a Δ–Y or Y–Δ transformer bank; the negative-sequence line-to-neutral voltages and line currents are shifted 90° in the direction opposite to the positive-sequence shift in passing through the same bank. Whether the shift for positive-sequence currents and voltages is 90° forward or 90° backward will depend upon the manner of connecting the windings, the positive-sequence voltage vector diagram being the same as the transformer connection diagram. The direction of the 90° shift is unimportant in determining currents and voltages during faults on systems already in operation, unless the relative phases of currents and voltages on the two sides of the bank are to be compared; it is very important to operating engineers when two circuits, each with a Δ–Y bank, are connected in parallel.

In solving short-circuit problems in which the connection diagram of the Δ–Y bank, is not given, it will be assumed that positive-sequence line currents and voltages to neutral are shifted 90°, either forward or backward, and that negative-sequence line currents and voltages to neutral are shifted 90° in the direction opposite to the positive-sequence shift. The only difference in rotating positive-sequence components forward 90° and negative-sequence components backward 90°, instead of rotations in the opposite directions for each, will be a difference in sign for all phase currents and voltages whose components have been rotated. This is illustrated in Problem 6.

Figure 20(*a*) shows a one-line diagram of a system consisting of generator, transformer bank, and transmission line in series, with the distant end of the line open. The connection diagram is given by Fig. 20(*b*). The transformer bank is connected Δ–Y with the Y on the line side grounded through impedance Z_n. The generator is Δ-connected. The positive- and negative-sequence diagrams are given by Figs. 20(*c*) and (*d*), respectively. In these diagrams, transformer exciting current is neglected and the transformer bank is replaced by its equivalent Y–Y bank. Line capacitance is neglected, and the line replaced by its series sequence impedances. Points P, Q, and F in all diagrams correspond to points P, Q, and F in the one-line system diagram. The generator, transformer, and line impedances are connected in series in the positive- and negative-sequence diagrams. In the zero-sequence impedance diagram, Fig. 20(*e*), the point P between two Δ-connected windings is represented as a point of open circuit, since its zero-sequence impedance is infinite in both directions. Should zero-sequence voltages be present in the generator, they would cause circulating currents in the Δ-connected windings, but no zero-sequence currents could flow from the generator into the line and no zero-sequence voltages could exist across the Δ-connected windings. The

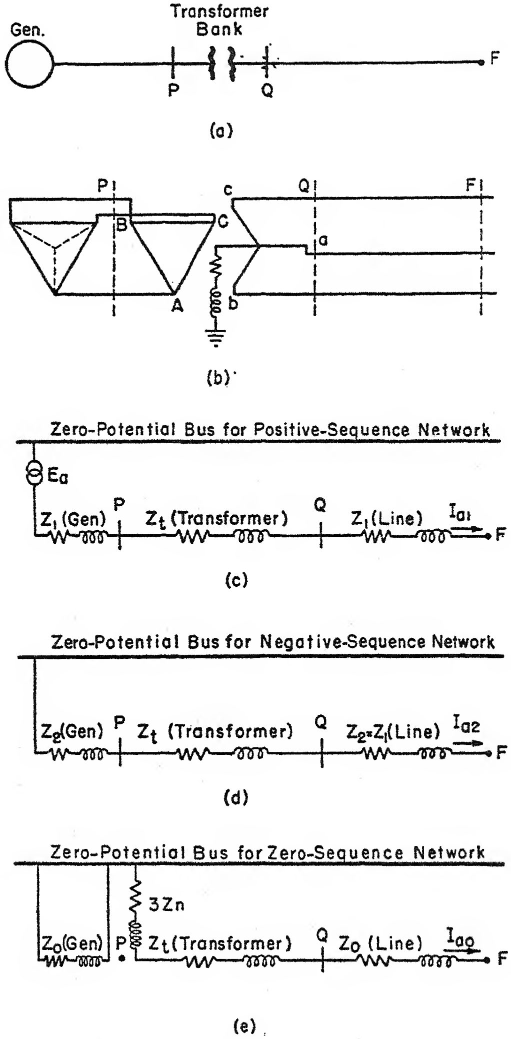

Gen.
Transformer Bank
P
Q
F
(a)
P
c
Q
F
B
C
a
A
b
(b)
Zero-Potential Bus for Positive-Sequence Network
Ea
Z1 (Gen)
P
Zt (Transformer)
Q
Z1 (Line)
Ia1
F
(c)
Zero-Potential Bus for Negative-Sequence Network
Z2(Gen)
P
Zt (Transformer)
Q
Z2=Z1(Line)
Ia2
F
(d)
Zero-Potential Bus for Zero-Sequence Network
3Zn
Z0(Gen)
P
Zt (Transformer)
Q
Z0 (Line)
Ia0
F
(e)

FIG. 20. Diagrams for Problem 6. (*a*) One-line diagram. (*b*) Three-line diagram. (*c*), (*d*), (*e*) Positive-, negative-, and zero-sequence networks, respectively.

generator zero-sequence impedance is represented as shorted to the zero-potential bus for the network, indicating that there may be zero-sequence currents in the Δ-connected windings but there will be no resultant zero-sequence voltages across them. For the case under consideration, that of a symmetrical generator with balanced generated voltages, there will be no generated zero-sequence voltages and no zero-sequence currents in the generator.

When the sequence impedance diagrams are expressed in per cent or per unit, the same diagrams can be used to determine fundamental-frequency currents and voltages throughout the system with a fault at any location. In sequence impedance diagrams of power system, impedances (or equivalent impedances) are per phase and therefore base kva in these diagrams is kva per phase and base voltage is line-to-neutral voltage.

Problem 6. With a line-to-ground fault at F in the system shown in Fig. 20(a), determine line-to-ground voltages at F and voltages across the Δ-connected windings of the generator and transformer bank. Draw a three-line diagram showing currents in amperes in the fault, in the transmission line, at the generator terminals, and in the Δ-connected windings of the generator and transformer bank. Neglect resistance, transformer exciting current, and line capacitance.

Data. Three-phase generator rated 30,000 kva, 13.8 kv. Positive- and negative-sequence reactances are 35% and 50%, respectively, based on the generator rating. Step-up transformer bank of three single-phase units, each unit rated 10,000 kva, 13.2 – 66.4/115 Y kv, leakage reactance 10%, neutral of Y grounded through a reactance of 15 ohms. Three-phase transmission line 25 miles in length, with positive- and negative-sequence reactances of 0.8 ohm per mile and zero-sequence reactance of 2.7 ohms per mile. *Operating condition:* The line-to-line voltage in the transmission line was 115 kv before the fault.

Solution. Calculations will be made in per unit on a 30,000 kva base with a base voltage of 115 kv in the transmission circuit.

Base three-phase kva = 30,000 kva (10,000 kva per phase)

Base line-to-line voltage in transmission line = 115 kv line-to-line ($115/\sqrt{3} = 66.4$ kv line-to-neutral)

$$\text{Base line current in transmission line} = \frac{10{,}000}{66.4} = 151 \text{ amp}$$

Base line-to-line voltage at P (base voltages in Δ circuits) =

$$115 \times \frac{13.2}{115} = 13.2 \text{ kv}$$

$$\text{Base line-to-neutral voltage at } P = \frac{13.2}{\sqrt{3}} = 7.62 \text{ kv}$$

$$\text{Base line current at } P = \frac{10{,}000}{7.62} = 1313 \text{ amp}$$

$$\text{Base current in } \Delta\text{-connected circuits} = \frac{10{,}000}{13.2} = 758 \text{ amp}$$

At no load, the internal voltage of the generator is the same as its terminal voltage. For the given operating condition, a terminal voltage of 13.2 kv is required to produce 115 kv on the transmission line at no load. For this operating condition the generator terminal voltage is less than its rated voltage. The equivalent line-to-neutral voltage of the generator to be used in the positive-sequence network with the Δ–Y transformer banks replaced by its equivalent Y–Y bank is

$$E_a = \frac{13.2}{\sqrt{3}} \text{ kv} = 7.62 \text{ kv} = 1.0 \text{ in per unit}$$

Note. In the work which follows the numerical values of per unit quantities will be written without the words *per unit.*

Applying [27] and [31] of Chapter I, the system reactances expressed in per unit on the chosen base quantities are

$$x_1 \text{ (line)} = x_2 \text{ (line)} = \frac{25 \times 0.8 \times 30{,}000}{(115)^2 \times 10^3} = 0.045$$

$$x_0 \text{ (line)} = \frac{25 \times 2.7 \times 30{,}000}{(115)^2 \times 10^3} = 0.153$$

$$3x_n = \frac{3 \times 15 \times 30{,}000}{(115)^2 \times 10^3} = 0.102$$

$$x_1 \text{ (generator)} = 0.35 \times \left(\frac{13.8}{13.2}\right)^2 = 0.382$$

$$x_2 \text{ (generator)} = 0.50 \times \left(\frac{13.8}{13.2}\right)^2 = 0.545$$

$$x_t \text{ (transformer)} = 0.10$$

With a fault at F, the terminals of the transmission line, positive- and negative-sequence currents flowing from the generator to the fault meet the impedances of the generator, transformer bank, and line in series. Zero-sequence currents, starting at the grounded reactance of the transformer bank, flow through this reactance, the reactance of the transformer bank, and the reactance of the line in series to the fault and return through the ground. The total positive-, negative-, and zero-sequence impedances are

$$Z_1 = j(0.382 + 0.10 + 0.045) = j0.527$$

$$Z_2 = j(0.545 + 0.10 + 0.045) = j0.690$$

$$Z_0 = j(0.102 + 0.10 + 0.153) = j0.355$$

Applying [30] for a line-to-ground fault at the terminals of an unloaded generator, the per unit components of current of phase a in the transmission line with E_a as reference vector are

$$I_{a1} = I_{a2} = I_{a0} = \frac{E_a}{Z_1 + Z_2 + Z_0} = \frac{1.0}{j1.572} = -j0.636$$

$$I_a = I_{a1} + I_{a2} + I_{a0} = -j1.908$$

$$I_b = I_c = 0$$

$$I_{\text{reactor}} = 3I_{a0} = -j1.908$$

In amperes: $I_a = I_{\text{reactor}} = -j1.908 \times 151 = -j288$ amp.

The line currents are also the currents in the Y-connected transformer windings. When expressed in per unit with transformer exciting currents neglected, they are also the per unit currents in the Δ-connected transformer windings (with directions indicated by arrows in Fig. 21).

$$I_{CB} = I_a = -j1.908$$
$$I_{BA} = I_c = 0$$
$$I_{AC} = I_b = 0$$

In amperes: $I_{CB} = -j1.908 \times 758 = -j1446$ amp; $I_{BA} = I_{AC} = 0$.

There are no zero-sequence components of line current at P. The transformer connection diagram given in Fig. 20(*b*) corresponds to that of Fig. 19(*b*), in which the positive- and negative-sequence components of line current and line-to-neutral voltage on the Δ side of the Δ–Y transformer bank are 90° behind and 90° ahead, respectively, of those on the Y side of the bank. The per unit components of line current at P are therefore

$$I_{A0} = 0$$
$$I_{A1} = -jI_{a1} = -j(-j0.636) = -0.636$$
$$I_{A2} = +jI_{a2} = +j(-j0.636) = 0.636$$

The per unit line currents at P are

$$I_A = I_{A1} + I_{A2} = 0$$
$$I_B = a^2 I_{A1} + aI_{A2} = j1.103$$
$$I_C = aI_{A1} + a^2 I_{A2} = -j1.103$$

In amperes: $I_B = -I_C = j1.103 \times 1313 = j1446$ amp.

The per unit positive- and negative-sequence components of the current I_{BC} in the Δ-connected generator are the positive- and negative-sequence components of the line current I_A flowing from the Δ turned 90° forward and 90° backward, respectively. (See [62*a*].)

$$I_{BC1} \text{ (generator)} = jI_{A1} = -j0.636$$
$$I_{BC2} \text{ (generator)} = -jI_{A2} = -j0.636$$

From [58],

$$I_{BC} \text{ (generator)} = I_{BC1} + I_{BC2} = -j1.272$$
$$I_{CA} \text{ (generator)} = a^2 I_{BC1} + aI_{BC2} = j0.636$$
$$I_{AB} \text{ (generator)} = aI_{BC1} + a^2 I_{BC2} = j0.636$$

In amperes: $I_{BC} = -j1.272 \times 758 = -j964$ amp; $I_{CA} = I_{AB} = j482$ amp.

Figure 21 is a three-line current diagram for Problem 6 showing the currents in amperes with E_a as reference vector. Arrows in Fig. 21 show directions of currents which correspond to their calculated values. The direction of any calculated current may be reversed by reversing the assumed direction of current flow and changing the phase of the calculated current by 180° (i.e., multiplying it by -1). If the directions of I_{CA} and I_{AB} in the generator and I_B in the line calculated above had been reversed, their calculated values would have been multiplied by -1.

From [8]–[10], the components of V_a at the fault are

$$V_{a1} = E_a - I_{a1}Z_1 = 1 - (-j0.636)(j0.527) = 0.664$$
$$V_{a2} = -I_{a2}Z_2 = -(-j0.636)(j0.690) = -0.438$$
$$V_{a0} = -I_{a0}Z_0 = -(-j0.636)(j0.355) = -0.226$$

The line-to-ground voltages at the fault from [7], [25], and [26] of Chapter II are

$$V_a = 0.664 - 0.436 - 0.226 = 0$$
$$V_b = -0.226 - \tfrac{1}{2}(0.226) + j\frac{\sqrt{3}}{2}(1.102) = -0.339 + j0.955 = 1.02\underline{/109.5^\circ}$$
$$V_c = -0.339 - j0.955 = 1.02\overline{/70.5^\circ}$$

In kilovolts: $V_a = 0$; $V_b = 1.02\underline{/109.5^\circ} \times 66.4 = 6.77\underline{/109.5^\circ}$ kv; $V_c = 6.77\overline{/70.5^\circ}$ kv.

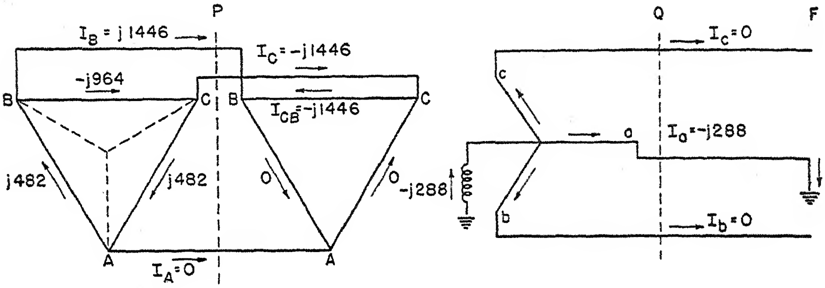


FIG. 21. Three-line diagram showing currents in amperes for Problem 6.

There are no zero-sequence voltages at P. The positive- and negative- sequence components of the voltage to ground of phase a at P, calculated on an equivalent Y–Y basis, can be obtained from the positive- and negative-sequence networks. Let V'_{A1} and V'_{A2} represent these components. Then at the generator terminals from [8] and [9],

$$V'_{A1} = E_{a1} - I_{a1}Z_1 = 1 - (-j0.636)(j0.382) = 0.757$$
$$V'_{A2} = -I_{a2}Z_2 = -(-j0.636)(j0.545) = -0.346$$

For the connection diagram of Fig. 20(b), positive- and negative-sequence components of line-to-neutral voltages on the Δ side of a Δ–Y bank are 90° behind and 90° ahead, respectively, of their components calculated on an equivalent Y–Y basis. Therefore at P,

$$V_{A0} = 0$$
$$V_{A1} = -jV'_{A1} = -j0.757$$
$$V_{A2} = jV'_{A2} = -j0.346$$

From [7], [25], and [26], of Chapter II,

$$V_A = -j1.103 = 1.103\overline{/90^\circ}$$
$$V_B = -0.356 + j0.552 = 0.657\underline{/122.8}$$
$$V_C = 0.356 + j0.552 = 0.657\underline{/32.8}$$

In kv: $|V_A| = 1.103 \times 7.62 = 8.41$ kv; $|V_B| = |V_C| = 5.04$ kv.

The voltages across the deltas of the generator and transformer bank are the line-to-line voltages at P which can be determined in per unit of *line-to-neutral voltage* by substituting the above values of V_A, V_B, and V_C in [11]. An alternate solution will be given in terms of per unit line-to-line symmetrical components. From [55],

$$V_{BC0} = 0$$
$$V_{BC1} = jV_{A1} = 0.757$$
$$V_{BC2} = -jV_{A2} = -0.346$$

From [54],

$$V_{BC} = 0.411$$
$$V_{CA} = -0.205 - j0.956 = 0.98\overline{/102.1^\circ}$$
$$V_{AB} = -0.205 + j0.956 = 0.98\underline{/102.1^\circ}$$

In kilovolts: $|V_{BC}| = 0.411 \times 13.2 = 5.42$ kv; $|V_{CA}| = V_{AB} = 12.9$ kv.

If the Δ–Y transformer bank had been connected as in Fig. 19 (*a*), or if the connection diagram of the Δ–Y bank had not been given and the connection of Fig. 19 (*a*) had been assumed, the positive- and negative-sequence voltages at P would be determined from V'_{A1} and V'_{A2}, the positive- and negative-sequence voltages at A calculated on the equivalent Y–Y basis, by the equations

$$V_{A1} = jV'_{A1} = j0.757$$
$$V_{A2} = -jV'_{A2} = j0.346$$

The phase voltages at A would then become

$$V_A = j1.103$$
$$V_B = 0.356 - j0.552$$
$$V_C = -0.356 - j0.552$$

These voltages differ from those calculated for the given connection diagram by 180°. This is true also for the line currents and the currents and voltages in the Δ-connected windings.

If the line-to-ground fault is at point Q, the procedure is similar to that with a fault at F except that the transmission line impedances are not included in calculating the total sequence impedances between the generator and fault. If the fault is at P, the zero-sequence impedance is infinite and no current will flow into the fault. The phase voltages at P are the same as those of the ungrounded Y-connected generators with a line-to-ground fault at its terminals.

Problem 7. Solve Problem 2 for a double line-to-ground fault: (*a*) generator Y-connected with neutral solidly grounded; (*b*) generator Δ-connected.

Problem 8. With phase a as reference phase, derive equations relating the symmetrical components of I_a flowing into the fault and the line-to-ground voltage V_a at the fault for: (1) a line-to-ground fault on phase b; (2) a line-to-line fault between phases a and c; (3) a double line-to-ground fault on phases a and b. (For check see Table I, Chapter VII.)

Problem 9. A generator having a solidly grounded neutral and rated 10,000 kva, 13.8 kv has positive-, negative-, and zero-sequence reactances of 30, 40, and 5%, respectively. (*a*) What reactance must be placed in the generator neutral so that the fault current for a line-to-ground fault of zero fault impedance will not exceed rated line current? (*b*) What value of resistance in the neutral will serve the same purpose? Express resistance and reactance values in per unit and also in ohms.

Problem 10. (*a*) What reactance must be placed in the neutral of the generator of Problem 9 to reduce the ground current to rated line current for a double line-to-ground fault? (*b*) What will be the magnitudes of the line currents when the ground current is so reduced? (*c*) As the reactance in the neutral is indefinitely increased, what are the limiting minimum values of the line currents?

Problem 11. Solve Problem 4 for a line-to-ground fault, assuming the generator Δ-connected.

Problem 12. Solve Problem 6, Fig. 20, for (1) a line-to-line fault at *F*, (2) a line-to-ground fault at *Q*, (3) a double line-to-ground fault at *P*, using the transformer connection diagram of Fig. 19(*a*).

CHAPTER IV

UNSYMMETRICAL FAULTS ON NORMALLY BALANCED THREE-PHASE SYSTEMS

In a large three-phase power transmission system there may be circuits which are unsymmetrical, and therefore do not carry balanced currents even under normal operation. However, transmission systems as a whole are approximately symmetrical, and the main transmission circuits under normal operation have substantially balanced voltages and currents. In this chapter, the circuits discussed are assumed symmetrical; therefore, under normal operating conditions, the currents and voltages are balanced. In cases where an unsymmetrical circuit appreciably affects the operation of the rest of the system, or when the performance of the circuit itself is under consideration, it becomes necessary to investigate such circuits. Chapter VIII is devoted to the discussion of unsymmetrical three-phase circuits.

Two types of unsymmetrical faults on normally balanced power systems will be considered: short circuits and open conductors.

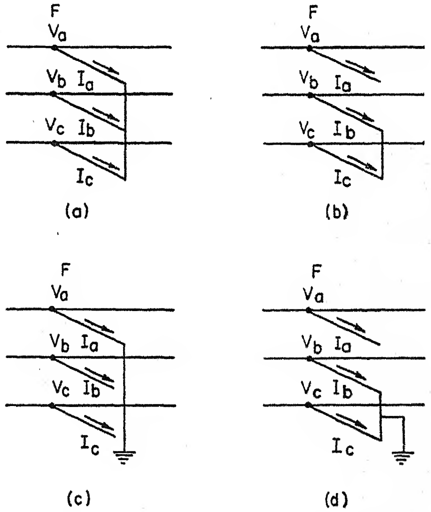


FIG. 1. Currents flowing into the fault at any point F of a three-phase system. (*a*) Three-phase fault. (*b*) Line-to-line fault. (*c*) Single line-to-ground fault. (*d*) Double line-to-ground fault.

SHORT CIRCUITS

I_a, I_b, and I_c, which were used to represent the line currents in Chapter III, will now be used to indicate the currents flowing into the fault from the three phases *a*, *b*, and *c*, respectively; V_a, V_b, and V_c as before will represent the voltages to ground at the point of fault F. In Fig. 1 the fault currents are represented as flowing in hypothetical stub connections from the conductor, so that they will not be confused with line currents. I_{a1}, I_{a2}, and I_{a0} are the positive-, negative-, and zero-sequence components, respectively, of the fault current I_a; and V_{a1}, V_{a2}, and V_{a0} are the corresponding components of V_a, the voltage to ground of phase a at F. Positive direction of current flow for fault currents and their sequence components will be taken from the conductors into the fault.

If the conditions at the fault for the four types of short circuits shown in Fig. 1 are expressed in equations, it will be seen that the equations are the same as those for faults at the terminals of an unloaded generator given in Chapter III. The relations between the symmetrical components of current and of voltage will therefore be the same, the only difference being that the symmetrical components of current are those for the fault current I_a instead of the line current I_a. The symmetrical components of voltage are, as before, components of the line-to-ground voltage of phase a at the point of fault.

Six equations are needed to determine the six unknown symmetrical components, I_{a1}, I_{a2}, I_{a0}, V_{a1}, V_{a2}, and V_{a0}. The conditions at the fault provide three equations involving phase currents and voltages from which three equations can be obtained connecting symmetrical components of current or of voltage of phase a at the point of fault. The other three equations should express the positive-, negative-, and zero-sequence components of voltage, respectively, of the reference phase a at the fault in terms of the corresponding sequence current flowing into the fault and the impedance associated with it. For a system with one unloaded symmetrical generator these three equations are given by [8]–[10] of Chapter III. For the general case, these three equations can be obtained from the three sequence networks, each considered separately. Figure 2(a) gives a one-line diagram of a symmetrical power system; the sequence networks of this system are given in parts (b), (c), and (d) of Fig. 2.

Positive-Sequence Network. In a large three-phase power system, there are usually several generating stations connected by transformers and transmission line, with load tapped off at various points. Under load, the relative angular positions of the internal voltages of the synchronous generators and motors of the system in the positive-sequence network will depend upon operating conditions. In Fig. 2(b), the internal voltages E_g, E_m, and E_n can be calculated when operating conditions are known. At no load, neglecting charging currents, all voltages, when expressed in per unit of their respective base voltages, will be equal and in phase. This statement is based upon the use of equivalent Y–Y transformer banks to replace Δ–Y or Y–Δ connected banks. The shift in phase because of Δ–Y or Y–Δ transformer banks can be taken into account when required. (See Chapter III.)

Before the occurrence of a fault, there is no positive-sequence fault current and the voltages at the point of fault F are balanced positive-sequence voltages. Let the voltage of phase a at F before the fault be V_f. I_{a1} is the positive-sequence current flowing from phase a into the fault and V_{a1} is the positive-sequence voltage of phase a at the fault.

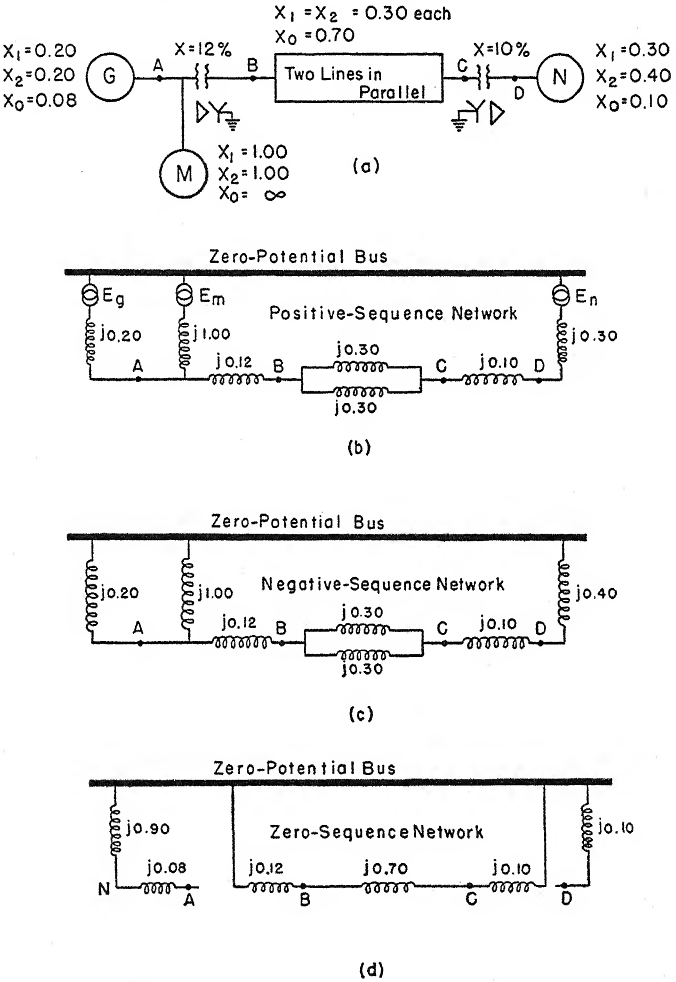

X₁ = X₂ = 0.30 each
X₀ = 0.70
X₁=0.20
X₂=0.20
X₀=0.08
G
A
X=12%
B
Two Lines in Parallel
X=10%
C
D
N
X₁=0.30
X₂=0.40
X₀=0.10
M
X₁ = 1.00
X₂ = 1.00
X₀ = ∞
(a)
Zero-Potential Bus
Eg
Em
En
Positive-Sequence Network
j0.20
j1.00
j0.30
j0.12
j0.30
j0.10
j0.30
(b)
Zero-Potential Bus
Negative-Sequence Network
j0.20
j1.00
j0.40
j0.12
j0.30
j0.10
j0.30
(c)
Zero-Potential Bus
Zero-Sequence Network
j0.90
j0.08
j0.12
j0.70
j0.10
j0.10
(d)

FIG. 2. (*a*) One-line diagram of a symmetrical three-phase power system. (*b*), (*c*), and (*d*) Positive-, negative-, and zero-sequence networks for system in (*a*).

The effect of a fault of any type on the positive-sequence network is to change the positive-sequence fault current from 0 to I_{a1} and the positive-sequence voltage at the point of fault F from V_f to V_{a1}. The change in voltage, or the voltage required to change the fault voltage from V_f to V_{a1}, is $-(V_f - V_{a1})$. By the principle of superposition, the rms initial symmetrical positive-sequence current at any point in the system can be determined by adding to the rms load current at that point the current resulting from the voltage $-(V_f - V_{a1})$ applied at F, with all other applied voltages equated to zero. If the voltage $-(V_f - V_{a1})$ is applied at point F in the positive-sequence network, with all generated voltages reduced to zero, the positive-sequence current ΔI_f flowing *into the network* from F, given by [52], Chapter I, is

$$\Delta I_f = \frac{-(V_f - V_{a1})}{A_{ff}} = \frac{-(V_f - V_{a1})}{Z_1}$$

where $Z_1 = A_{ff}$ = driving-point impedance at the fault in the positive-sequence network or the positive-sequence impedance of the system viewed from the fault.

I_{a1} has been defined as the positive-sequence current flowing into the fault. I_{a1} is therefore equal in magnitude and opposite in sign to ΔI_f. Replacing ΔI_f by $-I_{a1}$ in the above equation and solving for V_{a1},

$$V_{a1} = V_f - I_{a1}Z_1 \tag{1}$$

An alternate method of obtaining [1] is by the superposition of voltages instead of currents. I_{a1} flowing from the zero-potential bus of the positive-sequence network through the network to the fault meets Z_1, the positive-sequence impedance viewed from the fault. The voltage drop produced is $I_{a1}Z_1$; the voltage rise is $-I_{a1}Z_1$. Adding $-I_{a1}Z_1$ or subtracting $I_{a1}Z_1$ from V_f, [1] is obtained.

V_f and Z_1 are known, or can be determined. V_{a1} and I_{a1} are unknown; but, regardless of their values, [1] gives the relation existing between them.

Negative-Sequence Network. When there are no negative-sequence voltages generated in a system, and none induced from outside sources, the only negative-sequence voltages associated with the network will be those resulting from the flow of negative-sequence currents. The negative-sequence voltage of phase a at the fault is zero before the occurrence of the fault and V_{a2} after the fault occurs; the *change in voltage* resulting from the fault will be V_{a2}. The negative-sequence current entering the negative-sequence network because of the fault may be determined by applying V_{a2} to the negative-sequence network between the fault point and the zero-potential bus, with no other voltages applied. The applied voltage V_{a2} will send current *into the network*; I_{a2} is defined as the negative-sequence current flowing *from the network into the fault.* If Z_2 is the negative-sequence impedance of the system viewed from the fault, or the driving-point impedance at the fault in the negative-sequence network,

$$-I_{a2} = \frac{V_{a2}}{Z_2}, \quad \text{or} \quad V_{a2} = -I_{a2}Z_2$$

The relation between V_{a2} and I_{a2} can also be obtained by adding the voltage rise, caused by I_{a2} flowing from the zero-potential bus of the negative-sequence network to the fault, to the negative-sequence voltage at the fault point before the fault occurred. The voltage rise is $-I_{a2}Z_2$, negative-sequence fault voltage was zero; therefore

$$V_{a2} = 0 - I_{a2}Z_2 = -I_{a2}Z_2 \qquad [2]$$

In [2], Z_2 is known or can be determined; V_{a2} and I_{a2} are unknown, but [2] expresses the relation between them.

Zero-Sequence Network. Assuming no generated and no induced zero-sequence voltages, the relation between the fault current and voltage in the zero-sequence network is similar to that in the negative-sequence network. If V_{a0} is the zero-sequence voltage of phase a at the fault, I_{a0} the zero-sequence current from phase a flowing into the fault, and Z_0 the zero-sequence impedance of the system viewed from the fault, then

$$V_{a0} = -I_{a0}Z_0 \qquad [3]$$

In [3], Z_0 is known or can be determined; I_{a0} and V_{a0} are unknown, but [3] expresses the relation between them.

Solution of Simultaneous Symmetrical Component Equations. Equations [1], [2], and [3], together with the three equations expressing relations between the symmetrical components of current or of voltage of phase a at the fault, provide the six simultaneous equations needed for determining the six symmetrical components I_{a1}, I_{a2}, I_{a0}, V_{a1}, V_{a2}, and V_{a0}.

A simple way of solving these six equations is to select I_{a1} and V_{a1} as the unknowns which are first to be determined. This selection is made because in a normally balanced system the positive-sequence network is the only network in which there are generated voltages. To determine I_{a1} and V_{a1}, two equations are required. One of these equations is given by [1]. The other is obtained by eliminating the four unknowns V_{a2}, V_{a0}, I_{a2}, and I_{a0} from the five equations consisting of [2], [3], and the three fault equations. The remaining equation in terms of I_{a1}, V_{a1}, Z_2, and Z_0 can be written

$$V_{a1} = I_{a1}K \qquad [4]$$

where K is a function of Z_2 and Z_0, depending upon the type of fault.

From [1] and [4], V_{a1} is eliminated and I_{a1} obtained. Knowing I_{a1}, V_{a1} and the other components are readily obtained.

System Replaced by Equivalent Generator for Determining Initial Symmetrical Rms Fault Currents and Voltages. If [1], [2], and [3] are compared with [8], [9], and [10] of Chapter III, respectively, it will be noted that the relations between the symmetrical components of the fault voltage and current of phase a for each of the three sequences are the same for the system as for the unloaded generator. E_a in [8] is equal to V_f, the voltage of phase a at the fault before the fault occurred, and Z_1, Z_2, and Z_0 in [8], [9], and [10], respectively, are the positive-, negative-, and zero-sequence impedances viewed from the fault. Since the equations connecting the current and voltage at the point of fault in each of the three sequence networks is the same for a system as for an unloaded generator, the relations developed in Chapter III for a generator may be extended by analogy to a system. Thus, to determine the *initial symmetrical rms fault currents and voltages*, the system may be treated as an unloaded generator with internal voltage equal to V_f, the prefault voltage at the point of fault, and sequence impedances equal to those of the system viewed from the fault.

To determine the current and voltage distribution in the system, the distribution in each of the sequence networks must first be determined. The fault currents I_{a1}, I_{a2}, and I_{a0}, flowing from the zero-potential bus to the fault in their respective networks, divide among the various parallel paths inversely in proportion to the impedances of these paths. In the negative- or zero-sequence networks, the voltage rise from the zero-potential bus to any system point will give the negative- or zero-sequence voltage, respectively, at that point.

In the positive network the currents throughout the system *due to the fault* can be added to the load currents before the fault to give total positive-sequence currents. The positive-sequence voltage at any point P in the system may be determined by adding to the positive-sequence voltage at the fault the voltage drop between P and the fault caused by the *total* positive-sequence current; or by superposing upon the positive-sequence voltage which existed at P before the fault the voltage rise resulting from the flow of the positive-sequence *fault* current. The line currents in the three phases and the phase voltages to ground at any point in the system are obtained by substituting the symmetrical components of I_a and V_a, respectively, at that point in [19]–[21] and [7]–[9] of Chapter II.

Determination of Currents and Voltages Following Initial Values. When a power system is operating under normal conditions, neglecting fluctuations in load, the rotors of the various synchronous machines on the system have fixed angular positions with respect to each other, which depends upon the load and the excitations of the machines. In the positive-sequence network, the angular displacement of the rotors is represented as angular displacements between the internal voltages of the machines, and the magnitudes of the voltage vectors are determined by the excitations. Under normal operation there exists a condition of equilibrium, the electrical output of each machine being equal to its mechanical input less losses. This applies to a synchronous motor or condenser as well as to a generator, if the electrical output and mechanical input are both considered negative. When a disturbance occurs, such as a short circuit, the electrical outputs of some, if not all, of the machines of the system change instantly, the change depending upon the severity of the disturbance. For example, a three-phase fault at the terminals of a machine reduces its electrical output immediately to resistance losses. The mechanical input, controlled by governors, does not change immediately. A machine will attempt to speed up if its mechanical input exceeds its electrical output, and to slow down if the reverse is true. Its acceleration depends upon its inertia as well as upon the difference between mechanical input and electrical output. If the disturbance is not severe enough to cause the machines to lose synchronism, the rotors will eventually take up new fixed relative angular positions, and a condition of equilibrium will again exist. If the disturbance is a fault, followed by the switching out of service of a circuit, and it is required to determine the currents and voltages during the disturbance, transient reactances are used in the positive sequence network and generated voltages behind transient reactances. If it is known that the disturbance is such that the relative angular positions of the rotors change but slightly, as an approximation, the procedure given above for determining initial symmetrical rms currents and voltages can be used to determine rms transient currents and voltages of fundamental frequency, the only difference being the use of transient reactance viewed from the fault in [1] instead of subtransient reactance. Equations [2] and [3] for the negative- and zero-sequence networks, respectively, apply for subtransient, transient, or steady-state conditions.

When the change in relative angular positions of the rotors must be considered, as in the case of transient stability studies, [1], which gives the relation between V_{a1} and I_{a1} as a function of V_f, can be used if V_f is redefined as the voltage which would exist at the fault point with the fault removed and with the generated voltages of the machines given the angular positions which they have at the instant under consideration. Calculation of the relative angular positions of the generated voltages in the positive-sequence network following a fault is simplified if the fault is replaced by an equivalent circuit in the positive-sequence network.

EQUIVALENT CIRCUIT TO REPLACE FAULT IN POSITIVE-SEQUENCE NETWORK — CONNECTIONS OF SEQUENCE NETWORKS TO REPRESENT FAULT CONDITIONS

Line-to-Line Fault. Figure 1(*b*) shows a line-to-line fault between phases *b* and *c* at some point *F* of a previously balanced three-phase system. I_a, I_b, I_c, V_a, V_b, and V_c represent the three fault currents and the three voltages to ground of phases *a*, *b*, and *c*, respectively, at the point of fault *F*. The fault conditions are

$$I_a = 0 \qquad I_b = -I_c \qquad V_b = V_c$$

These conditions are the same as those for a line-to-line fault on an unloaded generator (Chapter III). Therefore, following the same procedure,

$$I_{a0} = 0,$$

and

$$I_{a1} = -I_{a2} \tag{5}$$

$$V_{a1} = V_{a2} \tag{6}$$

For a line-to-line fault through zero fault impedance, the positive- and negative-sequence components of fault current in the unfaulted phase are equal in magnitude but opposite in phase, while the positive- and negative-sequence components of voltage at the fault are equal both in magnitude and phase.

Replacing I_{a2} in [2] by $-I_{a1}$ from [5] and substituting [2] in [6],

$$V_{a1} = V_{a2} = -I_{a2}Z_2 = I_{a1}Z_2 \tag{7}$$

Solving [1] and [7] for I_{a1},

$$I_{a1} = \frac{V_f}{Z_1 + Z_2} \tag{8}$$

Equivalent Circuit to Replace the Line-to-Line Fault in the Positive-Sequence Network. Since $V_{a1} = I_{a1}Z_2$ in [7], it is evident that V_{a1} and I_{a1} will be correctly determined if the impedance Z_2 is inserted between the fault point and the zero-potential bus in the positive-sequence network. The equivalent circuit to replace a line-to-line fault at F in the positive-sequence network is therefore Z_2, the impedance of the negative-sequence network viewed from the fault, as shown in Fig. 3.

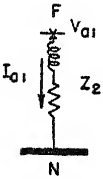


FIG. 3. Equivalent circuit to replace a line-to-line fault in the positive-sequence network.

An equivalent circuit of a single lumped impedance to replace the fault in the positive-sequence network is particularly useful in analytical calculations. When either an a-c or d-c calculating table is used to determine current and voltage distribution, the equivalent circuit consisting of the complete negative-sequence network may be used to advantage; then the current and voltage distribution for the negative- as well as for the positive-sequence network will be obtained. The currents and voltages so determined will be those for the reference phase a.

Connection of Sequence Networks for Solution of Line-to-Line Faults on an A-C Calculating Table. Assume a line-to-line fault to occur at B in Fig. 2(a). Since the positive- and negative-sequence components of voltage at the point of fault are equal from [6], the positive- and negative-sequence networks should be connected with their fault points together and also their zero-potential buses, as in Fig. 4. This connection will also satisfy the condition of [5] that $I_{a1} = -I_{a2}$ in the fault, as may be seen by an inspection of Fig. 4. The current I_{a1} flows from the positive-sequence zero-potential bus through the network into the fault, then (as $-I_{a2}$) through the negative-sequence network from the fault to the zero-potential bus of the negative-sequence network, and finally back to the zero-potential bus for the positive-sequence network. The voltages in each sequence network are measured from the zero-potential bus for the network to the point under consideration.

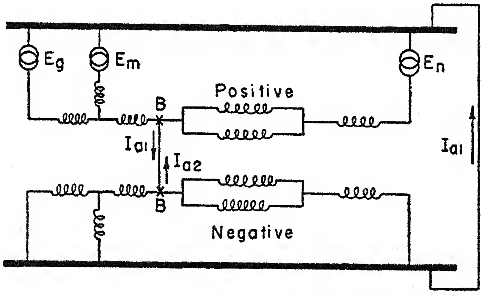


FIG. 4. Connection of positive- and negative-sequence networks of Fig. 2 for a line-to-line fault at B.

Problem 1. The generator and motor described in Problem 2, Chapter I, Fig 18 (*a*), have negative-sequence reactances of 40 and 210%, respectively, on the chosen kva base. The operating conditions previous to the fault are the same as those given in Problem 2, Chapter I. A line-to-line fault occurs on the bus T between phases *b* and *c*.

(*a*) Determine the initial symmetrical rms currents flowing from the three phases into the fault, the three line currents in the generator and in the motor, and the line-to-ground voltages at the point of fault.

(*b*) Draw a three-line diagram showing line and fault currents.

(*c*) The positive-sequence diagram is given in Chapter I, Fig. 18 (*b*). Draw the negative-sequence diagram and indicate how the two sequence networks should be connected for use on an a-c calculating table to determine the sequence currents and voltages for the reference phase *a*.

Solution. (*a*) From the given conditions,

$$V_f = V_t = 0.98 + j0$$

$$Z_1 = \frac{j0.40 \times j2.00}{j2.40} = j0.333$$

$$Z_2 = \frac{j0.40 \times j2.10}{j2.50} = j0.336$$

From [5] and [8],

$$I_{a0} = 0$$

$$I_{a1} = \frac{V_f}{Z_1 + Z_2} = \frac{0.98 + j0}{j0.669} = -j1.465$$

$$I_{a2} = -I_{a1} = j1.465$$

The three fault currents are

$$I_a = I_{a0} + I_{a1} + I_{a2} = 0$$

$$I_b = (I_{a0} + a^2 I_{a1} + aI_{a2}) = (a^2 - a)I_{a1} = -j\sqrt{3}I_{a1} = -2.537 + j0$$

$$I_c = -I_b = 2.537 + j0$$

The positive- and negative-sequence currents due to the fault will flow from generator and motor inversely in proportion to their respective impedances; therefore,

$$I_{a1} \text{ (from generator)} = -j1.465 \times \frac{2.00}{2.40} = -j1.221$$

$$I_{a1} \text{ (from motor)} = -j1.465 \times \frac{0.40}{2.40} = -j0.244$$

$$I_{a2} \text{ (from generator)} = j1.465 \times \frac{2.10}{2.50} = j1.231$$

$$I_{a2} \text{ (from motor)} = j1.465 \times \frac{0.40}{2.50} = j0.234$$

Adding the load currents in generator and motor to the positive-sequence currents due to the fault, initial symmetrical rms positive-sequence currents are obtained:

$$\text{Total } I_{a1} \text{ (from generator)} = 0.5 - j1.221$$

$$\text{Total } I_{a1} \text{ (from motor)} = -0.5 - j0.244$$

Combining the sequence currents in generator and motor by [19]–[21] of Chapter II, the following currents are obtained. From the generator:

$$I_a = I_{a0} + I_{a1} + I_{a2} = 0 + 0.5 - j1.221 + j1.231 = 0.5 + j0.010$$

$$\begin{aligned} I_b &= I_{a0} + a^2 I_{a1} + a I_{a2} \\ &= 0 + (-0.5 - j0.866)(0.5 - j1.221) + (-0.5 + j0.866)(j1.231) \\ &= -2.373 - j0.438 \end{aligned}$$

$$I_c = I_{a0} + a I_{a1} + a^2 I_{a2} = 1.873 + j0.428$$

From the motor:

$$I_a = I_{a0} + I_{a1} + I_{a2} = 0 - 0.5 - j0.244 + j0.234 = -0.5 - j0.010$$

$$I_b = a^2 I_{a1} + a I_{a2} = -0.165 + j0.438$$

$$I_c = a I_{a1} + a^2 I_{a2} = 0.665 - j0.428$$

In a symmetrical system with $I_{a0} = 0$, $V_{a0} = 0$. From [7],

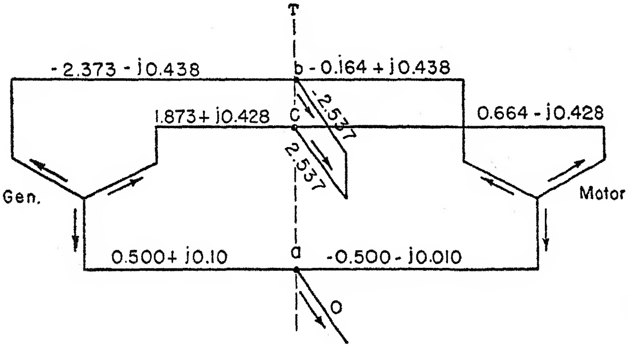
$$V_{a1} = V_{a2} = I_{a1} Z_2 = (-j1.465)(j0.336) = 0.482 + j0$$

Combining the sequence voltages, the line-to-ground voltages at T are

$$V_a = V_{a0} + V_{a1} + V_{a2} = 0 + 0.482 + j0 + 0.482 + j0 = 0.964 + j0$$

$$V_b = V_{a0} + a^2 V_{a1} + a V_{a2} = (a^2 + a) V_{a1} = -V_{a1} = -0.482 + j0$$

$$V_c = V_{a0} + a V_{a1} + a^2 V_{a2} = (a + a^2) V_{a1} = -V_{a1} = -0.482 + j0$$



(a)

FIG. 5(*a*). Line currents for Problem 1. Line-to-line fault at T.

(*b*) A three-line diagram of the system showing the line currents is given in Fig. 5(*a*). Positive direction of line currents is taken from the machine neutrals towards the fault and is indicated by arrows. The voltage of phase *a* at T before the fault is reference vector.

(*c*) The connection of the sequence networks for solution on the a-c network analyzer is shown in Fig. 5(*b*). Since the fault points T of both networks are connected, at the fault $V_{a1} = V_{a2}$ and $I_{a1} = -I_{a2}$, positive direction of current flow being from the system into the fault in both networks. The internal voltages of the generator and motor in the positive-sequence network correspond to the operating condition before the fault given in Problem 2, Chapter I, with the voltage at T before the fault as reference vector.

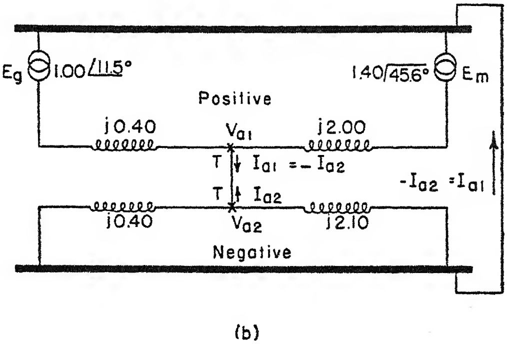


FIG. 5(*b*). Connection of the sequence networks for Problem 1.

Line-to-Ground Fault. Figure 1(*c*) shows a line-to-ground fault through zero impedance on phase *a* at some point *F* in a grounded system. I_a, I_b, and I_c represent fault currents, and V_a, V_b, and V_c voltages to ground. The fault conditions are

$$I_b = 0 \qquad I_c = 0 \qquad V_a = 0$$

These conditions are the same as those for a line-to-ground fault on an unloaded generator, given in Chapter III. Therefore, by the same procedure,

$$I_{a0} = I_{a1} = I_{a2} \tag{9}$$

$$V_{a1} = -(V_{a0} + V_{a2}) \tag{10}$$

For a line-to-ground fault of zero fault impedance on a grounded system, the zero-, positive-, and negative-sequence components of fault current in the faulted phase are equal in magnitude and phase, and the positive-sequence component of voltage to ground at the fault is equal in magnitude and opposite in phase to the sum of the zero- and negative-sequence components.

Substituting V_{a2} and V_{a0} from [2] and [3] in [10], then replacing I_{a2} and I_{a0} by I_{a1} from [9],

$$V_{a1} = -(V_{a0} + V_{a2}) = I_{a1}(Z_0 + Z_2) \tag{11}$$

Solving [1] and [11] for I_{a1},

$$I_{a1} = \frac{V_f}{Z_1 + Z_2 + Z_0} \tag{12}$$

Equivalent Circuit to Replace a Line-to-Ground Fault in the Positive-Sequence Network. From [11], $V_{a1} = I_{a1}(Z_0 + Z_2)$. It follows, therefore, that the equivalent circuit to replace the fault in the positive-sequence network is the sum of Z_0 and Z_2, this impedance to be inserted between the point of fault and the zero-potential bus for the network, as indicated in Fig. 6. Instead of inserting a lumped impedance equal to $Z_0 + Z_2$, the negative- and zero-sequence networks can be connected in series between the point of fault and the zero-potential bus.

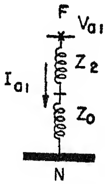


FIG. 6. Equivalent circuit to replace a single line-to-ground fault in the positive-sequence network.

Connections of Sequence Networks for Solution of a Line-to-Ground Fault on an A-C Calculating Table. Equations [9] and [10] determine the manner in which the networks must be connected to satisfy the conditions for the signs of the currents and voltages. For the system shown in Fig. 2(*a*) with a line-to-ground fault at *B*, the connection of the sequence networks is given in Fig. 7. This connection satisfies the condition that the fault currents in the three sequence networks are equal, since the same current flows into the fault from all three networks. The voltage condition that $V_{a1} = -(V_{a0} + V_{a2})$ is also satisfied. The current and voltage distribution in the three sequence networks is that of the reference phase *a*.

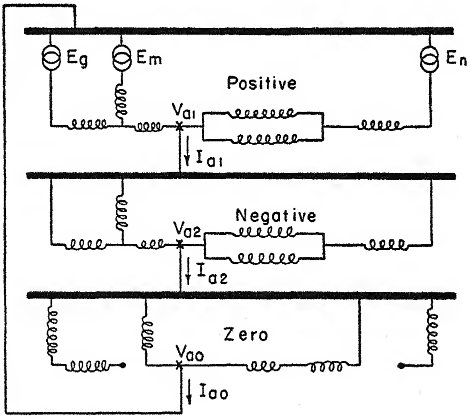


FIG. 7. Connection of sequence networks of Fig. 2 for line-to-ground fault at *B*.

Double Line-to-Ground Fault. Figure 1(*d*) shows a double line-to-ground fault through zero impedance on phases *b* and *c* at some point *F* in a grounded system. The fault conditions are

$$I_a = 0 \qquad V_b = 0 \qquad V_c = 0$$

These conditions are the same as those given in Chapter III; therefore, by the same procedure,

$$V_{a0} = V_{a1} = V_{a2} \tag{13}$$

$$I_{a1} = -(I_{a0} + I_{a2}) \tag{14}$$

$$I_{a0} = -I_{a1}\frac{Z_2}{Z_0 + Z_2} \tag{15}$$

$$I_{a2} = -I_{a1}\frac{Z_0}{Z_0 + Z_2} \tag{16}$$

For a double line-to-ground fault of zero fault impedance on a grounded system, the positive-, negative-, and zero-sequence components of voltage to ground of the unfaulted phase are equal in magnitude and phase, while the positive-sequence component of current in the fault is equal in magnitude and opposite in phase to the sum of the zero- and negative-sequence components, these components varying inversely as their sequence impedances viewed from the fault.

Substituting [16] in [2] and [2] in [13],

$$V_{a0} = V_{a1} = V_{a2} = I_{a1}\frac{Z_0Z_2}{Z_0 + Z_2} \tag{17}$$

Solving [1] and [17] for I_{a1},

$$I_{a1} = \frac{V_f}{Z_1 + \dfrac{Z_0Z_2}{Z_0 + Z_2}} \tag{18}$$

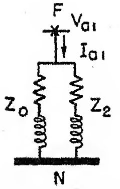


FIG. 8. Equivalent circuit to replace a double line-to-ground fault in the positive-sequence network.

Equivalent Circuit to Replace the Double Line-to-Ground Fault in the Positive-Sequence Network. From [17], $V_{a1} = I_{a1}\dfrac{Z_0Z_2}{Z_0 + Z_2}$. The equivalent circuit to replace the fault in the positive-sequence network is, therefore, the parallel value of the impedances Z_0 and Z_2 as shown in Fig. 8. If this value of impedance is inserted in the positive-sequence network between the point of fault and the zero-potential bus for the network, the positive-sequence current and voltage distribution will be correctly determined. Instead of inserting the impedance $Z_0Z_2/(Z_0 + Z_2)$, the same results will be obtained if the negative- and zero-sequence networks are connected in parallel between the point of fault and zero-potential bus for the positive-sequence network.

Connection of Sequence Networks for Solution of Double Line-to-Ground Faults on an A-C Calculating Table. Equations [13]–[17] determine the manner in which the networks should be connected to satisfy the conditions for the signs of the currents and voltages. The connection for a double line-to-ground fault at *B* in the system shown in Fig. 2(*a*) is given in Fig. 9. This connection satisfies the conditions that $V_{a1} = V_{a2} = V_{a0}$, that $I_{a1} = -(I_{a2} + I_{a0})$, and that I_{a0} and I_{a2} divide inversely in proportion to their sequence impedances viewed from the fault.

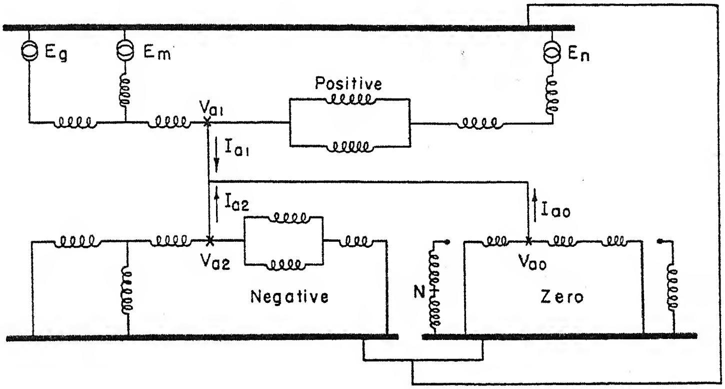


FIG. 9. Connection of sequence networks of Fig. 2 for a double line-to-ground fault at *B*.

Determination of Fault Currents by D-C Calculating Table. A d-c calculating table may be used to determine initial symmetrical rms current distribution for unsymmetrical as well as for three-phase faults when resistance and capacitance are neglected and the system is operated at no load. In this case the internal voltages of the machines in the positive-sequence network behind subtransient reactance in per unit will be equal and in phase and therefore can be represented on a d-c table. It may also be used under load conditions, when resistance and capacitance are neglected, to determine the currents due to the fault, the applied voltage in the positive-sequence network being V_f, the prefault voltage at the point of fault. This voltage on the d-c table will be applied between the neutrals of the machines, connected at a common point, and the zero potential bus for the positive-sequence network. Load currents at any point *P* can be added to currents at *P* due to the fault to give total currents. Since load currents are usually small relative to short-circuit currents and are not in phase with them, their inclusion will not, in general, appreciably affect the magnitudes of the short-circuit currents.

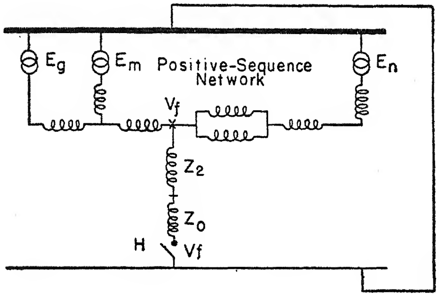
Two methods of determining system voltages and currents during short circuits have been given, both based upon *superposition*. The principles underlying these two methods are explained in Chapter I, and discussed in connection with their application to the calculation of system currents and voltages during three-phase short circuits. The method of superposition is a powerful tool, frequently used in a-c power transmission problems. For this reason, the two methods of calculation based on superposition, described in Chapter I, will be reviewed in their application to unsymmetrical faults.

Assume a line-to-ground fault to occur at B in the system of Fig. 2(a). It has been shown that the positive-sequence currents and voltages in the fault and in the positive-sequence network will be correctly determined if the impedance $Z_2 + Z_0$, the sum of the negative- and zero-sequence impedances viewed from the fault, is inserted in the positive-sequence network between the fault point B and the zero-potential bus for the network. If one terminal of the impedance $Z_2 + Z_0$ is connected at the fault point in the positive-sequence network with the other terminal H isolated as in Fig. 10(a), the system operating conditions before the fault are in no way affected. An open switch is indicated at H with a connection to the zero-potential bus of the positive-sequence network. The voltage rise across the switch measured from the zero-potential bus to H is V_f. The fault is replaced by its equivalent circuit if the switch is closed at H in Fig. 10(a).

By the method called the *first method* in Chapter I, the currents resulting from the voltages E_g, E_m, and E_n, applied one at a time in Fig. 10(a) with the switch closed, are superposed to give total currents in accordance with equations [33] of Chapter I. When calculations are made on an a-c calculating table, and the negative- and zero-sequence networks set up and connected as in Fig. 7, the currents and voltages in the three sequence networks are those resulting from all the applied voltages acting simultaneously. With the a-c calculating table, superposition is automatically applied.

In the method called the *second method* in Chapter I, the *changes* in currents resulting from the fault are determined, not the total currents; load currents are not included. When the switch is closed in Fig. 10(a) to represent fault conditions, the voltage across it becomes zero. The voltage across the switch can be made zero, if a voltage $-V_f$ is applied across it. The currents resulting from the voltage $-V_f$ are the changes in the currents resulting from the fault; and, as indicated in equations

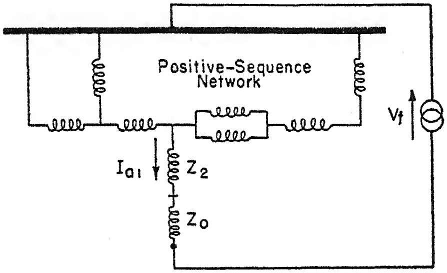
[52] of Chapter I, are determined with $-V_f$ acting alone. The voltage $-V_f$ is the only voltage applied to the network, but, for simplicity, the voltage rise $-V_f$ between the zero-potential bus and H is represented in Fig. 10(b) as a voltage rise V_f in the opposite direction. In Fig. 10(b), V_f is shown outside the positive-sequence network. With this arrangement, fault currents will be correctly determined; but the



Eg
Em
Positive-Sequence Network
En
Vf
Z2
Z0
H
Vf

(a)

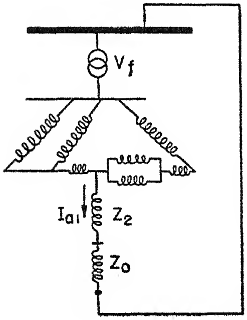
(a) Line-to-ground fault replaced by an equivalent circuit in the positive-sequence network by closing the switch at H.



Positive-Sequence Network
Vf
Ia1
Z2
Z0

(b)

(b) Connection of sequence networks to obtain changes in currents and voltages resulting from the fault.



(c)

(c) Connection for d-c calculating table, where Z_2 and Z_0 can be replaced by the negative- and zero-sequence networks, respectively.

FIG. 10.

positive-sequence voltage at the fault referred to the zero-potential bus for the network will be $-I_{a1}Z_1$ and not V_{a1}. If V_f is placed between the zero-potential bus for the positive-sequence network and the impedances of the synchronous machines connected at a common point as in Fig. 10(*c*), the voltage at the fault in the positive-sequence network will be V_{a1}. The connection of the synchronous machines shown in Fig. 10(*c*) is that used for the d-c calculating table.

It should be noted that the initial symmetrical rms currents in the fault, voltages at the fault, and currents and voltages in the negative- and zero-sequence networks, calculated by the two methods, are identical, provided V_f in Fig. 10(*c*) is the same as V_f in Fig. 10(*a*) before the fault is applied. Since the load currents and internal machine voltages are included in the first method and not in the second, the currents and voltages in the positive-sequence network will be different. The second method is preferable for an analytic solution, because the internal voltages of the various symmetrical machines on the system under load are not required. The voltage V_f under load, if not given, can usually be closely estimated by one familiar with the power system. The load currents can also be estimated and added to the currents due to the fault to obtain total currents.

In problems where part of the system is switched out of service following a fault, it is necessary to consider the internal voltages of the machines in both parts of the system. Problems of this nature frequently arise. For example, if one machine and a part of the system with an unsymmetrical fault are cut off from the rest of the system by switching, the currents and voltages in the fault and in the severed part of the system are determined by using the internal voltage of the machine and the sequence impedances of the part of the system connected to it.

The following problem illustrates the application of the second method to the analytic determination of initial symmetrical rms currents and voltages at the fault, negative- and zero-currents and voltages in the system, positive-sequence currents resulting from the fault, and phase currents in the system with load currents neglected.

Problem 2. A double line-to-ground fault occurs at B in the system shown by one-line diagram in Fig. 2(*a*). Determine the per unit initial symmetrical rms phase voltages and currents at the fault and the phase currents in the system resulting from the fault.

Solution. The operating conditions previous to the fault are not given; therefore load currents cannot be included. It will be assumed that the voltage V_f at B before the fault was equal to base voltage in the transmission circuit. Under this assumption V_f is unity in magnitude. This is a reasonable assumption. By the second method described above, currents and voltages in Figs. 10(*c*) or (*b*) vary directly with V_f; and therefore, if it is later learned that V_f differed from unity, the currents and voltages obtained with $V_f = 1$ can be accurately determined if they are multiplied by the known per unit value of V_f. Load currents can also be added later, if given.

With V_f as reference vector,

$$V_f = 1\,\underline{/0^\circ} = 1 + j0$$

The positive-sequence impedance viewed from the fault B in Fig. 2(*b*) is

$$Z_1 = \frac{\left(\dfrac{j1.0 \times j0.20}{j1.20} + j0.12\right)(j0.15 + j0.10 + j0.30)}{\left(\dfrac{j1.0 \times j0.20}{j1.20} + j0.12\right) + (j0.15 + j0.10 + j0.30)} = j0.188$$

The negative-sequence impedance viewed from the fault B in Fig. 2(*c*) is

$$Z_2 = \frac{\left(\dfrac{j1.0 \times j0.20}{j1.20} + j0.12\right)(j0.15 + j0.10 + j0.40)}{\left(\dfrac{j1.0 \times j0.20}{j1.20} + j0.12\right) + (j0.15 + j0.10 + j0.40)} = j0.199$$

The zero-sequence impedance viewed from the fault B in Fig. 2(*d*) is

$$Z_0 = \frac{(j0.12)(j0.70 + j0.10)}{(j0.12 + j0.70 + j0.10)} = j0.104$$

From [15], [16], and [18], the sequence components of the fault currents are

$$I_{a1} = \frac{V_f}{Z_1 + \dfrac{Z_0 Z_2}{Z_0 + Z_2}} = \frac{1.0 + j0}{j0.188 + \dfrac{j0.104 \times j0.199}{j0.303}} = -j3.900$$

$$I_{a2} = -I_{a1}\frac{Z_0}{Z_0 + Z_2} = j3.900\frac{j0.104}{j0.303} = j1.34$$

$$I_{a0} = -I_{a1}\frac{Z_2}{Z_0 + Z_2} = j3.900\frac{j0.199}{j0.303} = j2.56$$

Substituting I_{a1}, I_{a2}, and I_{a0} in [19], [29], and [30] of Chapter II, the three fault currents are

$$I_a = -j3.90 + 1.34 + j2.56 = 0$$

$$I_b = -\tfrac{1}{2}(-j3.90 + j1.34) - j\frac{\sqrt{3}}{2}(-j3.90 - j1.34) + j2.56 = -4.54 + j3.84$$

$$I_c = -\tfrac{1}{2}(-j3.90 + j1.34) + j\frac{\sqrt{3}}{2}(-j3.90 - j1.34) + j2.56 = 4.54 + j3.84$$

The ground current at the fault is

$$I_g = 3I_{a0} = j7.68$$

The sequence voltages at the fault from [17] are

$$V_{a1} = V_{a2} = V_{a0} = I_{a1}\frac{Z_0Z_2}{Z_0 + Z_2} = -j3.90 \times j0.0684 = 0.267 + j0$$

The phase voltages to ground at the fault are

$$V_a = V_{a0} + V_{a1} + V_{a2} = 3V_{a0} = 0.801 + j0$$

$$V_b = V_c = 0$$

The sequence currents due to the fault will flow from the right and left of B inversely in proportion to the respective impedances in the two directions.

Positive-sequence current distribution in Fig. 2 (*b*):

$$I_{a1} \text{ (from right of } B) = -j3.90 \times \frac{0.287}{0.837} = -j1.33$$

$$I_{a1} \text{ (from left of } B) = -j3.90 \times \frac{0.55}{0.837} = -j2.57$$

$$I_{a1} \text{ (from generator } G) = -j2.57 \times \frac{1.0}{1.20} = -j2.142$$

$$I_{a1} \text{ (from motor } M) = -j2.57 \times \frac{0.20}{1.20} = -j0.428$$

Negative-sequence current distribution in Fig. 2 (*c*):

$$I_{a2} \text{ (from right of } B) = j1.34 \times \frac{0.287}{0.937} = j0.41$$

$$I_{a2} \text{ (from left of } B) = j1.34 \times \frac{0.65}{0.937} = j0.93$$

$$I_{a2} \text{ (from generator } G) = j0.93 \times \frac{1.0}{1.20} = j0.775$$

$$I_{a2} \text{ (from motor } M) = j0.93 \times \frac{0.20}{1.20} = j0.155$$

Zero-sequence current distribution in Fig. 2 (*d*):

$$I_{a0} \text{ (from left of } B) = j2.56 \times \frac{0.80}{0.92} = j2.23$$

$$3I_{a0} = I_n \text{ in neutral connection } = j6.69$$

$$I_{a0} \text{ (from right of } B) = j2.56 \times \frac{0.12}{0.92} = j0.33$$

$$3I_{a0} = I_n \text{ in neutral connection } = j0.99$$

Figure 11 (*a*) shows the flow of sequence currents resulting from the fault, determined analytically by applying V_f as in Fig. 10 (*b*). Positive direction for current flow is indicated by arrows. If the direction of an arrow is reversed, the sign of the current to which the arrow is attached must also be reversed. The currents in Fig. 11 (*a*) are the *changes* in the sequence currents because of the fault. Combining the components of current, using [19], [29], and [30] of Chapter II, the phase currents are obtained.

The currents flowing toward B from the right are the currents which flow in the Y-connected windings of the transformer bank at C and through the transmission lines in parallel. They are

$$I_a = -j1.33 + j0.41 + j0.33 = -j0.59$$

$$I_b = -\tfrac{1}{2}(-j1.33 + j0.41) - j\frac{\sqrt{3}}{2}(-j1.33 - j0.41) + j0.33 = -1.51 + j0.79$$

$$I_c = -\tfrac{1}{2}(-j1.33 + j0.41) + j\frac{\sqrt{3}}{2}(-j1.33 - j0.41) + j0.33 = 1.51 + j0.79$$

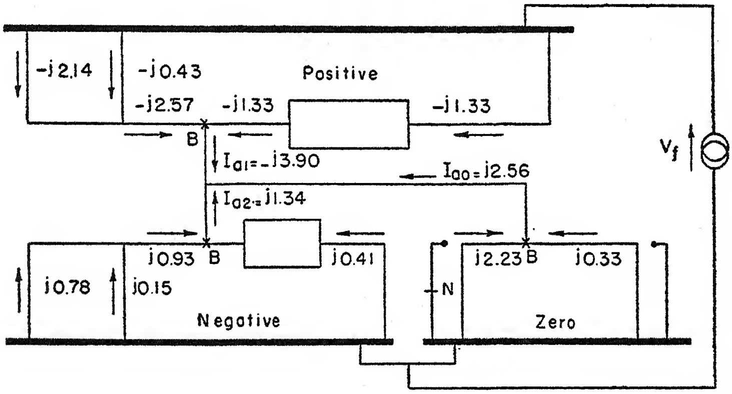
The currents flowing toward B from the left are the currents in the Y-connected windings of the transformer bank at B.

$$I_a = -j2.57 + j0.93 + j2.23 = j0.59$$

$$I_b = -\tfrac{1}{2}(-j2.57 + j0.93) - j\frac{\sqrt{3}}{2}(-j2.57 - j0.93) + j2.23 = -3.03 + j3.05$$

$$I_c = -\tfrac{1}{2}(-j2.57 + j0.93) + j\frac{\sqrt{3}}{2}(-j2.57 - j0.93) + j2.23 = 3.03 + j3.05$$

$$I_a + I_b + I_c = 3I_{a0} = j6.69$$



(a)

FIG. 11(*a*). Sequence currents for Problem 2. Double line-to-ground fault at B in system of Fig. 2 operating at no load.

There are no zero-sequence line currents on the Δ sides of the transformer bank. The positive- and negative-sequence currents of Fig. 11(*a*) in the three machines (assumed Y-connected) are given in terms of equivalent Y–Y transformer banks; the shift in phase in passing through the bank is not included. Turning the positive- and negative-sequence currents in the three machines, calculated on the basis of equivalent Y–Y transformer banks, 90° forward and 90° backward, respectively, the line currents are obtained. (See Chapter III.) In machine N:

$$I_{a1} = j(-j1.33) = 1.33$$
$$I_{a2} = -j(j0.41) = 0.41$$
$$I_a = 1.74$$
$$I_b = -\tfrac{1}{2}(1.33 + 0.41) - j\frac{\sqrt{3}}{2}(1.33 - 0.41) = -0.87 - j0.80$$
$$I_c = -0.87 + j0.80$$

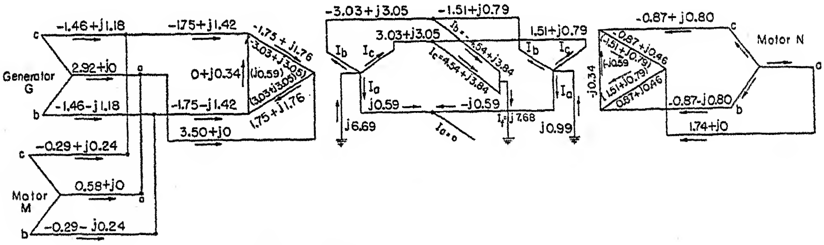
In machine G:

$$I_{a1} = j(-j2.142) = 2.142$$
$$I_{a2} = -j(j0.775) = 0.775$$
$$I_a = 2.92$$
$$I_b = -1.46 - j1.18$$
$$I_c = -1.46 + j1.18$$

In machine M:

$$I_{a1} = j(-j0.428) = 0.428$$
$$I_{a2} = -j(j0.155) = 0.155$$
$$I_a = 0.583 + j0$$
$$I_b = -0.29 - j0.236$$
$$I_c = -0.29 + j0.236$$

Currents in the three phases of the system of Fig. 2(a) with a double line-to-ground fault at B are shown in Fig. 11(b). Currents in Δ-connected windings of the transformer banks, in per unit of base current in the Δ circuits, are equal in magnitude and phase to the currents in the Y-connected windings in per unit of base current in the Y flowing in the opposite directions. If Δ currents are expressed in per unit of base Δ current and line currents in per unit of base line current, the sum of the per unit currents flowing into a Δ terminal will not add to zero. Per unit Δ currents are converted to per unit line currents if they are divided by $\sqrt{3}$; and per unit line currents to per unit Δ currents if they are multiplied by $\sqrt{3}$. In Fig. 11(b), Δ currents in per



-1.46+j1.18
c
Generator G
2.92+j0
a
b
-1.46-j1.18
-1.75+j1.42
-1.75+j1.76
-3.03+j3.05)
0+j0.34
(j0.59)
(3.03+j3.05)
-1.75-j1.42
1.75+j1.76
3.50+j0
-0.29+j0.24
c
Motor M
0.58+j0
a
b
-0.29-j0.24
-3.03+j3.05
-1.51+j0.79
3.03+j3.05
Ib
Ic
Ib=-4.54+j3.84
Ic=4.54+j3.84
Ia
j0.59
-j0.59
j6.69
Ia=0
If=j7.68
1.51+j0.79
Ib
Ic
Ia
j0.99
-j0.34
-0.87+j0.80
-0.87+j0.46
(-1.51+j0.79)
(-j0.59)
(1.51+j0.79)
0.87+j0.46
c
Motor N
a
-0.87-j0.80
b
1.74+j0

FIG. 11(b). Phase currents for Problem 2.

unit of base Δ currents are given in parentheses inside the Δ's; Δ currents in per unit of base line currents are given outside the Δ's. Using the latter, the sum of the currents flowing into a Δ terminal is zero. To satisfy this condition, currents must be expressed in amperes, or in per unit of the same base current.

FAULTS THROUGH IMPEDANCE

The most common type of ground faults occur on transmission circuits when there is a flashover between one or two conductors and a tower. In such cases, the impedances in the arc or arcs, the tower, and the tower footing influence the currents and voltages obtained. The impedance of the tower itself is negligible but the tower footing resistance, which is the resistance between the tower footing and true ground, may vary from a low value of 3 to 20 ohms in exceptionally moist earth, or when care has been taken to secure low resistance, up to 300 ohms or more in rocky soil where nothing has been done to lower the resistance. A line-to-line fault may occur directly between two conductors, or through an arc when there is a flashover between conductors not involving ground.

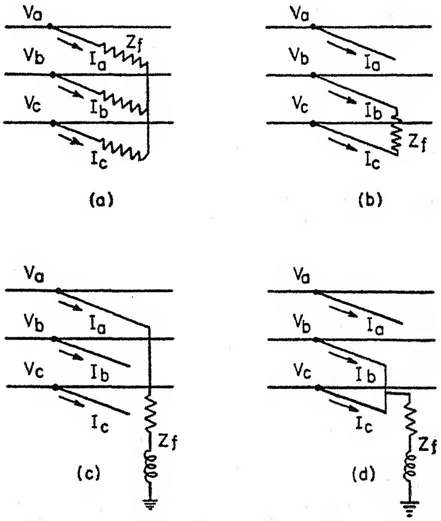


FIG. 12. Currents flowing into impedance faults. (*a*) Three-phase fault. (*b*) Line-to-line fault. (*c*) Single line-to-ground fault. (*d*) Double line-to-ground fault.

Figure 12 shows the currents I_a, I_b, and I_c flowing from the three phases *a*, *b*, and *c*, respectively, through hypothetical stub connections into the fault for the four types of faults. Z_f is the fault impedance. V_a, V_b, and V_c are the phase voltages to ground at the fault. In Fig. 12, the fault impedance is shown as an equal impedance in the three phases for a three-phase fault; for the line-to-line fault, the impedance is shown between conductors; for the single line-to-ground fault, it is between the conductor and ground; for the double line-to-ground fault, it is placed between the conductors and ground, the impedance between conductors being neglected. The fault impedances of Fig. 12 are practical, but others are possible. With fault impedances as indicated in Fig. 12, if V_f is the prefault voltage at the fault point F, Z_1, Z_2, and Z_0 the positive-, negative-, and zero-sequence impedances, respectively, viewed from the fault, and Z_f the fault impedance, the initial symmetrical rms currents flowing into the fault and the voltages to ground at the fault can be obtained in a manner similar to that used for faults through zero fault impedance.

Three-Phase Fault. In Fig. 12(a), the fault impedance Z_f is assumed equal in the three phases. The system, therefore, is not unbalanced by the three-phase fault, the currents and voltages remain of positive sequence, and

$$\begin{aligned} I_{a1} &= \frac{V_f}{Z_1 + Z_f} \\ V_{a1} &= I_{a1} Z_f \end{aligned} \qquad [19]$$

From [19] the equivalent circuit to replace the fault in the positive-sequence network is the fault impedance Z_f. This impedance is connected between the fault point and the zero-potential bus for the network as indicated in Fig. 13(a).

Line-to-Line Fault through Impedance Z_f. The fault conditions from Fig. 12(b) are

$$I_a = 0 \qquad I_b = -I_c \qquad V_b - V_c = I_b Z_f$$

Substituting $I_a = 0$ and $I_b = -I_c$ in [22]–[24] of Chapter II,

$$\begin{aligned} I_{a0} &= 0 \\ I_{a2} &= -I_{a1} \end{aligned} \qquad [20]$$

Replacing V_b and V_c in the fault equation by [8] and [9] of Chapter II, respectively,

$$V_b - V_c = (a^2 - a)V_{a1} - (a^2 - a)V_{a2} = I_b Z_f \qquad [21]$$

Replacing I_b in [21] by $a^2 I_{a1} + a I_{a2} = I_{a1}(a^2 - a)$, and dividing the equation by $(a^2 - a)$,

$$V_{a1} = V_{a2} + I_{a1} Z_f \qquad [22]$$

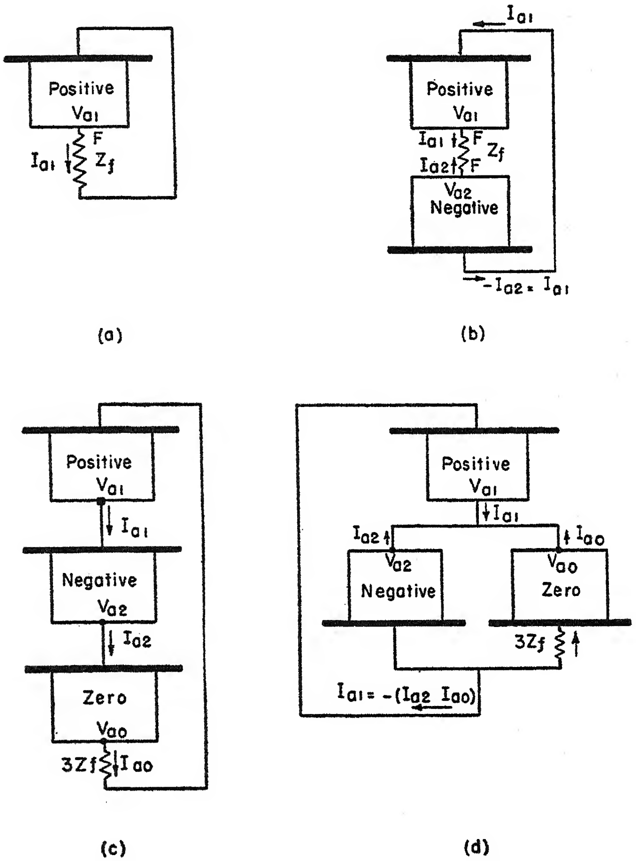
Replacing I_{a2} in [2] by $-I_{a1}$, and then substituting [2] in [22],

$$V_{a1} = I_{a1}(Z_2 + Z_f) \qquad [23]$$

Substituting [1] in [23], and solving for I_{a1},

$$I_{a1} = \frac{V_f}{Z_1 + Z_2 + Z_f} = -I_{a2} \qquad [24]$$

From [23], $V_{a1} = I_{a1}(Z_2 + Z_f)$. It follows, therefore, that the equivalent circuit to replace the fault in the positive-sequence network is the impedance $(Z_2 + Z_f)$, this impedance to be inserted between the fault point and the zero-potential bus for the network.


Positive
Va1
F
Ia1
Zf
Ia1
Positive
Va1
Ia1
F
Zf
Ia2
F
Va2
Negative
-Ia2 = Ia1
Positive
Va1
Ia1
Negative
Va2
Ia2
Zero
Va0
3Zf
Ia0
Positive
Va1
Ia1
Ia2
Ia0
Va2
Negative
Va0
Zero
3Zf
Ia1 = -(Ia2 Ia0)

(a) (b)

(c) (d)

FIG. 13. Connections of sequence networks for faults through impedance as indicated in Figs. 12 (*a*), (*b*), (*c*), and (*d*), respectively.

Instead of using an equivalent circuit in the positive-sequence network to obtain positive-sequence currents and voltages, the positive- and negative-sequence networks can be connected to satisfy [20] and [23], as shown in Fig. 13(*b*).

Line-to-Ground Fault through Impedance Z_f. From Fig. 12(*c*), the fault conditions are

$$I_b = 0 \qquad I_c = 0 \qquad V_a = I_a Z_f$$

Substituting $I_b = I_c = 0$ in [22]–[24] of Chapter II,

$$I_{a1} = I_{a2} = I_{a0} \tag{25}$$

$$I_a = I_{a1} + I_{a2} + I_{a0} = 3I_{a1}$$

$$V_a = V_{a1} + V_{a2} + V_{a0} = I_aZ_f = 3I_{a1}Z_f$$

Therefore

$$V_{a1} = -V_{a2} - V_{a0} + I_{a1}(3Z_f) \tag{26}$$

Substituting [2] and [3] in [26], and replacing I_{a2} and I_{a0} by I_{a1},

$$V_{a1} = I_{a1}(Z_2 + Z_0 + 3Z_f) \tag{27}$$

Substituting [1] in [27], and solving for I_{a1},

$$I_{a1} = \frac{V_f}{Z_1 + Z_2 + Z_0 + 3Z_f} \tag{28}$$

From [27], the equivalent circuit to replace the fault in the positive-sequence network is the impedance $(Z_2 + Z_0 + 3Z_f)$, this impedance to be inserted between the fault point and the zero-potential bus for the network.

The manner of connecting the sequence networks to satisfy [25] and [26] is shown in Fig. 13(*c*).

Double Line-to-Ground Fault through Impedance Z_f. From Fig. 12(*d*), the fault conditions are

$$I_a = 0 \qquad V_c = V_b = (I_b + I_c)Z_f$$

With $I_a = 0$,

$$I_{a1} + I_{a2} + I_{a0} = 0$$

or

$$I_{a1} = -(I_{a2} + I_{a0}) \tag{29}$$

Substituting $I_a = 0$ in [22] of Chapter II,

$$I_{a0} = \tfrac{1}{3}(I_b + I_c)$$

or

$$I_b + I_c = 3I_{a0} \tag{30}$$

With $V_b = V_c$ in [10]–[12] of Chapter II,

$$V_{a0} = \tfrac{1}{3}(V_a + 2V_b) \tag{31}$$

$$V_{a1} = V_{a2} = \tfrac{1}{3}(V_a - V_b) \tag{32}$$

Subtracting [32] from [31], and replacing V_b by $(I_b + I_c)Z_f = 3I_{a0}Z_f$,

$$V_{a0} - V_{a1} = V_b = 3I_{a0}Z_f$$

or

$$V_{a1} = V_{a0} - 3I_{a0}Z_f \tag{33}$$

Replacing V_{a2} and V_{a0} in [32] and [33] by their values from [2] and [3],

$$V_{a1} = V_{a2} = -I_{a2}Z_2$$

$$V_{a1} = -I_{a0}(Z_0 + 3Z_f)$$

Therefore

$$I_{a2} = -\frac{V_{a1}}{Z_2} \qquad [34]$$

and

$$I_{a0} = -\frac{V_{a1}}{Z_0 + 3Z_f} \qquad [35]$$

Substituting [34] and [35] in [29],

$$I_{a1} = V_{a1}\left[\frac{1}{Z_2} + \frac{1}{Z_0 + 3Z_f}\right]$$

Therefore

$$V_{a1} = I_{a1}\frac{Z_2(Z_0 + 3Z_f)}{Z_2 + Z_0 + 3Z_f} \qquad [36]$$

Substituting [36] in [34] and [35], I_{a2} and I_{a0} are expressed in terms of I_{a1}:

$$I_{a2} = -I_{a1}\frac{Z_0 + 3Z_f}{Z_2 + Z_0 + 3Z_f} \qquad [37]$$

$$I_{a0} = -I_{a1}\frac{Z_2}{Z_2 + Z_0 + 3Z_f} \qquad [38]$$

Replacing V_{a1} in [36] by its value from [1], and solving for I_{a1},

$$I_{a1} = \frac{V_f}{Z_1 + \dfrac{Z_2(Z_0 + 3Z_f)}{Z_2 + Z_0 + 3Z_f}} \qquad [39]$$

From [36], the equivalent circuit to replace the fault in the positive-sequence network is the impedance $Z_2(Z_0 + 3Z_f)/(Z_2 + Z_0 + 3Z_f)$, this impedance to be connected in the positive-sequence network between the fault point and the zero-potential bus for the network. In Fig. 13(*d*), the sequence networks are connected to satisfy equations [29], [32], and [33].

When the symmetrical components of current flowing into the fault and voltages to ground at the fault for the reference phase *a* have been determined for any type of fault, then by the use of [7]–[9] and [19]–[21] of Chapter II, the fault currents and voltages to ground at the fault

TABLE I

INITIAL SYMMETRICAL RMS FAULT CURRENTS AND VOLTAGES AT THE FAULT AND THEIR SYMMETRICAL COMPONENTS FOR VARIOUS TYPES OF FAULTS

The voltage to ground of phase *a* at the point of fault *F* before the fault occurred is V_f. The positive-, negative-, and zero-sequence impedances viewed from the fault are Z_1, Z_2, and Z_0, respectively. Z_f is the fault impedance. Z_{eq} is the equivalent impedance to replace the fault in the positive-sequence network.

	Three-phase fault through three-phase fault impedance, Z_f	Line-to-line fault. Phases *b* and *c* shorted through fault impedance, Z_f	Line-to-ground fault. Phase *a* grounded through fault impedance, Z_f	Double line-to-ground fault. Phases *b* and *c* shorted, then grounded through fault impedance, Z_f
I_{a1}	$I_{a1} = \dfrac{V_f}{Z_1 + Z_f}$	$I_{a1} = -I_{a2} = \dfrac{V_f}{Z_1 + Z_2 + Z_f}$	$I_{a1} = I_{a2} = I_{a0}$ $= \dfrac{V_f}{Z_0 + Z_1 + Z_2 + 3Z_f}$	$I_{a1} = -(I_{a2} + I_{a0})$ $= \dfrac{V_f}{Z_1 + \dfrac{Z_2(Z_0 + 3Z_f)}{Z_2 + Z_0 + 3Z_f}}$
I_{a2}	$I_{a2} = 0$	$I_{a2} = -I_{a1}$	$I_{a2} = I_{a1}$	$I_{a2} = -I_{a1} \dfrac{Z_0 + 3Z_f}{Z_2 + Z_0 + 3Z_f}$
I_{a0}	$I_{a0} = 0$	$I_{a0} = 0$	$I_{a0} = I_{a1}$	$I_{a0} = -I_{a1} \dfrac{Z_2}{Z_2 + Z_0 + 3Z_f}$
V_{a1}	$V_{a1} = I_{a1}Z_f$	$V_{a1} = V_{a2} + I_{a1}Z_f$ $= I_{a1}(Z_2 + Z_f)$	$V_{a1} = -(V_{a2} + V_{a0}) + I_{a1}(3Z_f)$ $= I_{a1}(Z_0 + Z_2 + 3Z_f)$	$V_{a1} = V_{a2} = V_{a0} - 3I_{a0}Z_f$ $= I_{a1} \dfrac{Z_2(Z_0 + 3Z_f)}{Z_2 + Z_0 + 3Z_f}$
V_{a2}	$V_{a2} = 0$	$V_{a2} = -I_{a2}Z_2 = I_{a1}Z_2$	$V_{a2} = -I_{a2}Z_2 = -I_{a1}Z_2$	$V_{a2} = -I_{a2}Z_2$ $= I_{a1} \dfrac{Z_2(Z_0 + 3Z_f)}{Z_2 + Z_0 + 3Z_f}$
V_{a0}	$V_{a0} = 0$	$V_{a0} = 0$	$V_{a0} = -I_{a0}Z_0$ $= -I_{a1}Z_0$	$V_{a0} = -I_{a0}Z_0$ $= I_{a1} \dfrac{Z_0Z_2}{Z_2 + Z_0 + 3Z_f}$
Z_{eg}	$Z_{eq} = Z_f$	$Z_{eq} = Z_2 + Z_f$	$Z_{eq} = Z_0 + Z_2 + 3Z_f$	$Z_{eq} = \dfrac{Z_2(Z_0 + 3Z_f)}{Z_2 + Z_0 + 3Z_f}$

TABLE I (*Continued*)

	Three-phase fault through three-phase fault impedance, Z_f	Line-to-line fault. Phases *b* and *c* shorted through fault impedance, Z_f	Line-to-ground fault. Phase *a* grounded through fault impedance, Z_f	Double line-to-ground fault. Phases *b* and *c* shorted, then grounded through fault impedance, Z_f
I_a	$\frac{V_f}{Z_1 + Z_f}$	0	$\frac{3V_f}{Z_0 + Z_1 + Z_2 + 3Z_f}$	0
I_b	$\frac{a^2V_f}{Z_1 + Z_f}$	$-j\sqrt{3}\frac{V_f}{Z_1 + Z_2 + Z_f}$	0	$-j\sqrt{3}V_f\frac{Z_0 + 3Z_f - aZ_2}{Z_1Z_2 + (Z_1 + Z_2)(Z_0 + 3Z_f)}$
I_c	$\frac{aV_f}{Z_1 + Z_f}$	$j\sqrt{3}\frac{V_f}{Z_1 + Z_2 + Z_f}$	0	$j\sqrt{3}V_f\frac{Z_0 + 3Z_f - a^2Z_2}{Z_1Z_2 + (Z_1 + Z_2)(Z_0 + 3Z_f)}$
V_a	$V_f\frac{Z_f}{Z_1 + Z_f}$	$V_f\frac{2Z_2 + Z_f}{Z_1 + Z_2 + Z_f}$	$V_f\frac{3Z_f}{Z_0 + Z_1 + Z_2 + 3Z_f}$	$V_f\frac{3Z_2(Z_0 + 2Z_f)}{Z_1Z_2 + (Z_1 + Z_2)(Z_0 + 3Z_f)}$
V_b	$V_f\frac{a^2Z_f}{Z_1 + Z_f}$	$V_f\frac{a^2Z_f - Z_2}{Z_1 + Z_2 + Z_f}$	$V_f\frac{3a^2Z_f - j\sqrt{3}(Z_2 - aZ_0)}{Z_0 + Z_1 + Z_2 + 3Z_f}$	$V_f\frac{-3Z_fZ_2}{Z_1Z_2 + (Z_1 + Z_2)(Z_0 + 3Z_f)}$
V_c	$V_f\frac{aZ_f}{Z_1 + Z_f}$	$V_f\frac{aZ_f - Z_2}{Z_1 + Z_2 + Z_f}$	$V_f\frac{3aZ_f + j\sqrt{3}(Z_2 - a^2Z_0)}{Z_0 + Z_1 + Z_2 + 3Z_f}$	$V_f\frac{-3Z_fZ_2}{Z_1Z_2 + (Z_1 + Z_2)(Z_0 + 3Z_f)}$
V_{bc}	$j\sqrt{3}V_f\frac{Z_f}{Z_1 + Z_f}$	$j\sqrt{3}V_f\frac{Z_f}{Z_1 + Z_2 + Z_f}$	$j\sqrt{3}V_f\frac{3Z_f + Z_0 + 2Z_2}{Z_0 + Z_1 + Z_2 + 3Z_f}$	0
V_{ca}	$j\sqrt{3}V_f\frac{a^2Z_f}{Z_1 + Z_f}$	$j\sqrt{3}V_f\frac{a^2Z_f - j\sqrt{3}Z_2}{Z_1 + Z_2 + Z_f}$	$j\sqrt{3}V_f\frac{a^2(3Z_f + Z_0) - Z_2}{Z_0 + Z_1 + Z_2 + 3Z_f}$	$\sqrt{3}V_f\frac{\sqrt{3}Z_2(Z_0 + 3Z_f)}{Z_1Z_2 + (Z_1 + Z_2)(Z_0 + 3Z_f)}$
V_{ab}	$j\sqrt{3}V_f\frac{aZ_f}{Z_1 + Z_f}$	$j\sqrt{3}V_f\frac{aZ_f + j\sqrt{3}Z_2}{Z_1 + Z_2 + Z_f}$	$j\sqrt{3}V_f\frac{a(3Z_f + Z_0) - Z_2}{Z_0 + Z_1 + Z_2 + 3Z_f}$	$-\sqrt{3}V_f\frac{\sqrt{3}Z_2(Z_0 + 3Z_f)}{Z_1Z_2 + (Z_1 + Z_2)(Z_0 + 3Z_f)}$

for the three phases can be obtained. The line-to-line voltages at the fault can be obtained from [11] of Chapter III, or by taking the differences of the line-to-ground voltages.

Fault Currents and Voltages from Formulas. For convenience in determining fault currents and voltages, Table I of this chapter has been prepared. This table gives I_{a1}, the positive-sequence component of initial symmetrical rms current flowing from the reference phase a into the fault, for various types of fault in terms of V_f, the voltage at the fault before the fault occurred, Z_1, Z_2, Z_0, the positive-, negative-, and zero-sequence impedances, respectively, viewed from the fault, and Z_f, the fault impedance. The other symmetrical components of fault current and the symmetrical components of voltage to ground at the fault are given in terms of I_{a1}. The currents flowing into the fault from the three phases and the line-to-ground and line-to-line voltages at the fault are expressed in terms of V_f, Z_1, Z_2, Z_0, and Z_f.

Table I can also be used for determining transient or sustained currents and voltages of fundamental frequency at the fault in cases where the rotors of the machines on the system do not materially change their relative angular positions. If this is the case, transient or equivalent steady-state impedance, respectively, is used for Z_1, the positive-sequence impedance viewed from the fault.

A study of Table I shows that the effect of fault impedance is to reduce the fault current. It may either increase or decrease fault voltages, depending upon its magnitude relative to the magnitudes of the sequence impedances and the amount of resistance in the system exclusive of the fault resistance.

Table I can be used for determining fault currents and voltages when there is no impedance in the fault if Z_f is replaced by zero.

OPEN CONDUCTORS

When circuits are controlled by fuses or any device which does not open all three phases, one or two phases of the circuit may be open while the other phases or phase is closed. This condition may also occur with one or two broken conductors. The case of a conductor breaking and one end falling to ground is considered in Chapters VII and X. If both ends fall to ground, the condition is that of a line-to-ground fault.

When the system impedances and admittances are constant, i.e., do not vary with the voltages and currents associated with them, the method of symmetrical components can be used to determine fundamental-frequency currents and voltages in the system with one or two open conductors. The exciting impedance of a transformer is an example of an impedance which is not constant, but varies with the applied voltage. In transformers under load, the exciting impedances can usually be neglected. An open conductor in a circuit supplying an unloaded or lightly loaded transformer bank is discussed in Vol. II. Constant system impedances and admittances are assumed in the following discussion.

One Open Conductor. Figure 14(*a*) shows a section of a three-phase system with phase *a* open between points P and Q. Let I_a, I_b, I_c be the *line currents* in phases *a*, *b*, *c*, respectively, with positive direction from P to Q; V_a, V_b, V_c, the voltages to ground at P, and V'_a, V'_b, V'_c the voltages to ground at Q. From Fig. 14(*a*),

$$I_a = 0$$

$$V_a - V'_a = v_a = \text{series voltage drop between } P \text{ and } Q \text{ in phase } a$$

$$V_b - V'_b = v_b = 0 = \text{series voltage drop between } P \text{ and } Q \text{ in phase } b$$

$$V_c - V'_c = v_c = 0 = \text{series voltage drop between } P \text{ and } Q \text{ in phase } c$$

In Chapter II, the fundamental symmetrical component equations for three-phase voltage and current vectors are given by [7]–[12] and [19]–[24]. It was pointed out that in these equations the voltage vectors V_a, V_b, and V_c and the current vectors I_a, I_b, and I_c can be used to represent any three voltage vectors and any three current vectors revolving at the same rate which are associated with the three phases of a three-phase system. These voltage equations will now be applied to v_a, v_b, and v_c, the series voltage drops between P and Q in phases *a*, *b*, and *c*, respectively, and the current equations to the line currents I_a, I_b, and I_c flowing in phases *a*, *b*, and *c*, respectively, from P to Q.

Resolving the series voltage drops v_a, v_b, v_c into their symmetrical components by [10]–[12] of Chapter II, with $v_b = v_c = 0$,

$$v_{a0} = V_{a0} - V'_{a0} = \tfrac{1}{3}(v_a + v_b + v_c) = \tfrac{1}{3}v_a$$

$$v_{a1} = V_{a1} - V'_{a1} = \tfrac{1}{3}(v_a + av_b + a^2v_c) = \tfrac{1}{3}v_a$$

$$v_{a2} = V_{a2} - V'_{a2} = \tfrac{1}{3}(v_a + a^2v_b + av_c) = \tfrac{1}{3}v_a$$

Therefore

$$v_{a0} = v_{a1} = v_{a2} \qquad [40]$$

Since $I_a = 0 = I_{a1} + I_{a2} + I_{a0}$,

$$I_{a1} = -(I_{a2} + I_{a0}) \qquad [41]$$

From [40], the open conductor introduces equal *series voltage drops* into each of the sequence networks at the opening in the direction of current flow, i.e., from P to Q. Stated in another way, the opening introduces equal series voltage rises in the three sequence networks in

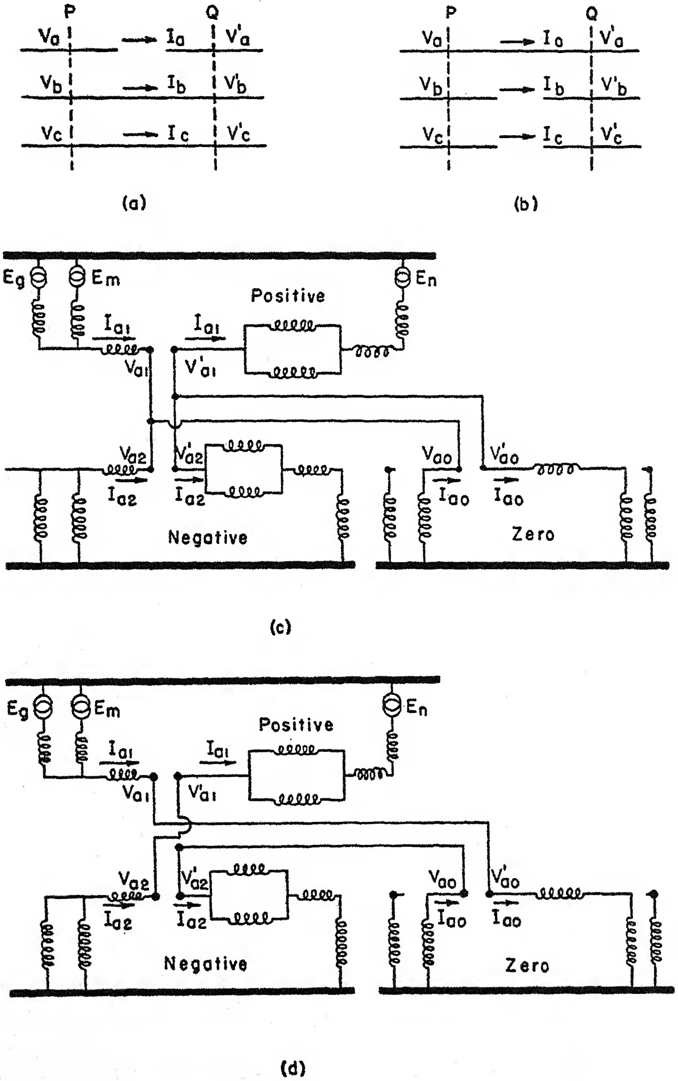

P
Q
Va
Ia
V'a
Vb
Ib
V'b
Vc
Ic
V'c
(a)
(b)
Eg
Em
En
Positive
Ia1
Va1
V'a1
Va2
V'a2
Ia2
Vao
V'ao
Iao
Negative
Zero
(c)
(d)

FIG. 14. Section of three-phase system between P and Q. (a) Phase a open. (b) Phases b and c open. Connection of sequence networks of Fig. 2 for open conductors at B. (c) Phase a open. (d) Phases b and c open.

the direction QP, which tend to send currents from Q to P. These generated voltages meet *series impedances*, as distinguished from impedances to neutral or to ground.

In a three-phase system with linear impedances, symmetrical except for an open conductor, let z_0 and z_2, respectively, represent the series impedances in the zero- and negative-sequence networks viewed from the opening. Consider, for example, the system shown by the one-line diagram of Fig. 2(*a*) with the sequence networks given by parts (*b*), (*c*), and (*d*) of Fig. 2. The negative-sequence series impedance viewed from any point B can be determined from Fig. 2(*c*) if a series voltage is inserted at B and the current read, the ratio of the series voltage at B to the current at B being the series impedance z_2 viewed from the opening at B. From Fig. 2(*c*), with the opening at B, C, or D,

$$z_2 = \left(j\,\frac{1.00 \times 0.20}{1.20} + j0.12 + j0.15 + j0.10 + j0.40\right) = j0.936$$

If the opening is at A in Fig. 2(*c*),

$$z_2 = j0.20 + j\,\frac{1.00(0.12 + 0.15 + 0.10 + 0.40)}{1.00 + 0.77} = j0.635$$

The zero-sequence series impedance z_0 is determined from Fig. 2(*d*) in a manner similar to that used to determine z_2. With the opening at A or D, $z_0 = \infty$. With the opening at B, or C,

$$z_0 = j(0.12 + 0.70 + 0.10) = j0.92$$

With balanced generated voltages, there are no negative- or zero-sequence voltages applied to the system. The only voltages causing negative- and zero-sequence currents to flow are those resulting from the opening. The sum of the voltage drops in any closed circuit is zero; therefore, the series negative- (or zero-) sequence voltage drop between P and Q plus the negative- (or zero-) sequence voltage drop in the negative- (or zero-) sequence network is zero. The equations are

$$v_{a0} + I_{a0}z_0 = 0 \qquad [42]$$

$$v_{a2} + I_{a2}z_2 = 0 \qquad [43]$$

From [40], [42], and [43],

$$I_{a0} = I_{a2}\,\frac{z_2}{z_0} \qquad [44]$$

From [41] and [44],

$$I_{a1} = -(I_{a2} + I_{a0}) = -\,\frac{z_0 + z_2}{z_0}\,I_{a2}$$

Therefore

$$I_{a2} = -\frac{z_0}{z_0 + z_2} I_{a1} \qquad [45]$$

$$I_{a0} = -\frac{z_2}{z_0 + z_2} I_{a1} \qquad [46]$$

From [40], [43], and [45],

$$v_{a1} = v_{a2} = v_{a0} = -I_{a2}z_2 = \frac{z_2 z_0}{z_0 + z_2} I_{a1} \qquad [47]$$

From [47] it follows that v_{a1}, the positive-sequence series voltage drop between P and Q, will be obtained if the impedance $z_2z_0/(z_0 + z_2)$ is inserted in the positive-sequence network between points P and Q. This impedance, which is the negative- and zero-sequence series impedances connected in parallel, is the equivalent circuit which replaces the opening in the positive-sequence network. Instead of inserting the lumped impedance $z_2z_0/(z_0 + z_2)$, the negative- and zero-sequence networks can be connected in parallel between P and Q in the positive-sequence network.

Figure 14(*c*) shows the connection of the sequence networks of Fig. 2 with one conductor open at point B for solution on the a-c network analyzer. This connection satisfies [40] and [41].

Two Open Conductors. Figure 14(*b*) shows a section of a three-phase system with phases b and c open between points P and Q. With the notation used for one open conductor,

$$I_b = I_c = 0$$

$$V_a - V_a' = v_a = 0$$

Then by resolving the line currents into their symmetrical components by equations [22]–[24] of Chapter II,

$$I_{a1} = I_{a2} = I_{a0} = \frac{I_a}{3} \qquad [48]$$

Since $v_a = 0$,

$$v_{a1} = -(v_{a2} + v_{a0}) \qquad [49]$$

Replacing v_{a2} and v_{a0} in [49] by their values $-I_{a0}z_0$ and $-I_{a2}z_2$ from [42] and [43], and substituting I_{a1} for I_{a2} and I_{a0},

$$v_{a1} = V_{a1} - V_{a1}' = -(v_{a2} + v_{a0}) = I_{a0}z_0 + I_{a2}z_2 = I_{a1}(z_0 + z_2) \qquad [50]$$

From [50], the positive-sequence series voltage drop v_{a1} across the opening will be obtained if the impedance $(z_0 + z_2)$ is inserted in the opening in the positive-sequence network. This impedance, which is the negative- and zero-sequence series impedances connected in series, is the equivalent circuit which replaces the opening in the positive-sequence network.

Figure 14(*d*) shows the connection of the sequence networks of Fig. 2 with two conductors open at *B* for solution on the a-c network analyzer. This connection satisfies [48] and [49]. If the zero-sequence series impedance viewed from the opening is infinite, the impedance across the opening will be infinite, and no currents will flow between *P* and *Q* when there are two open conductors.

Since one or two open conductors introduce an impedance in the positive-sequence network, the effect of the fault is to increase the transfer impedances between machines on opposite sides of the opening and therefore to decrease the power which can be transferred between them for given internal voltages with constant angular displacement. At the first instant, the rotors of the machines do not change their relative angular positions. Initial symmetrical rms currents and voltages of fundamental frequency can therefore be determined by using subtransient machine reactances in the positive-sequence network and the voltages behind these reactances corresponding to the operating conditions previous to the occurrence of the fault. When an analytic solution is made, the positive-sequence currents due to the internal voltage of each machine can be determined separately; then, by superposition, the total positive-sequence currents will be the sum of the several separate currents. From the positive-sequence line currents at *P* and *Q* (see Figs. 14(*a*) and (*b*)) the negative- and zero-sequence line currents at *P* and *Q* are obtained from [44] and [45] or from [48].

When an a-c calculating table is available, the negative- and zero-sequence series systems are connected in parallel for one open conductor and in series for two open conductors and inserted in the positive-sequence network at the point where the disturbance occurs, as in Figs. 14(*c*) and (*d*). The sequence line currents for phase *a* are then read directly.

When machines are operating at no load, all internal voltages are equal and in phase and no currents will flow. With the d-c calculating table, internal voltages are assumed in phase and of unit or 100% magnitude. It cannot therefore be used in the usual manner to determine the currents with one or two open conductors. But it can be used if the internal voltages in the machines are applied one at a time and the currents recorded, the currents so determined to be multiplied by the per unit vector values of the corresponding applied internal voltages and added vectorially in each of the sequence networks.

Following the opening of a conductor or two conductors, the machines will change their relative angular positions, depending upon their inertias and the difference between their mechanical inputs and electrical outputs. For example, if a generator is supplying a motor through a transmission circuit and one conductor is opened, the power received by the motor becomes less, and it will fall back in phase relative to the generator. If there is a phase displacement between generator and motor internal voltages at which the electrical input to the motor equals its mechanical output under sustained conditions (with steady-state impedances and the voltages behind them), the motor will remain in synchronism with the generator, provided it did not fall out of step during the transient disturbance. (See Problem 3.)

Single-Phase Switching. In the case of single-phase switching, a faulted conductor is cut out of service by opening it at both ends. If capacitance is negligible, there is no current in an open conductor, whether it is open at both ends or only at one point, and its effect upon the current and voltages of the other two conductors is the same. In fact, as far as the other two conductors are concerned, it would make no difference whether the open conductor were removed entirely or left in place.

The discussion given above for one and two open conductors can be applied to single-phase switching if capacitance is negligible and it is assumed that the conductor or conductors are open at one point only, chosen at some convenient location in the line. It must be remembered, however, that the voltage along the open conductor itself would depend upon the location of the opening.

Problem 3. In the system shown in Fig. 2, with motor M disconnected and base power numerically equal to base kva, 40% of base system power is supplied to motor N at unity power factor and unit voltage on the line side of the transformer at C.

(*a*) One lead on the line side of the transformer at B is accidentally disconnected.

(*b*) Two leads are disconnected.

Assuming the motor N and generator G do not lose synchronism during the transient disturbance, will they remain in synchronism after the disturbance is over? The equivalent steady-state positive-sequence reactances of generator and motor are approximately 55% and 100%, respectively, based on system kva and voltage. The load on the motor is a constant power load.

Solution. Expressed in per unit, with the voltage at C as reference vector,

$$V_c = 1 + j0$$

$$I = 0.4 + j0$$

The internal voltages in generator and motor behind equivalent steady-state reactances, determined in per unit from Fig. 2 (*b*) with motor M disconnected and per unit positive-sequence generator and motor reactances of 0.55 and 1.00, respectively, are

$$E_g = 1 + (0.4)(j0.82) = 1 + j0.328 = 1.052\underline{/18.2^\circ}$$
$$E_n = 1 - (0.4)(j1.10) = 1 - j0.440 = 1.097\overline{\backslash 23.8^\circ}$$

The series negative- and zero-sequence impedances z_2 and z_0, determined from Figs. 2 (*c*) and (*d*) with motor *M* disconnected, are

$$z_2 = j(0.20 + 0.12 + 0.15 + 0.10 + 0.40) = j0.97$$
$$z_0 = j0.92$$

The equivalent impedances to replace one and two open conductors in the positive-sequence network are:

(*a*) With one open phase at *B*,

$$\frac{z_2 z_0}{z_2 + z_0} = j\frac{0.97 \times 0.92}{1.89} = j0.47$$

(*b*) With two open phases open at *B*,

$$z_2 + z_0 = j(0.97 + 0.92) = j1.89$$

During normal operation the transfer impedance (see Chapter I) in the positive-sequence network between internal voltages behind steady-state impedances in machines *G* and *N* is

$$Z_{gn} = j1.92$$

(*a*) With one phase open, the transfer impedance is

$$Z_{gn} = j(1.92 + 0.47) = j2.39$$

(*b*) With two phases open, it is

$$Z_{gn} = j(1.92 + 1.89) = j3.81$$

With the transfer impedance increased, the angle between the internal machine voltages must be increased to transfer the same power. With resistance neglected, there is no power loss in the negative- and zero-sequence networks and the power out of the generator and into the motor is determined from the positive-sequence network.

Let δ = angle between E_g and $E_n = \delta_g - \delta_n$; then, with E_n as reference vector,

$$E_n = 1.097\underline{/0^\circ} = 1.097$$
$$E_g = 1.052\underline{/\delta} = 1.052\ (\cos\delta + j\sin\delta)$$

The current *I*, flowing from the generator into the motor, is

$$I = \frac{E_g - E_n}{jx_{gn}} = \frac{1.052\sin\delta}{x_{gn}} + j\frac{(1.097 - 1.052\cos\delta)}{x_{gn}}$$

The power component of *I* referred to E_n is

$$\frac{1.052\sin\delta}{x_{gn}}$$

The power delivered by the generator and received by the motor with no resistance in the circuit is

$$P = (1.097)\frac{1.052 \sin \delta}{x_{gn}} = \frac{1.154 \sin \delta}{x_{gn}}$$

The constant per unit power load on the motor is 0.4. To deliver this power under steady-state conditions:

(*a*) With one phase open at B,

$$P = 0.4 = \frac{1.154 \sin \delta}{2.39} = 0.483 \sin \delta$$

$$\delta = \sin^{-1}\frac{0.4}{0.483} = 56.0°$$

If the motor and generator do not lose synchronism during the transient disturbance they will remain in synchronism and the constant per unit power load of 0.4 will be delivered to the motor; the angle between internal voltages of generator and motor will be 56° with one phase open at B.

(*b*) With two phases open at B,

$$P = 0.4 = \frac{1.154 \sin \delta}{3.81} = 0.303 \sin \delta$$

$$\delta = \sin^{-1}\frac{0.4}{0.303} = \sin^{-1} 1.32$$

Sin δ cannot be 1.32; and therefore, assuming the machines did not lose synchronism during the transient disturbance, their rotors cannot take up relative positions such that the power supplied to the motor is equal to its mechanical load. The machines lose synchronism.

Problem 4. The given system is shown in Fig. 2(*a*) with sequence networks in Figs. 2(*b*), (*c*), and (*d*). Determine the currents flowing from the three phases into fault and the phase voltages to ground at the fault for (1) a line-to-ground fault at A, (2) a double line-to-ground fault at C, (3) a line-to-line fault at D.

Problem 5. Solve Problem 2 for a line-to-ground fault at D.

CHAPTER V

TWO COMPONENT NETWORKS FOR THREE-PHASE SYSTEMS

Equations for Voltages and Currents in a Three-Phase System. The equations for the phase voltages to ground and line currents in terms of the symmetrical components of voltage and current, respectively, of phase a developed in Chapter II are

$$V_a = V_{a1} + V_{a2} + V_{a0} = (V_{a1} + V_{a2}) + V_{a0} \qquad [1]$$

$$\begin{aligned} V_b &= a^2 V_{a1} + a V_{a2} + V_{a0} \\ &= -\tfrac{1}{2}(V_{a1} + V_{a2}) - j\frac{\sqrt{3}}{2}(V_{a1} - V_{a2}) + V_{a0} \end{aligned} \qquad [2]$$

$$\begin{aligned} V_c &= a V_{a1} + a^2 V_{a2} + V_{a0} \\ &= -\tfrac{1}{2}(V_{a1} + V_{a2}) + j\frac{\sqrt{3}}{2}(V_{a1} - V_{a2}) + V_{a0} \end{aligned} \qquad [3]$$

$$I_a = I_{a1} + I_{a2} + I_{a0} = (I_{a1} + I_{a2}) + I_{a0} \qquad [4]$$

$$\begin{aligned} I_b &= a^2 I_{a1} + a I_{a2} + I_{a0} \\ &= -\tfrac{1}{2}(I_{a1} + I_{a2}) - j\frac{\sqrt{3}}{2}(I_{a1} - I_{a2}) + I_{a0} \end{aligned} \qquad [5]$$

$$\begin{aligned} I_c &= a I_{a1} + a^2 I_{a2} + I_{a0} \\ &= -\tfrac{1}{2}(I_{a1} + I_{a2}) + j\frac{\sqrt{3}}{2}(I_{a1} - I_{a2}) + I_{a0} \end{aligned} \qquad [6]$$

where V and I refer to voltage and current, subscripts a, b, and c refer to the three phases a, b, and c, and 0, 1, 2, to zero, positive, and negative sequence, respectively. The operator $a = \left(-\tfrac{1}{2} + j\frac{\sqrt{3}}{2}\right) = 1\underline{/120^\circ}$; $a^2 = \left(-\tfrac{1}{2} - j\frac{\sqrt{3}}{2}\right) = 1\overline{/120^\circ}$.

In [1]–[6] phase voltages to ground and line currents are expressed in terms of the symmetrical components of phase a, selected as reference phase, and also in terms of positive-plus-negative, positive-minus-negative, and zero-sequence components* of phase a. Equations [1]–[6] provide a simple transition from symmetrical components to other related components and can be used to advantage when there is a single dissymmetry, such as an unsymmetrical fault, provided the positive- and negative-sequence impedances in the system can be assumed equal. This assumption, while never strictly true for rotating machines, in many cases leads to but slight error when the impedances involved are transmission lines and transformers in series with rotating machines.

POSITIVE-PLUS-NEGATIVE, POSITIVE-MINUS-NEGATIVE, AND ZERO-SEQUENCE COMPONENTS OF CURRENT AND VOLTAGE

The use of the sum and difference of the positive- and negative-sequence components of currents and voltages, together with zero-sequence components of current and voltage, will be illustrated for the case of unsymmetrical short circuits on an otherwise symmetrical system in which the positive- and negative-sequence impedances are assumed equal; and the connections between the component networks to satisfy fault conditions will be determined for the three types of unsymmetrical short circuits.

Unsymmetrical Short Circuits. In a balanced three-phase power system, neither negative- nor zero-sequence voltages are generated. Assuming a system, balanced before the fault, let V_f represent the prefault voltage of phase a at the fault point F. Let I_a, I_b, and I_c be the currents from phases a, b, and c, respectively, flowing into the fault; and V_a, V_b, and V_c the voltages to ground of phases a, b, and c at the fault. If Z_1, Z_2, and Z_0 are the positive-, negative-, and zero-sequence impedances, respectively, viewed from the fault, then the positive-, negative-, and zero-sequence components of the voltage of phase a at the fault, with $Z_2 = Z_1$, will be

$$V_{a1} = V_f - I_{a1}Z_1 \qquad [7]$$

$$V_{a2} = \quad - I_{a2}Z_1 \qquad [8]$$

$$V_{a0} = \quad - I_{a0}Z_0 \qquad [9]$$

where positive direction for the symmetrical components of current is from the system into the fault.

Equations [7]–[9] are the same as [1]–[3] of Chapter IV except that Z_2 has been replaced by Z_1. V_f in [7] represents the prefault voltage of phase a at F for any given operating condition. If the system is operating at no load, neglecting charging current, the prefault voltage V_f in per cent will be the internal generated voltage E_a of phase a, which is the same in all synchronous machines of the system. Under load, V_f is determined by the internal generated voltages and the load distribution in the system corresponding to the given operating condition.

---

* Determination of Voltages and Currents during Unbalanced Faults, by Edith Clarke, *Gen. Elec. Rev.*, November, 1937, pp. 511–513.

Adding equations [7] and [8], and subtracting [8] from [7],

$$(V_{a1} + V_{a2}) = V_f - (I_{a1} + I_{a2})Z_1 \tag{10}$$

$$(V_{a1} - V_{a2}) = V_f - (I_{a1} - I_{a2})Z_1 \tag{11}$$

Equations [10] and [11] express the positive-plus-negative and positive-minus-negative components of voltage at the fault in terms of their respective components of fault current, prefault voltage, and impedance viewed from the fault. V_f is a positive-sequence voltage: as no negative-sequence voltages are generated, both positive-plus-negative and positive-minus-negative *generated voltages* are positive-sequence voltages. From [10] and [11], the impedance met by both $(I_{a1} + I_{a2})$ and $(I_{a1} - I_{a2})$ is Z_1, a positive-sequence impedance: in a system with equal positive- and negative-sequence impedances, the impedances met by currents consisting of positive- and negative-sequence components are positive-sequence impedances. With the same generated voltages and the same impedances, the positive-plus-negative network and the positive-minus-negative network are identical; and each is the same as the positive-sequence network. The positive-sequence network can therefore be used to determine both the positive-plus-negative and the positive-minus-negative components of current and voltage, provided the proper connections are made at the fault point. The positive-sequence network, representing both the positive-plus-negative and the positive-minus-negative networks, will be used twice with different connections at the fault. The use of the sum and difference of the positive- and negative-sequence components, instead of the positive- and negative-sequence components themselves, is of special advantage when a short-circuit study is made on a d-c calculating table or an a-c network analyzer, because it is necessary to set up only the positive- and zero-sequence networks, and numerical calculations do not require the use of the operator a. This allows a larger system to be set up on a given calculating table and simplifies numerical work.

Line-to-Ground Fault. Conditions at the fault (fault on phase a):

$$V_a = 0 \tag{12}$$

$$I_b = I_c = 0 \tag{13}$$

Substituting [12] in [1] and replacing I_b and I_c in [5] and [6] by zero, the following relations between the components at the fault are obtained:

$$(V_{a1} + V_{a2}) = -V_{a0} \tag{14}$$

$$(I_{a1} + I_{a2}) = 2I_{a0} \tag{15}$$

$$(I_{a1} - I_{a2}) = 0 \tag{16}$$

Substituting [16] in [11],

$$V_{a1} - V_{a2} = V_f \tag{17}$$

Replacing I_{a0} in [9] by $\frac{1}{2}(I_{a1} + I_{a2})$ from [15], then substituting [9] in [14],

$$(V_{a1} + V_{a2}) = -V_{a0} = (I_{a1} + I_{a2})\frac{Z_0}{2} \tag{18}$$

Equating $(V_{a1} + V_{a2})$ in [18] and [10] and solving for $(I_{a1} + I_{a2})$,

$$(I_{a1} + I_{a2}) = 2I_{a0} = \frac{V_f}{Z_1 + \frac{Z_0}{2}} = \frac{2V_f}{2Z_1 + Z_0} \tag{19}$$

Phase Currents and Voltages at the Fault. From the equation above, and [1]–[6], the currents flowing from the three phases into the fault and the line-to-ground voltages at the fault are

$$\begin{aligned} I_a &= 3I_{a0} = \frac{3V_f}{2Z_1 + Z_0} \\ I_b &= I_c = 0; \quad V_a = 0 \\ V_b &= \tfrac{3}{2}V_{a0} - j\frac{\sqrt{3}}{2}V_f = V_f\left[-\frac{3}{2}\left(\frac{Z_0}{2Z_1 + Z_0}\right) - j\frac{\sqrt{3}}{2}\right] \\ V_c &= \tfrac{3}{2}V_{a0} + j\frac{\sqrt{3}}{2}V_f = V_f\left[-\frac{3}{2}\left(\frac{Z_0}{2Z_1 + Z_0}\right) + j\frac{\sqrt{3}}{2}\right] \end{aligned} \tag{20}$$

where V_f is the prefault line-to-neutral voltage of phase a at the fault point F.

Equivalent Circuit for Line-to-Ground Fault. Dividing [18] by $(I_{a1} + I_{a2}) = 2I_{a0}$, and replacing V_{a0} by $-I_{a0}Z_0$ from [9],

$$\frac{(V_{a1} + V_{a2})}{(I_{a1} + I_{a2})} = \frac{-V_{a0}}{2I_{a0}} = \frac{Z_0}{2} \tag{21}$$

In [21], $(V_{a1} + V_{a2})$ is the voltage at the point of fault in the positive-plus-negative network, and $(I_{a1} + I_{a2})$ is the current flowing from this network into the fault. The impedance therefore to be placed between the fault point and the zero-potential bus for the network to give the correct ratio of $(V_{a1} + V_{a2})$ to $(I_{a1} + I_{a2})$ is $Z_0/2$. Instead of the lumped impedance $Z_0/2$, the zero-sequence network with all zero-sequence impedances divided by two can be used as the equivalent circuit. The current flowing into the equivalent circuit is $(I_{a1} + I_{a2})$, which from [15] is $2I_{a0}$. Twice zero-sequence current flowing through one-half zero-sequence impedance produces zero-sequence voltage.

If all zero-sequence impedances are divided by two, and the network then connected in series with the positive-plus-negative network (which is also the positive-sequence network) as in Fig. 1, an equivalent circuit is obtained which satisfies [18] and [19] at the fault. Part (*a*) of Fig. 1 gives positive-plus-negative components of current and voltage. The zero-sequence voltages and twice the zero-sequence currents are given by part (*b*) of Fig. 1. When an a-c network analyzer is used, positive-plus-negative currents and voltages, and zero-sequence voltages and twice zero-sequence currents, can be read directly from parts (*a*) and (*b*), respectively, of Fig. 1 at any point *P* in the system. If the three-phase system was operating under load before the fault, and the generated voltages were adjusted to give the operating conditions

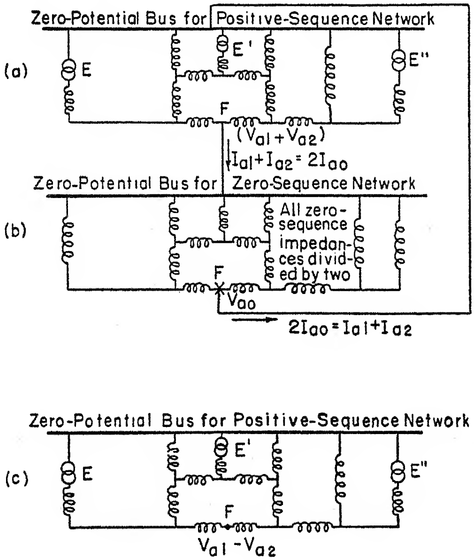

Zero-Potential Bus for Positive-Sequence Network
(a)
E
E'
E''
F
$(V_{a1}+V_{a2})$
$I_{a1}+I_{a2}=2I_{a0}$
Zero-Potential Bus for Zero-Sequence Network
(b)
All zero-sequence impedances divided by two
F
V_{a0}
$2I_{a0}=I_{a1}+I_{a2}$
Zero-Potential Bus for Positive-Sequence Network
(c)
E
E'
E''
F
$V_{a1}-V_{a2}$

FIG. 1. Equivalent circuit for line-to-ground fault. (*a*) Positive-plus-negative network. (*b*) Zero-sequence network with all impedances divided by two. (*c*) Positive-minus-negative network. The positive-sequence network is used for both (*a*) and (*c*).

which existed before the fault, the effect of load at any point in the system is included in part (*a*) of Fig. 1.

From [16] and [17], $(I_{a1} - I_{a2}) = 0$ and $(V_{a1} - V_{a2}) = V_f$. There is no current flowing into the fault from the positive-minus-negative network, and no change in voltage at the fault; consequently, there are no currents and no voltage drops in the network resulting from the fault. The positive-minus-negative network, which is unaffected by the line-to-ground fault, is shown in part (*c*) of Fig. 1. It is the positive-sequence network before the fault occurred. If load currents are considered, there will be currents in the network and voltage drops due to these currents. Load currents under normal operation are positive-sequence currents and cause positive-sequence voltage drops. They are determined from the positive-sequence network before the fault occurs, with generated voltages adjusted to give the operating conditions which existed before the fault.

At any point P in the system, the positive-minus-negative components of current and voltage are

$$(I_{a1} - I_{a2}) \text{ at } P = I_a \text{ at } P \text{ before the fault} \qquad [22]$$

$$(V_{a1} - V_{a2}) \text{ at } P = V_a \text{ at } P \text{ before the fault} \qquad [23]$$

If load currents are neglected and there is no capacitance,

$$(I_{a1} - I_{a2}) \text{ at } P = 0 \qquad [24]$$

Substituting zero-sequence components and the sum and difference of the positive- and negative-sequence components of current and voltage in [1]–[6], phase currents and voltages at any point P in the system are obtained.

Line-to-Line Fault. Conditions at the fault (fault on phases b and c):

$$I_a = 0 \qquad [25]$$

$$I_b = -I_c \qquad [26]$$

$$V_b = V_c \qquad [27]$$

From [25]–[27] and [2]–[6], the relations between the components at the fault are

$$I_{a0} = 0 \qquad [28]$$

$$(I_{a1} + I_{a2}) = 0 \qquad [29]$$

$$(V_{a1} - V_{a2}) = 0 \qquad [30]$$

Substituting [28] in [9], [29] in [10], and [30] in [11],

$$V_{a0} = 0 \tag{31}$$

$$(V_{a1} + V_{a2}) = V_f \tag{32}$$

$$(I_{a1} - I_{a2}) = \frac{V_f}{Z_1} \tag{33}$$

Phase Currents and Voltages at the Fault. Substituting [28]–[33] in [1]–[6], the phase currents flowing into the fault and the phase voltages to ground at the fault are

$$\left.\begin{aligned} I_a &= 0 \\ I_b &= -j\frac{\sqrt{3}}{2}\left(\frac{V_f}{Z_1}\right) \\ I_c &= j\frac{\sqrt{3}}{2}\left(\frac{V_f}{Z_1}\right) \\ V_a &= V_f \\ V_b &= V_c = -\tfrac{1}{2}V_f \end{aligned}\right. \tag{34}$$

Equivalent Circuit for Line-to-Line Fault. From [28] and [31], $I_{a0} = 0$ and $V_{a0} = 0$; therefore the zero-sequence network is not involved. From [30] and [33], $V_{a1} - V_{a2} = 0$ and $(I_{a1} - I_{a2}) = V_f/Z_1$. These two equations are satisfied if in the positive-minus-negative network (which is also the positive-sequence network) the fault point F is shorted to the zero-potential bus as in Fig. 2, part (a). It will be noted that with a three-phase fault, $V_{a1} = 0$ and $I_{a1} = V_f/Z_1$. The positive-minus-negative components of current and voltage for a line-to-line fault between phases b and c with phase a as reference phase are just the same as those of phase a for the three-phase short circuit. Positive-minus-negative currents and volt-

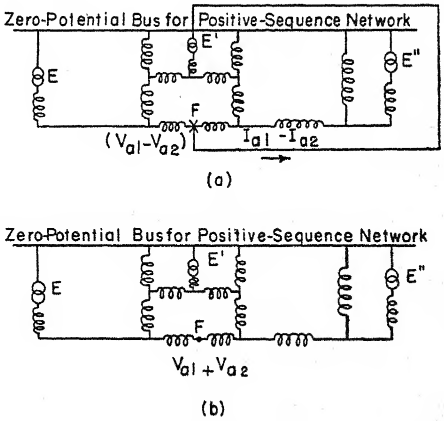

Zero-Potential Bus for Positive-Sequence Network
E
E'
E"
F
(Va1-Va2)
Ia1 -Ia2
(a)
Zero-Potential Bus for Positive-Sequence Network
E
E'
E"
F
Va1 +Va2
(b)

FIG. 2. Equivalent circuit for line-to-line fault. (a) Positive-minus-negative network. (b) Positive-plus-negative network. The positive-sequence network is used for both (a) and (b).

ages at any point in the system can be calculated from Fig. 2(*a*), or read directly if a calculating table is used.

From [29] and [32], $(I_{a1} + I_{a2}) = 0$ and $V_{a1} + V_{a2} = V_f$ at the fault. The positive-plus-negative network (which is also the positive-sequence network) is unaffected by the line-to-line fault. This is shown in part (*b*) of Fig. 2. At any point P in the system, the positive-plus-negative components of currents and voltages are

$$(I_{a1} + I_{a2}) \text{ at } P = I_a \text{ at } P \text{ before the fault} \qquad [35]$$

$$(V_{a1} + V_{a2}) = V_a \text{ at } P \text{ before the fault} \qquad [36]$$

If loads are neglected and there is no capacitance,

$$(I_{a1} + I_{a2}) \text{ at } P = 0 \qquad [37]$$

Substituting $V_{a0} = 0$, $I_{a0} = 0$, and the sum and difference of the positive- and negative-sequence components of current and voltage in [1]–[6], phase currents and voltages at any desired point P of the system are obtained.

Double Line-to-Ground Fault. Conditions at the fault (fault on phases b and c):

$$I_a = 0 \qquad [38]$$

$$V_b = V_c = 0 \qquad [39]$$

Substituting [38] in [4], and [39] in [2] and [3], the following relations between the components at the fault are obtained:

$$(I_{a1} + I_{a2}) = -I_{a0} \qquad [40]$$

$$(V_{a1} + V_{a2}) = 2V_{a0} \qquad [41]$$

$$(V_{a1} - V_{a2}) = 0 \qquad [42]$$

Substituting [42] in [11],

$$(I_{a1} - I_{a2}) = \frac{V_f}{Z_1} \qquad [43]$$

Replacing $-I_{a0}$ in [9] by $(I_{a1} + I_{a2})$ given in [40], and substituting [9] in [41],

$$(V_{a1} + V_{a2}) = 2V_{a0} = -I_{a0}(2Z_0) = (I_{a1} + I_{a2})(2Z_0) \qquad [44]$$

Solving [10] and [44] for $(I_{a1} + I_{a2})$,

$$(I_{a1} + I_{a2}) = -I_{a0} = \frac{V_f}{Z_1 + 2Z_0} \qquad [45]$$

Phase Currents and Voltages at the Fault. From the above equation and [1]–[6], the line currents flowing into the fault and the voltages to ground at the fault are

$$
\begin{aligned}
I_a &= 0 \\
I_b &= \tfrac{3}{2} I_{a0} - j\frac{\sqrt{3}}{2}\left(\frac{V_f}{Z_1}\right) = V_f\left[-\frac{\frac{3}{2}}{Z_1 + 2Z_0} - j\frac{\sqrt{3}}{2Z_1}\right] \\
I_c &= \tfrac{3}{2} I_{a0} + j\frac{\sqrt{3}}{2}\left(\frac{V_f}{Z_1}\right) = V_f\left[-\frac{\frac{3}{2}}{Z_1 + 2Z_0} + j\frac{\sqrt{3}}{2Z_1}\right] \qquad [46] \\
V_a &= 3V_{a0} = V_f\left(\frac{3Z_0}{Z_1 + 2Z_0}\right) \\
V_b &= V_c = 0
\end{aligned}
$$

Equivalent Circuit for Double Line-to-Ground Fault. Dividing [44] by $(I_{a1} + I_{a2}) = -I_{a0}$, and replacing V_{a0} by $-I_{a0}Z_0$ from [9],

$$\frac{(V_{a1} + V_{a2})}{(I_{a1} + I_{a2})} = \frac{2V_{a0}}{-I_{a0}} = 2Z_0 \qquad [47]$$

From [47], the equivalent circuit to be placed between the fault point and the zero-potential bus of the positive-plus-negative network to give the correct ratio of $(V_{a1} + V_{a2})$ to $(I_{a1} + I_{a2})$ is $2Z_0$. The zero-sequence network with all impedances multiplied by two can be used as the equivalent circuit. The current $(I_{a1} + I_{a2})$ flowing into the equivalent circuit from [40] is $-I_{a0}$. Zero-sequence current flowing through twice zero-sequence impedance produces twice zero-sequence voltage. The (arbitrary) positive direction for phase current and their components is from the network into the fault; therefore the zero-sequence network must be connected so that $(I_{a1} + I_{a2})$ flowing from the positive-plus-negative network into the fault, traverses the zero-sequence network in the negative direction.

If all zero-sequence impedances are multiplied by two and the network then connected in series with the positive-plus-negative network as in Figs. 3(*a*) and (*b*), an equivalent circuit is obtained which satisfies [40], [41], and [47] at the fault. If a calculating table is used, at any point *P* of the system, the positive-plus-negative components of current and voltage and twice the zero-sequence voltage and the zero-sequence current can be read directly from parts (*a*) and (*b*), respectively, of Fig. 3.

From [42] and [43], $(V_{a1} - V_{a2}) = 0$ and $(I_{a1} - I_{a2}) = V_f/Z_1$. This condition is satisfied if the fault point in the positive-minus-negative network is shorted to the zero-potential bus of the network, as in Fig. 3(*c*). Figure 3(*c*) is just the same as Fig. 2(*a*). Positive-minus-negative currents and voltages at any point *P* of the system can be calculated from Fig. 3(*c*), or read directly if a calculating table is used.

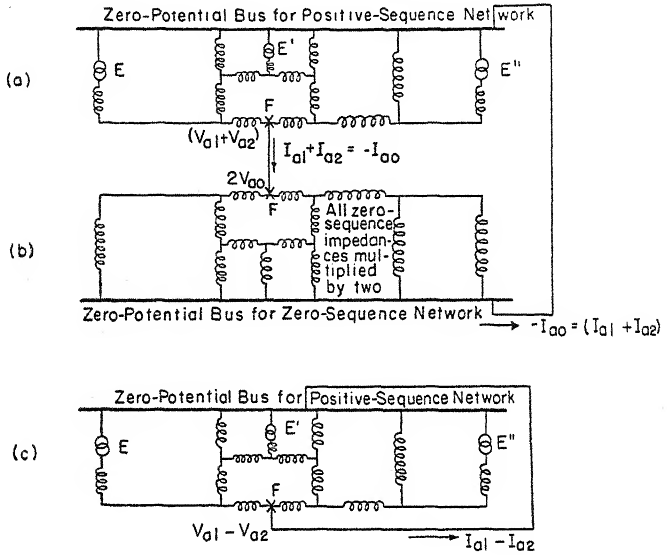

Zero-Potential Bus for Positive-Sequence Network
(a)
E
E'
E''
F
$(V_{a1}+V_{a2})$
$I_{a1}+I_{a2} = -I_{a0}$
$2V_{a0}$
F
All zero-sequence impedances multiplied by two
(b)
Zero-Potential Bus for Zero-Sequence Network
$-I_{a0} = (I_{a1}+I_{a2})$
Zero-Potential Bus for Positive-Sequence Network
(c)
E
E'
E''
F
$V_{a1}-V_{a2}$
$I_{a1}-I_{a2}$

FIG. 3. Equivalent circuit for double line-to-ground fault. (*a*) Positive-plus-negative network. (*b*) Zero-sequence network with all impedances multiplied by two. (*c*) Positive-minus-negative network. The positive-sequence network is used for both (*a*) and (*c*).

Substituting zero-sequence components and the sum and difference of the positive- and negative-sequence components of current and voltage in [1]–[6], phase currents and voltages at any point *P* in the system are obtained.

Average Power during Unsymmetrical Faults. The average power at any point in a system in per unit of three-phase base power (see Chapter II) is

$$\begin{aligned} P &= V_{a1}\cdot I_{a1} + V_{a2}\cdot I_{a2} + V_{a0}\cdot I_{a0} \\ &= \tfrac{1}{2}[(V_{a1}+V_{a2})\cdot(I_{a1}+I_{a2}) + (V_{a1}-V_{a2})\cdot(I_{a1}-I_{a2})] \\ &\qquad + V_{a0}\cdot I_{a0} \end{aligned} \tag{48}$$

From [48], it follows that the average power at any given point in a three-phase system is the zero-sequence power at the point plus one-half the sum of the power in the positive-plus-negative and the positive-minus-negative networks at the given point.

Problem 1. Determine the connection of the positive-plus-negative and zero-sequence networks for one and two open conductors in a symmetrical system with equal positive- and negative-sequence impedances. Is the positive-minus-negative network affected by an open conductor in phase *a*? by open conductors in phases *b* and *c*? (*Note.* This problem is solved in Chapter X by α, β, and 0 components, but positive-plus-negative, positive-minus-negative, and zero-sequence components can be used as well.)

Problem 2. Check the phase currents and voltages at the fault given by equations [20], [34], and [46] with those given in Table I of Chapter IV when $Z_1 = Z_2$ and the fault impedance $Z_f = 0$.

CHAPTER VI

TRANSMISSION CIRCUITS WITH DISTRIBUTED CONSTANTS

In system studies in which transmission circuits are of short or moderate length, capacitance can frequently be neglected without appreciable error. This is usually the case in short-circuit calculations in which currents and voltages of fundamental frequency only are to be determined. On the other hand, there are certain problems in which even small capacitances must be taken into consideration. In the systems discussed in preceding chapters, capacitance has not been included. The connections of the sequence networks to represent unsymmetrical faults will be the same with capacitance included as with it neglected, but the equivalent circuits which replace an actual circuit in the sequence networks will be different in the two cases.

EQUIVALENT CIRCUITS FOR SYMMETRICAL TRANSMISSION CIRCUITS WITH DISTRIBUTED CONSTANTS

Let Fig. 1(*a*) indicate one phase of a symmetrical single-phase or three-phase transmission circuit, with or without ground wires. The terminals are *S* and *R*. *S* is the sending end under normal operation, and *R* the receiving end. The equivalent circuit which replaces the transmission line between the points *S* and *R* in any one of the sequence networks is a three-terminal circuit. One terminal is at *S*, one at *R*, and one is connected to zero potential for the network. In the positive- and negative-sequence networks all neutral points are at zero potential. In the zero-sequence network of a grounded system, the ground at any given point is the zero potential from which voltages at that point are measured.

A three-terminal circuit can be replaced by an equivalent Y or an equivalent Δ for calculating conditions at its terminals. (See Chapter I.) The equivalent circuits for transmission circuits, developed[1] by Dr. A. E. Kennelly, are called T-lines and Π-lines. The T-line is an equivalent Y, and the Π-line an equivalent Δ. Positive- and negative-sequence self-impedances and admittances of transmission lines are equal; therefore the positive- and negative-sequence equivalent circuits are the same. Zero-sequence equivalent circuits for the transmission line are similar to the positive, but, since zero-sequence impedances and admittances differ from those of positive sequence, the branches of the equivalent T or Π of the zero-sequence equivalent circuit will differ from those of the positive. Let

$$Z = \ell z = \ell(r + jx) = \ell(r + j2\pi fL)$$
$$= \text{total series impedance per phase}$$

$$Y = \ell y = \ell(g + jb) = \ell(g + j2\pi fC)$$
$$= \text{total shunt admittance per phase}$$

where ℓ is length of line in miles; r, L, C, and g are resistance, inductance, capacitance, and leakance, respectively, per mile; f is frequency

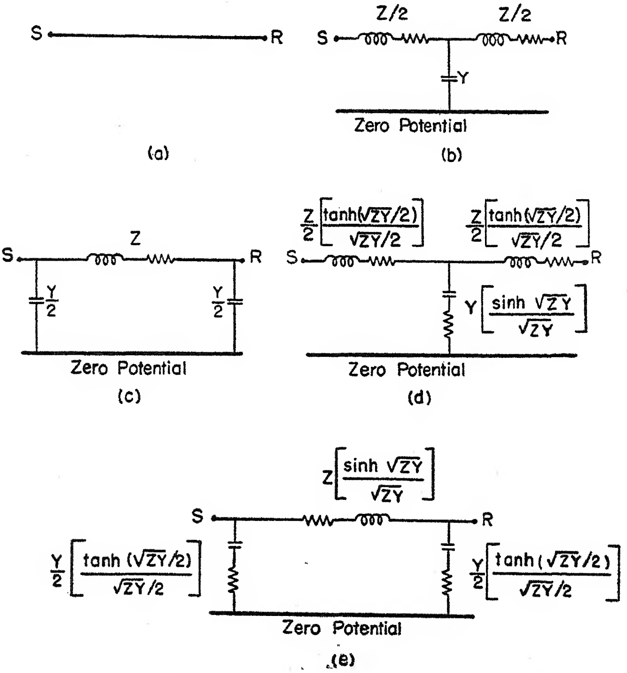

S
R
Z/2
Z/2
S
R
Y
Zero Potential
(a)
(b)
Z
S
R
Y/2
Y/2
Zero Potential
(c)
Z/2 [tanh(√ZY/2) / √ZY/2]
Z/2 [tanh(√ZY/2) / √ZY/2]
S
R
Y [sinh √ZY / √ZY]
Zero Potential
(d)
Z [sinh √ZY / √ZY]
S
R
Y/2 [tanh (√ZY/2) / √ZY/2]
Y/2 [tanh (√ZY/2) / √ZY/2]
Zero Potential
(e)

FIG. 1. (*a*) One-line diagram of single-phase or three-phase transmission circuit, with or without ground wires. (*b*) Nominal T. (*c*) Nominal Π. (*d*) Equivalent T. (*e*) Equivalent Π.

in cycles per second. C is in farads per mile; L is in henries per mile; r, x, and z are in ohms per mile; g, b, and y are in mhos per mile. The subscripts 1, 2, and 0 with the above symbols will be used to indicate positive, negative, and zero sequence, respectively.

Methods for determining the sequence constants per unit length of transmission circuits are given in later chapters. For the present, let it be assumed that these constants are known or can be readily determined. In the usual overhead transmission line during normal operation there is no corona, and leakance across insulators is negligible; therefore $g = 0$ and $Y = j\ell b = j2\pi f\ell C$, where $b = 2\pi fC$ = capacitive susceptance in mhos per mile.

Nominal T or Π. As a first approximation, the transmission line can be represented by a nominal T or Π as shown in Figs. 1(*b*) and (*c*). For the nominal T as shown in Fig. 1(*b*), one-half the total series impedance Z is placed in each arm of the T and the total shunt admittance Y in the staff. For the nominal Π as shown in Fig. 1(*c*), the total series impedance Z is placed in the architrave of the Π and one-half the total shunt admittance Y in each of the pillars of the Π.

Equivalent T or Π. The equivalent T or Π can be obtained from the nominal T or Π by applying[1] correcting factors as indicated in Figs. 1(*d*) and (*e*). The correcting factors are hyperbolic functions of $\sqrt{ZY}$ which depend upon frequency, length of line, and line constants. In addition to the correcting factors indicated in Figs. 1(*d*) and (*e*), $\cosh\sqrt{ZY}$ is also used in transmission calculations. $\operatorname{Cosh}\sqrt{ZY}$ and the correcting factors $\dfrac{\sinh\sqrt{ZY}}{\sqrt{ZY}}$ and $\dfrac{\tanh\dfrac{\sqrt{ZY}}{2}}{\dfrac{\sqrt{ZY}}{2}}$ are complex numbers which may be calculated or read from charts[2,3,4]. The calculation of complex hyperbolic functions by the use of mathematical tables of hyperbolic and circular functions is discussed later and illustrated in Problem 3. An alternate method is by substitution of Z and Y in following series.*

$$\cosh\sqrt{ZY} = 1 + \frac{ZY}{2} + \frac{Z^2Y^2}{24} + \frac{Z^3Y^3}{720} + \cdots \qquad [1]$$

$$\frac{\sinh\sqrt{ZY}}{\sqrt{ZY}} = 1 + \frac{ZY}{6} + \frac{Z^2Y^2}{120} + \frac{Z^3Y^3}{5040} + \cdots \qquad [2]$$

$$\frac{\tanh\dfrac{\sqrt{ZY}}{2}}{\dfrac{\sqrt{ZY}}{2}} = 1 - \frac{ZY}{12} + \frac{Z^2Y^2}{120} - \frac{17Z^3Y^3}{20{,}160} + \cdots \qquad [3]$$

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* *A Short Table of Integrals*, by B. O. Peirce, equations 790–792, Ginn and Company.

Length of line, frequency, and required degree of precision determine the number of terms which must be retained in the above series. In many problems, only the first two terms need be used. If more than three terms are required, the method illustrated in Problem 3 will be found less laborious.

Hyperbolic Functions from Charts. Figures 2(*a*) and (*b*) of this chapter, taken from reference 3, give the real and imaginary parts of the hyperbolic functions of $\sqrt{ZY}$ listed in [1]–[3]. Let

$$\cosh\sqrt{ZY} = A = |A| \underline{/\phi_a} = a_1 + ja_2 \qquad [4]$$

$$\frac{\sinh\sqrt{ZY}}{\sqrt{ZY}} = \beta = |\beta| \underline{/\phi_\beta} = \beta_1 + j\beta_2 \qquad [5]$$

$$\frac{\tanh\dfrac{\sqrt{ZY}}{2}}{\dfrac{\sqrt{ZY}}{2}} = \gamma = |\gamma| \underline{/\phi_\gamma} = \gamma_1 + j\gamma_2 \qquad [6]$$

$|A|$, $|\beta|$, and $|\gamma|$ in [4]–[6] are the magnitudes of the hyperbolic functions when expressed in polar form; ϕ_a, ϕ_β, ϕ_γ are the corresponding angles. Expressed in complex form, a_1, β_1, γ_1 are the real parts and a_2, β_2, and γ_2 the corresponding imaginary parts. (The subscripts 1 and 2 have no connection here with positive and negative sequence.) The total series impedance Z can be written

$$Z = \ell(r + jx) = j\ell x\left(1 - j\frac{r}{x}\right) = j2\pi f\ell L\left(1 - j\frac{r}{x}\right) \qquad [7]$$

With leakance g neglected, the total shunt admittance Y becomes $Y = j\ell b = j2\pi f\ell C$. Then

$$\sqrt{ZY} = j2\pi f\ell\sqrt{LC}\sqrt{1 - j\frac{r}{x}} \qquad [8]$$

From [8], $\sqrt{ZY}$ and the hyperbolic functions of $\sqrt{ZY}$ in [4]–[6] are functions of:

(1) $f\ell$, the product of frequency in cycles per second and length of line in miles.

(2) r/x, the ratio of resistance to reactance.

(3) $\sqrt{LC}$, the square root of the product of inductance and capacitance.

The curves of Figs. 2(*a*) and (*b*) are based on a constant value of $LC = 30 \times 10^{-12}$, which is a representative or average value of the product of the positive-sequence inductance L and the positive-sequence capacitance C of conventional overhead lines, where L is in henries per mile and C in farads per mile. The parameter is r/x. To make the curves applicable to all values of the product LC, whether of positive or of zero sequence, an abscissa labeled $(f\ell)'$ is used which is the product of frequency in cycles per second, length of line in miles, and a factor determined as follows. Equation [8] may be rewritten

$$\sqrt{ZY} = j2\pi\sqrt{30 \times 10^{-12}}\,\sqrt{1 - j\frac{r}{x}}\left(f\ell\sqrt{\frac{LC}{30 \times 10^{-12}}}\right) \qquad [9]$$

$$= j2\pi\sqrt{30 \times 10^{-12}}\,\sqrt{1 - j\frac{r}{x}}\,(f\ell)' \qquad [10]$$

From [9] and [10]

$$(f\ell)' = f\ell\sqrt{\frac{LC}{30 \times 10^{-12}}} = f\ell\sqrt{\frac{\dfrac{x}{2\pi f}\,\dfrac{b}{2\pi f}}{30 \times 10^{-12}}} = f\ell\sqrt{\frac{xb10^6}{4.26}\left(\frac{60}{f}\right)^2} \qquad [11]$$

In terms of an equivalent 60-cycle circuit, in which $LC = 30 \times 10^{-12}$, or $xb = 4.26 \times 10^{-6}$,

$$(f\ell)' = 60\ell\sqrt{\frac{xb10^6}{4.26}} \qquad [12]$$

where $x = 2\pi fL$ and $b = 2\pi fC$, and f is the given frequency.

In Figs. 2(*a*) and (*b*), $(f\ell)'$ is abscissa. The real parts of the hyperbolic functions α_1, β_1, and γ_1 are given for several values of the parameter r/x, and may be read directly corresponding to calculated values of r/x and $(f\ell)'$; the imaginary parts α_2, β_2, and γ_2 are determined from $\dfrac{\alpha_2}{r/x}$, $\dfrac{\beta_2}{r/x}$, and $\dfrac{\gamma_2}{r/x}$, read directly corresponding to $(f\ell)'$, and multiplied by r/x.

Problem 1. Find A and the positive-sequence correcting factors β and γ for a three-phase, 60-cycle transmission circuit, 250 miles in length; $z_1 = 0.12 + j0.80$ ohm per mile; $y_1 = j5.1 \times 10^{-6}$ mho per mile.

Solution. $\dfrac{r}{x} = \dfrac{0.12}{0.80} = 0.15$; from [12],

$$(f\ell)' = 60 \times 250\sqrt{\frac{(0.8)(5.1)}{4.26}} = 15{,}000 \times 0.98 = 14{,}700$$

Read from Fig. 2 (*a*) with $r/x = 0.15$ and $(f\ell)' = 14{,}700$,

$$A = a_1 + ja_2 = 0.875 + j(0.1220 \times 0.15) = 0.875 + j0.0183 = 0.875 \underline{/1.3°}$$

$$\beta = \beta_1 + j\beta_2 = 0.958 + j(0.0415 \times 0.15) = 0.958 + j0.0062 = 0.958 \underline{/0.4°}$$

$$\gamma = \gamma_1 + j\gamma_2 = 1.022 - j(0.0225 \times 0.15) = 1.022 - j0.0034 = 1.022 \overline{\backslash 0.2°}$$

In this problem, $LC = 28.7 \times 10^{-12}$ for the given line — not 30×10^{-12}, the value upon which the curves of Figs. 2(*a*) and (*b*) are based. Yet the correcting factors β and γ for the positive-sequence network would have but slight error if read at the value of $f\ell$ (= 15,000) corresponding to the actual product of frequency and length of line. Except for very long lines, the correcting factors β and γ corresponding to the actual value of $f\ell$ are approximately correct for positive and negative sequence.

Zero-Sequence Equivalent Circuit. The curves of Figs. 2(*a*) and (*b*) can be used to determine the correcting factors β and γ to be used in the equivalent T or Π to replace the transmission circuit in the zero-sequence network. The correcting factors, as for the positive-sequence network, are functions of $\sqrt{ZY}$, where Z and Y are the total zero-sequence series impedance and shunt admittance, respectively. But, whereas for the positive-sequence network $LC = 30 \times 10^{-12}$ is a representative value, for the zero-sequence network this is not the case. The zero-sequence inductance of overhead transmission circuits varies approximately from two to four times the positive-sequence inductance, while the zero-sequence capacitance may be less than half or approximately 75% of the positive-sequence capacitance. However, if r/x is calculated as the ratio of the zero-sequence resistance to reactance, and $(f\ell)'$ is determined from [12], using zero-sequence inductance and capacitance, the curves of Figs. 2(*a*) and (*b*) are applicable.

Problem 2. Given a three-phase, 60-cycle transmission circuit, 250 miles in length; $z_0 = 0.40 + j2.2$ ohms per mile; $y_0 = j3.12 \times 10^{-6}$ mho per mile. Find the zero-sequence correcting factors β and γ from Fig. 2.

Solution. $\dfrac{r}{x} = \dfrac{0.40}{2.2} = 0.18$; from [12],

$$(f\ell)' = 60 \times 250 \sqrt{\frac{2.2 \times 3.12}{4.26}} = 19{,}000$$

Read from Fig. 2 (*a*), with $r/x = 0.18$ and $(f\ell)' = 19{,}000$,

$$\beta = 0.930 + j(0.068 \times 0.18) = 0.930 + j0.012 = 0.930 \underline{/0.7°}$$

$$\gamma = 1.037 - j(0.039 \times 0.18) = 1.037 - j0.007 = 1.037 \overline{\backslash 0.4°}$$

In Problems 1 and 2, the imaginary parts β_2 and γ_2 of the correcting factors are small, resulting in small angles with β and γ in polar form. The error in neglecting β_2 and γ_2 in Fig. 2(*a*) is small. For the longer lines given by Fig. 2(*b*), the imaginary parts become larger relative to the real parts. For values of $(f\ell)'$ higher than those given in Fig. 2(*b*), or where greater precision than that obtainable from reading curves is desired, the correcting factors can be calculated either from the series given in [2] and [3] or from hyperbolic and circular functions. The latter method, illustrated in Problem 3, is simpler for very high values of $(f\ell)'$.

Cable Circuits. The curves of Figs. 2(*a*) and (*b*) may also be used to determine the correcting factors to be used in the equivalent T or Π to replace cable circuits in the sequence networks if leakance can be neglected. The procedure is then analogous to that used for the zero-sequence network illustrated in Problem 2. If leakance g cannot be neglected, the correcting factors can be calculated with $Y = \ell(g + j2\pi fC)$.

Capacitive Impedances versus Admittances in Equivalent Circuits. In analytic calculations, as contrasted with calculations on an a-c calculating table, it is convenient to replace capacitive admittances by capacitive impedances. In the nominal T of Fig. 1(*b*), $Y = j\ell b$ is replaced by $Z_c = -j\dfrac{x_c}{\ell}$; in the nominal Π line of Fig. 1(*c*), $\dfrac{Y}{2} = j\dfrac{\ell b}{2}$ is replaced by $2Z_c = -j\dfrac{2x_c}{\ell}$. In these equations $x_c = \dfrac{1}{b} = \dfrac{1}{2\pi fC}$. Z_c and $2Z_c$ are *divided* by the correcting factors β and γ, respectively, to obtain the equivalent T and Π of Figs. 1(*d*) and (*e*) with capacitive impedances replacing capacitive admittances.

Nominal versus Equivalent T or Π. There are many problems in which capacitance is appreciable but correcting factors need not be applied, the nominal T or Π being adequate. The degree of precision required in calculations will determine whether nominal or equivalent circuits should be used. A glance at Fig. 2 shows the error in neglecting positive-sequence correcting factors. With $f\ell = 12{,}000$ (corresponding to a 200-mile line at 60 cycles, a 240-mile line at 50 cycles, or a 480-mile line at 25 cycles), the error in neglecting β is approximately 3%, and in neglecting γ less than 1.5%. With $f\ell = 30{,}000$ (500-mile line at 60 cycles), the departure of β and γ from unity is pronounced: $\beta_1 = 0.832$; $\gamma_1 = 1.099$. Comparing β and γ in Problems 1 and 2, the error in neglecting correcting factors in the zero-sequence network is larger than in the positive for the same conventional transmission circuit.

Sometimes in analytic calculations, and more frequently in calculations on the a-c network analyzer, it is convenient to replace a long line by two or more equal sections of such length that each section can be represented by its nominal T or Π without correcting factors. This method of representation has the advantage of providing points along the line at which currents and voltages can be calculated or measured.

Equations for Currents and Voltages in Symmetrical Transmission Circuits with Uniformly Distributed Constants

The following equations[1] in which E_r and E_s represent voltages to neutral or to ground and I_r and I_s line currents at the circuit terminals R and S, respectively, positive direction for currents being from S to R, are applicable to voltages and currents in the positive-, negative-, and zero-sequence networks of symmetrical three-phase transmission circuits at constant frequency f.

$$E_s = E_r \cosh \theta + I_r \sqrt{\frac{z}{y}} \sinh \theta \qquad [13]$$

$$I_s = I_r \cosh \theta + E_r \sqrt{\frac{y}{z}} \sinh \theta \qquad [14]$$

$$E_r = E_s \cosh \theta - I_s \sqrt{\frac{z}{y}} \sinh \theta \qquad [15]$$

$$I_r = I_s \cosh \theta - E_s \sqrt{\frac{y}{z}} \sinh \theta \qquad [16]$$

In [13]–[16],

$\theta = \ell\alpha =$ hyperbolic angle of the line

$\alpha = \sqrt{zy} =$ normal attenuation constant per unit length of line

$Z_S = \sqrt{\frac{z}{y}} = \sqrt{\frac{r + jx}{g + jb}} =$ surge impedance to neutral or to ground

With mile as the unit of length,

$$\alpha = \sqrt{(r + jx)(g + jb)} = \alpha_1 + j\alpha_2$$
$$= \text{normal attenuation constant per mile} \qquad [17]$$

Normal attenuation occurs when the impedance of the load is equal to the surge impedance. The real part α_1 determines the attenuation in magnitude of voltage and current per mile along the line, while α_2 determines the attenuation in phase. The distance in which the normal phase attenuation amounts to 360° or 2π radians is one wave length λ. With $\lambda\alpha_2 = 2\pi$,

$$\text{The wave length } \lambda = \frac{2\pi}{\alpha_2} \text{ miles}$$

$$\text{The apparent velocity of propagation } v = f\lambda$$

$$= \frac{2\pi f}{\alpha_2} \text{ miles per second} \qquad [18]$$

The surge impedance $Z_S = \sqrt{z/y}$ is independent of length of line. It may be written $\sqrt{z\ell/y\ell} = \sqrt{Z/Y} = Z/\sqrt{ZY}$, where $z\ell$ and $y\ell$ are replaced by Z and Y, the total impedance and admittance, respectively, per phase of ℓ miles of line. Likewise, the surge admittance $\sqrt{y/z}$ may be written $Y/\sqrt{ZY}$; and θ may be written $\sqrt{\ell z \ell y} = \sqrt{ZY}$. Making these substitutions, equations [13]–[16] become

$$E_s = E_r \cosh\sqrt{ZY} + I_r Z \frac{\sinh\sqrt{ZY}}{\sqrt{ZY}} = E_r A + I_r B \qquad [19]$$

$$I_s = I_r \cosh\sqrt{ZY} + E_r Y \frac{\sinh\sqrt{ZY}}{\sqrt{ZY}} = I_r D + E_r C \qquad [20]$$

$$E_r = E_s \cosh\sqrt{ZY} - I_s Z \frac{\sinh\sqrt{ZY}}{\sqrt{ZY}} = E_s A - I_s B \qquad [21]$$

$$I_r = I_s \cosh\sqrt{ZY} - E_s Y \frac{\sinh\sqrt{ZY}}{\sqrt{ZY}} = I_s D - E_s C \qquad [22]$$

The *circuit constants* A, B, C, D for symmetrical transmission circuits used in the construction of *circle diagrams*[5] are

$$A = D = \cosh\sqrt{ZY} = \cosh\theta = a_1 + ja_2$$

$$B = Z\frac{\sinh\sqrt{ZY}}{\sqrt{ZY}} = Z\frac{\sinh\theta}{\theta} = Z(\beta_1 + j\beta_2) \qquad [23]$$

$$C = Y\frac{\sinh\sqrt{ZY}}{\sqrt{ZY}} = Y\frac{\sinh\theta}{\theta} = Y(\beta_1 + j\beta_2)$$

The above equations on a per phase basis apply to either positive- (and negative-) or zero-sequence systems. As the positive- and zero-sequence line constants are different, their surge impedances, hyperbolic angles, wave lengths, and apparent velocities of propagation are different.

Third Method of Determining Complex Hyperbolic Functions. Expressed in complex form,

$$\theta = \sqrt{ZY} = \theta_1 + j\theta_2$$

With θ_1 and θ_2 in radians, the following equations* are applicable

$$\cosh\theta = \cosh\theta_1 \cos\theta_2 + j\sinh\theta_1 \sin\theta_2 \qquad [24]$$

$$\sinh\theta = \sinh\theta_1 \cos\theta_2 + j\cosh\theta_1 \sin\theta_2 \qquad [25]$$

$$\tanh\frac{\theta}{2} = \frac{\cosh\theta - 1}{\sinh\theta} = \frac{\sinh\theta}{\cosh\theta + 1} \qquad [26]$$

The application of these equations is illustrated in the following problem, in which tables of circular† functions to hundredths of a degree, and of circular and hyperbolic‡ functions to thousandths of a radian, are used.

Problem 3. A three-phase line 65 miles long is used to transmit signals at 600 cycles. The positive-sequence impedance and admittance per mile at 600 cycles are $z = 0.8 + j8.0$ ohms and $y = 0 + j52 \times 10^{-6}$ mho, respectively. Calculate Z_S, α, θ, λ, v, and the hyperbolic functions to be used in equations [19]–[22] and in the correcting factors defined in [5] and [6].

Solution.

$$Z_S = \sqrt{\frac{z}{y}} = 10^3\sqrt{\frac{0.8 + j8.0}{j52}} = 10^3\sqrt{\frac{8.040\,\underline{/84.294^\circ}}{52\,\underline{/90^\circ}}} = 393\,\overline{/2.85^\circ}\text{ ohms}$$

$$\alpha = \sqrt{zy} = 10^{-3}\sqrt{(0.8 + j8.0)(j52)} = 20.447(10^{-3})\,\underline{/87.147^\circ}$$
$$= 10^{-3}\,(1.0177 + j20.422)$$

$$\alpha_1 = 1.018 \times 10^{-3};\quad \alpha_2 = 20.42 \times 10^{-3}$$

$$\theta = \theta_1 + j\theta_2 = \ell(\alpha_1 + j\alpha_2) = 0.06615 + j1.3274$$

$\lambda = 308$ miles; $v = 185{,}000$ miles per second

The surge impedance of a conventional transmission line has a small negative angle which depends upon resistance. The magnitude of the surge impedance is but slightly affected by resistance and frequency; it is approximately equal to $\sqrt{L/C}$. The real part α_1 of the normal attenuation constant is influenced by resistance, but α_2 is approximately equal to $2\pi f\sqrt{LC}$, and v to $1/\sqrt{LC}$. From the Smithsonian Tables,

$$\cosh 0.06615 = 1.0022;\qquad \sinh 0.06615 = 0.06620$$
$$\cos 1.3274 = 0.2410;\qquad \sin 1.3274 = 0.9705$$

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* *A Short Table of Integrals*, by B. O. Peirce, equations 660, 661, and 668, Ginn and Company.

† *Five Place Table of Natural Trigonometric Functions to Hundredths of a Degree*, by Amelia DeLella, John Wiley and Sons.

‡ *Smithsonian Mathematical Tables — Hyperbolic Functions*, prepared by George F. Becker and C. E. Van Orstrand, Washington, D. C., published by the Smithsonian Institution, 1909.

Substituting these values in [24] and [25],

$$A = \cosh\theta = \cosh\sqrt{ZY} = 0.2415 + j0.0642$$

$$\sinh\theta = \sinh\sqrt{ZY} = 0.01595 + j0.9726$$

$$\beta = \frac{\sinh\theta}{\theta} = \frac{\sinh\sqrt{ZY}}{\sqrt{ZY}} = 0.7320 + j0.0245$$

From [26], knowing cosh θ and sinh θ,

$$\gamma = \frac{\tanh\frac{\theta}{2}}{\frac{\theta}{2}} = \frac{2\beta}{\cosh\theta + 1} = 1.178 - j0.0215$$

Read from Fig. 2 (*b*), with $r/x = 0.1$ and $(f\ell)' = 60 \times 65\sqrt{(8 \times 52)/4.26} = 38{,}500$

$$A = 0.241 + j0.0642$$

$$\beta = 0.732 + j0.0244$$

$$\gamma = 1.177 - j0.0215$$

Comparing the calculated correcting factors with those read from Fig. 2 (*b*), the differences are only those resulting from the scale used in plotting the curves. With the curves plotted to a larger scale, the agreement to any desired degree of precision can be obtained.

As a check on the calculation of Z_S and $\theta = \theta_1 + j\theta_2$ (and for those who prefer rectangular coordinates to polar) the following equations may be useful:

$$|z| = \sqrt{r^2 + x^2}$$

Let $\phi = \cos^{-1} x/|z|$; then*

$$\cos\frac{\phi}{2} = \sqrt{\frac{|z| + x}{2|z|}} \quad \text{and} \quad \sin\frac{\phi}{2} = \frac{r}{\sqrt{2|z|(|z| + x)}}$$

The surge impedance Z_S

$$Z_S = \sqrt{\frac{z}{y}} = \sqrt{\frac{|z|}{b}}\left(\cos\frac{\phi}{2} - j\sin\frac{\phi}{2}\right)$$

$$= \sqrt{\frac{|z| + x}{2b}} - j\frac{r}{\sqrt{2b(|z| + x)}} \tag{27}$$

The hyperbolic angle

$$\theta = \theta_1 + j\theta_2 = \ell\sqrt{zy}$$

$$= \ell\sqrt{|z|b}\left(\sin\frac{\phi}{2} + j\cos\frac{\phi}{2}\right)$$

$$= \ell\left(r\sqrt{\frac{b}{2(|z| + x)}} + j\sqrt{\frac{b(|z| + x)}{2}}\right) \tag{28}$$

* *A Short Table of Integrals*, by B. O. Peirce, equations 576–578, Ginn and Company.

Calculation of Currents and Voltages in Systems with Appreciable Capacitance

There is an important difference to be noted between the paralleling of two inductive impedances or two capacitive impedances and the paralleling of an inductive impedance and a capacitive impedance. For example, if $Z_a = j2$ and $Z_b = j8$, $Z_aZ_b/(Z_a + Z_b) = j1.6$. The impedance of $-j2$ and $-j8$ in parallel is $-j1.6$. But, if $Z_a = j2$ and $Z_b = -j8$, $Z_aZ_b/(Z_a + Z_b) = 16/(-j6) = j2.667$; if $Z_a = -j2$ and $Z_b = j8$, their parallel value is $-j2.667$. The equivalent impedance of an inductive and a capacitive impedance in parallel has the sign of the smaller of the two impedances and is larger in magnitude than the smaller. If $Z_a = j2$ and $Z_b = -j2$, their parallel value is infinite. The following problem illustrates the effects of capacitance upon system calculations.

Problem 4. Figure 3 (*a*) shows a large three-phase 60-cycle power system, replaced for purposes of calculation by a single equivalent synchronous machine *A*. The positive sequence subtransient reactance of the large system viewed from *F*, based on 100,000 kva and 110 kv in the line, is 10%. The negative- and zero-sequence reactances are 10% and 5%, respectively. Power is supplied to a second power system over one 110 kv three-phase transmission circuit, 200 miles in length. The line constants are

$$z_1 = 0.235 + j0.79 \text{ ohm per mile; } b_1 = 5.35 \times 10^{-6} \text{ mho per mile}$$

$$z_0 = 0.515 + j2.65 \text{ ohms per mile; } b_0 = 3.20 \times 10^{-6} \text{ mho per mile}$$

Loads are supplied from the 110-kv line at various points through Δ–Δ transformer banks with synchronous condensers at some of the stations to regulate line voltages. At the time of a line-to-ground fault at *F*, the system was lightly loaded and only a 15,000-kva condenser at *C* was in operation. The voltage at *F* before the fault was 110 kv. The positive- and negative-sequence transient reactances of the condenser at *C* viewed from the line through the Δ–Δ transformer bank are approximately 30% based on its rating, or 200% based on 100,000 kva. The equivalent per unit excitation voltage E_a of the condenser (voltage behind transient reactance) is 0.85. The condenser is underexcited to prevent voltage rise at *P*. The sequence networks and their connection for a line-to-ground fault on phase *a* at *F* are shown in Fig. 3 (*b*), with loads neglected. The negative-sequence network is the same as the positive with E_a and E_a' equated to zero. The zero-sequence network is open at *P*. The positive- and negative-sequence equivalent circuits for the transmission line are equivalent Π lines. The equivalent T-line is used in the zero-sequence network. The choice of a T or Π is arbitrary. For this particular problem, T-lines in all three networks would be preferable.

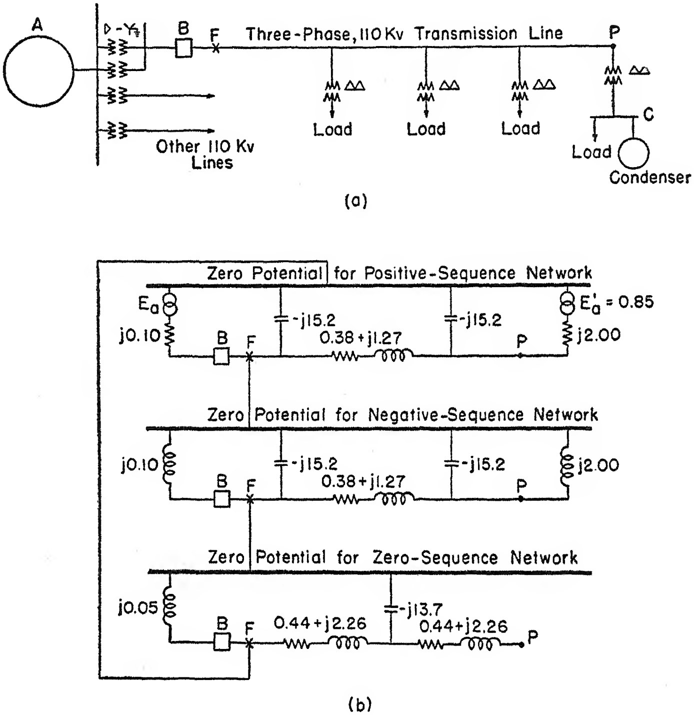
The positive- and negative-sequence equivalent Π lines shown in Fig. 3 (*b*) with impedances in per unit are determined as follows:

$$\frac{r}{x} = \frac{0.235}{0.79} = 0.30; \quad (fl)' = 12{,}000\sqrt{\frac{0.79 \times 5.35}{4.26}} = 12{,}000 \text{ approximately}$$

From Fig. 2 (*a*), neglecting β_2 and γ_2, $\beta = 0.972$ and $\gamma = 1.014$.

$$\beta Z = 200(0.235 + j0.79)0.972 = 45.7 + j153.5 \text{ ohms}$$

$$\gamma \frac{Y}{2} = 100(j5.35 \times 10^{-6})1.014 = j544 \times 10^{-6} \text{ mho}$$


A
Δ-Y
B
F
Three-Phase, 110 Kv Transmission Line
P
ΔΔ
Load
Load
Load
C
Load
Condenser
Other 110 Kv Lines
Zero Potential for Positive-Sequence Network
E_a
$E_a' = 0.85$
j0.10
-j15.2
0.38+j1.27
-j15.2
j2.00
Zero Potential for Negative-Sequence Network
j0.10
-j15.2
0.38+j1.27
-j15.2
j2.00
Zero Potential for Zero-Sequence Network
j0.05
-j13.7
0.44+j2.26
0.44+j2.26

(a)

(b)

FIG. 3. (*a*) One-line system diagram. (*b*) Connection of sequence networks of (*a*) for a line-to-ground fault at *F*. Impedances are in per unit based on 100,000/3 kva per phase and a base line-to-neutral voltage of $110/\sqrt{3}$ kilovolts in the transmission line. Loads are neglected.

To express impedances, given in ohms, in per unit based on 100,000/3 kva per phase and a line-to-neutral voltage of $110/\sqrt{3}$ kv (see [27], Chapter I), the multiplier is $100{,}000/(110^2 \times 10^3) = 0.826 \times 10^{-2}$.

$$\beta Z = (45.7 + j153.5) \times 0.826 \times 10^{-2} = 0.38 + j1.27 \text{ per unit}$$

$$\frac{2}{\gamma Y} = -j1840 \times 0.826 \times 10^{-2} = -j15.2 \text{ per unit}$$

The zero-sequence equivalent T-line is determined as follows:

$$\frac{r}{x} = \frac{0.515}{2.65} = 0.194; \quad (f\ell)' = 12{,}000 \sqrt{\frac{2.65 \times 3.20}{4.26}} = 17{,}000$$

From Fig. 2(*a*), neglecting β_2 and γ_2, $\beta = 0.944$ and $\gamma = 1.03$. In per unit,

$$\gamma \frac{Z}{2} = 100(0.515 + j2.65)1.03 \times (0.826 \times 10^{-2}) = 0.438 + j2.26$$

$$\frac{1}{\beta Y} = \frac{10^6 \times (0.826 \times 10^{-2})}{200(j3.20) \times 0.944} = -j13.7$$

Determine the initial symmetrical rms current in the fault and voltages to ground at the fault on phases b and c (with the circuit breaker at B closed). Assume as subtransient reactances for the condenser the given transient values.

Solution. The equivalent excitation voltage E_a is not given nor is it required, as the prefault line-to-line voltage at F is given. The line-to-neutral voltage $V_f = 110 \text{ kv}/\sqrt{3}$. In per unit with V_f as reference vector,

$$V_f = 1 \underline{/0^\circ}.$$

The positive- and negative-sequence subtransient impedances viewed from F are equal and are determined by paralleling the impedances viewed from the fault in the two directions. The impedance Z_1' to the left of F is $j0.10$; that to the right is

$$Z_1'' = \frac{\left[\frac{j2.00(-j15.2)}{j(2.00-15.2)} + 0.38 + j1.27\right](-j15.2)}{(j2.30 + 0.38 + j1.27) - j15.2} = \frac{(0.38 + j3.57)(-j15.2)}{0.38 - j11.63}$$

$$= \frac{(3.60 \underline{/83.9^\circ})(15.2 \overline{/90^\circ})}{11.64 \overline{/88.1^\circ}} = 4.70 \underline{/85.8^\circ} = 0.345 + j4.69$$

$$Z_1 = Z_2 = \frac{Z_1' \times Z_1''}{Z_1' + Z_1''} = \frac{(j0.10)(4.70 \underline{/85.8^\circ})}{0.345 + j4.79 (= 4.80 \underline{/85.9^\circ})} = 0.098 \underline{/89.9^\circ} = 0.00 + j0.098$$

= per unit positive- or negative-sequence impedance viewed from the fault

Viewed from F in the zero-sequence network,

$$Z_0' = j0.05 \quad \text{and} \quad Z_0'' = 0.44 + j2.26 - j13.7 = 0.44 - j11.44 = 11.45 \overline{/87.8^\circ}$$

$$Z_0 = \frac{Z_0' Z_0''}{Z_0' + Z_0{}'} = \frac{(0.05 \underline{/90^\circ})(11.45 \overline{/87.8^\circ})}{0.44 - j11.39 (= 11.40 \overline{/87.8^\circ})} = j0.0502 = \text{per unit zero-sequence impedances viewed from the fault}$$

It will be noted that Z_0 viewed from the fault is not appreciably affected by the impedance $Z_0{}'$ because of its high magnitude relative to $Z_0' = j0.05$. $Z_1 = Z_2$ is likewise but little affected by paralleling Z_1'' with $Z_1' = j0.10$. The impedances viewed towards the large system in this problem determine conditions at the fault. In cases such as this, where it is apparent that exact values of the sequence impedances in one direction are unimportant, they may be roughly estimated. Using transient reactance instead of subtransient for the synchronous condenser at C has negligible effect on the impedances Z_1 and Z_0 viewed from the fault.

Substituting V_f, Z_1, Z_2, and Z_0 in the equations of Table I, Chapter IV, with $R_f = 0$,

$$I_{a1} = I_{a2} = I_{a0} = \frac{1\,\underline{/0^\circ}}{j0.098 + j0.098 + j0.050} = -j4.06$$

$$I_f = 3I_{a0} = -j12.18$$

$$V_{a2} = -I_{a2}Z_2 = -0.398;\ V_{a0} = -I_{a0}Z_0 = -0.203;\ V_{a1} = -(V_{a2} + V_{a2}) = 0.601$$

$$V_b = -0.305 - j0.866 = 0.920\,\overline{/109.4^\circ}$$

$$V_c = -0.305 + j0.866 = 0.920\,\underline{/109.4^\circ}$$

V_b and V_c are both below normal line-to-neutral voltage.

Zero-Sequence Voltages at Points Distant from the Fault. In a system where all reactances are inductive, the zero-sequence voltage has its highest value at the fault. In circuits with capacitance this may not be the case. In Problem 4, the zero-sequence voltage V_{a0} at F is -0.203. Applying this voltage in the zero-sequence network between F and the zero-potential bus for the network, the zero-sequence voltage at P (neglecting resistance) is

$$V_{a0}\ (\text{at } P) = \frac{-0.203}{j(2.26 - 13.7)}(-j13.7) = -0.243$$

The zero-sequence voltage at P is approximately 20% higher than at F.

A Ground-Fault Neutralizer (Petersen Coil). When, because of lightning or other causes, a flashover between a conductor and tower occurs, the arc offers a relatively low impedance to fundamental-frequency currents, which will be inductive or capacitive, depending upon whether the system is operated with grounded or ungrounded neutral. A ground-fault neutralizer is a reactance, placed between neutral and ground in a system otherwise ungrounded, of such magnitude that the fundamental-frequency zero-sequence capacitive currents in a ground fault are neutralized by zero-sequence inductive currents passed by the ground-fault neutralizer. With little resultant fundamental-frequency current in the arc, it rapidly dies out, and switching operations are unnecessary.

A reactance in the neutral has an effective value of three times its actual value. (See equation [7], Chapter III.) Placed between the neutral of Y-connected transformer windings and ground, with its effective reactance $3X_L$ plus the transformer reactance x_t equal to the system capacitive reactance X_c (neglecting resistance and line reactance), the zero sequence impedance viewed from any system point will be infinite.

$$Z_0 = \frac{j(3X_L + x_t)(-jX_c)}{0} = \infty$$

Actually, resistance is present, and the effects of line reactance may be appreciable. But as a first approximation in determining the required neutral reactance, the total capacitive reactance X_c of all the lines is equated to $3X_L + x_t$. Thus,

$$3X_L + x_t = X_c \quad \text{and} \quad X_L = \tfrac{1}{3}(X_c - x_t)$$

When a line-to-ground fault occurs and $Z_0 = \infty$, the zero sequence voltage at the fault is $-V_a$, the voltage of phase a before the fault. (See Chapter III, equation [32] and Fig. 9.) No fundamental-frequency current will flow in the positive and negative sequence networks; but, with $V_{a0} = -V_a$ at the fault, both capacitive and inductive currents will flow in the zero-sequence network. In the fault these currents neutralize each other, except for their components resulting from resistance and imperfect tuning.

A neutralizer, in tune for a fault at one system point, is in tune for faults at other points as well. Therefore, the problem of locating the neutralizer on the system becomes that of determining the position where it will not be disconnected during system disturbances. Sometimes a system can be subdivided into areas, and each area provided with its own neutralizer. With this arrangement, the areas can be interconnected or separated and the system as a whole will still have protection.

When some of the lines are long, and the voltage high (110 kv or above), a rise in zero-sequence voltage at points distant from neutralizer may occur during ground faults, as explained in the above section. The use of two or more coils judiciously placed in the system will limit this voltage rise to a reasonable value.

CHARTS OF FUNDAMENTAL FREQUENCY LINE-TO-GROUND VOLTAGES DURING A LINE-TO-GROUND FAULT

During a line-to-ground fault, the voltages of the two unfaulted phases may be higher or lower than normal, depending upon the system impedances. Figures 4(*a*) and (*b*), taken from a paper[6] by Messrs. Hunter, Pragst, and Light, show the magnitudes of the fundamental-frequency voltages to ground of phases b and c, respectively, at the fault (on phase a) in terms of the positive- and zero-sequence system impedances viewed from the fault. The positive- and negative-sequence impedances are assumed equal, and the effects of corona and saturation are neglected.

The curves give only the fundamental-frequency dynamic voltages. Harmonic voltages resulting from unbalanced currents in synchronous machines with unequal reactances in the direct and quadrature axes

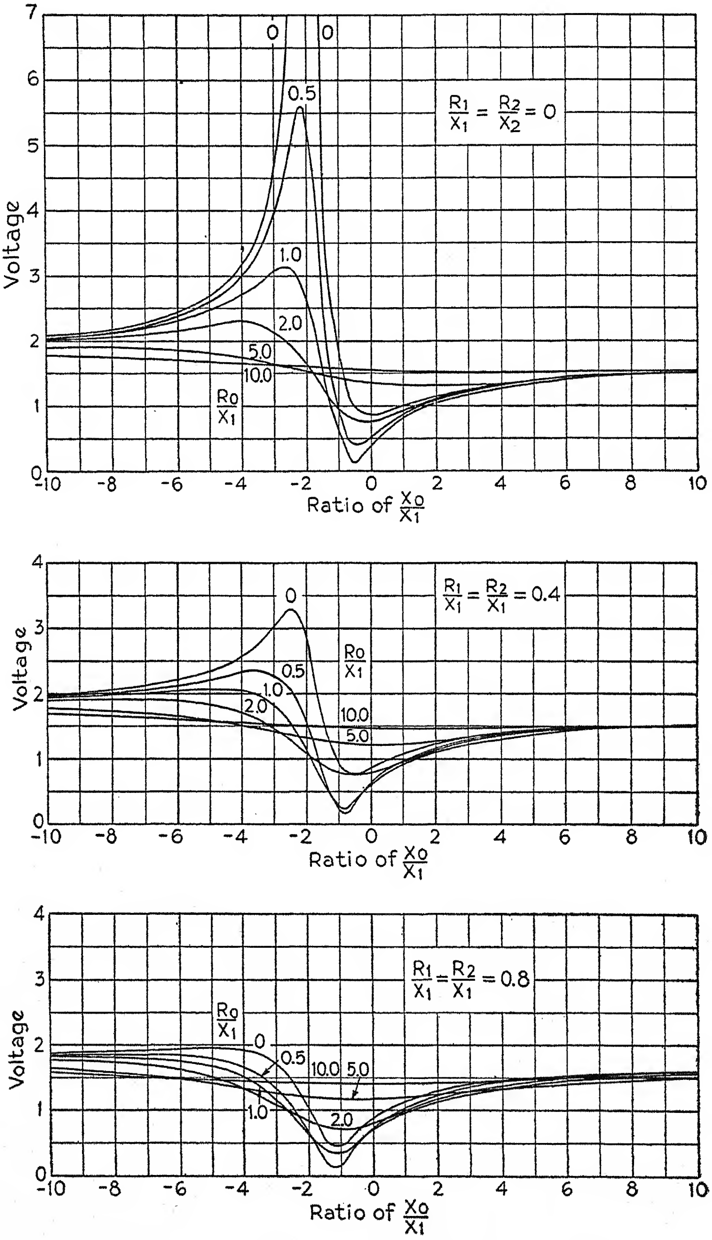


FIG. 4(*a*). Phase *b* voltage to ground at point of fault in per unit of normal voltage to neutral versus X_0/X_1 with conductor-to-ground fault on phase *a*.

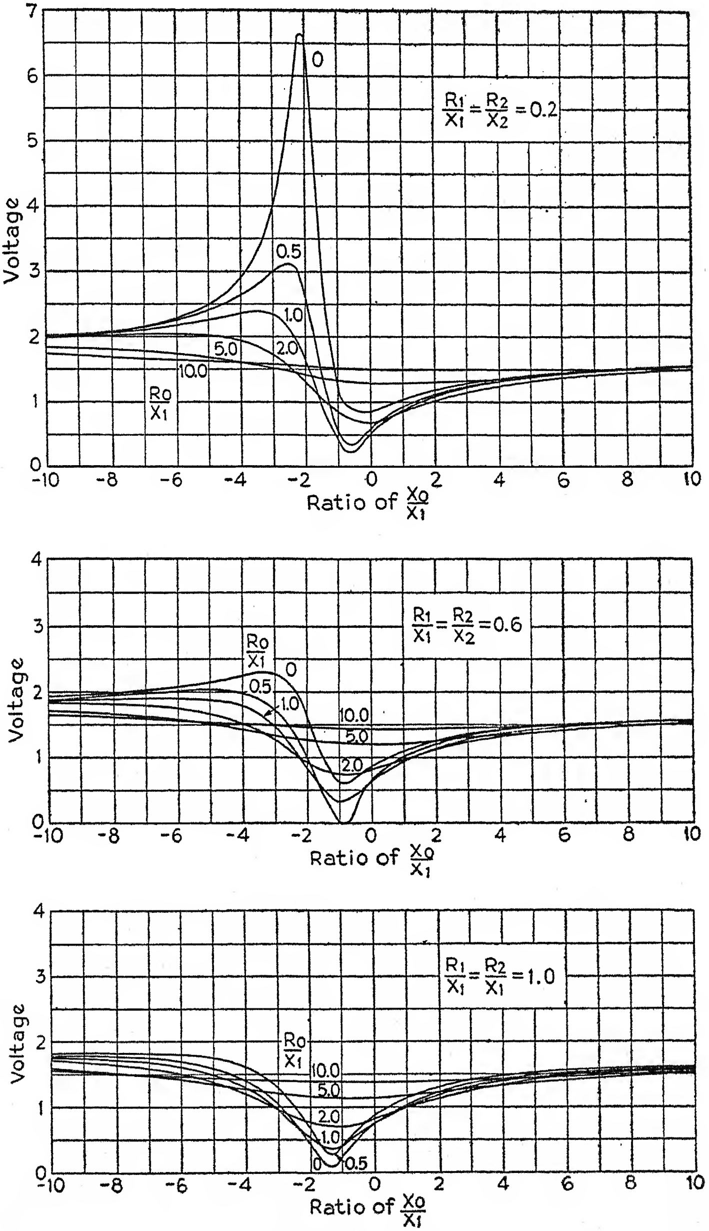


FIG. 4(*a*) *continued*

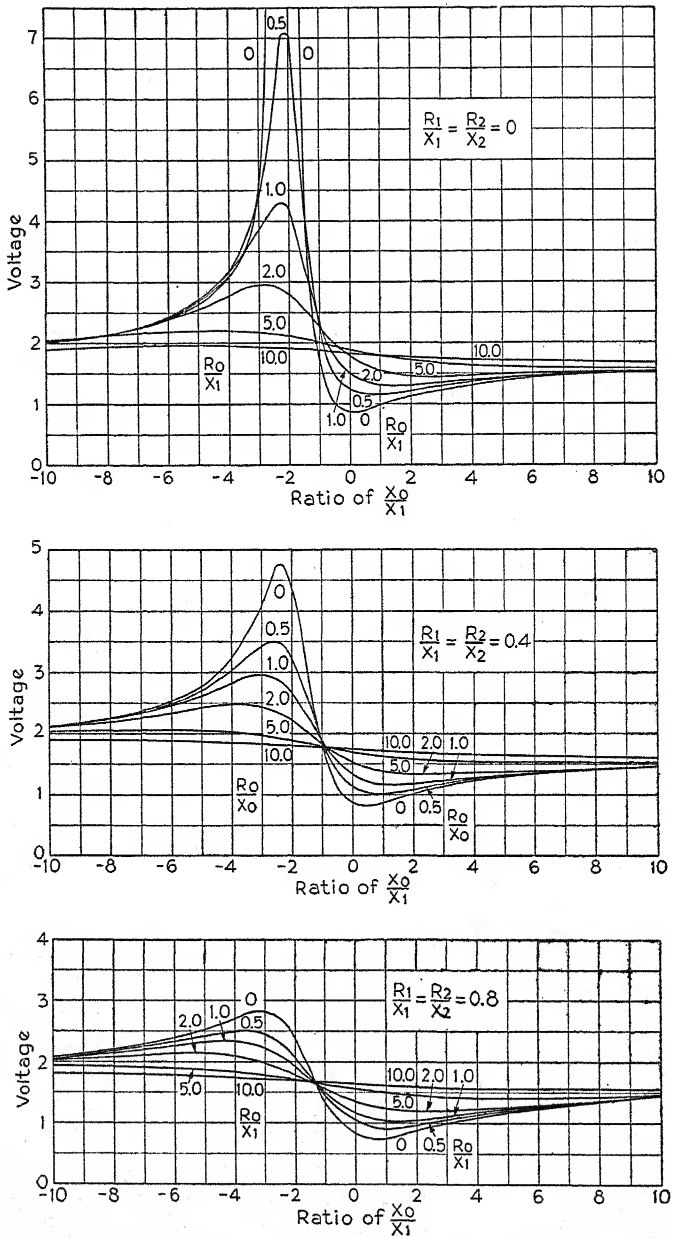
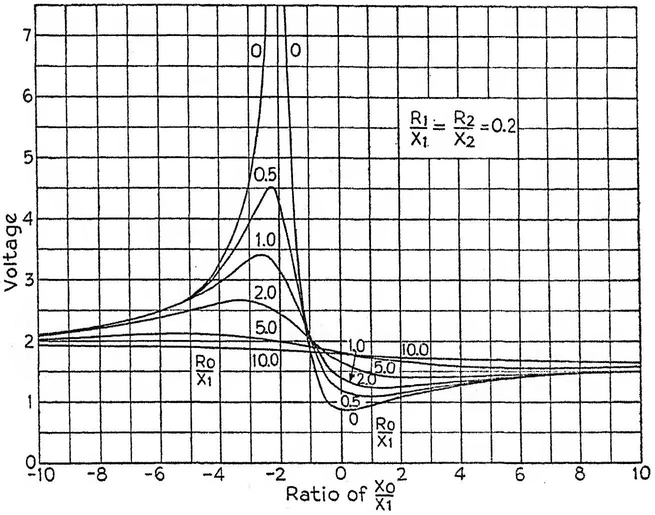
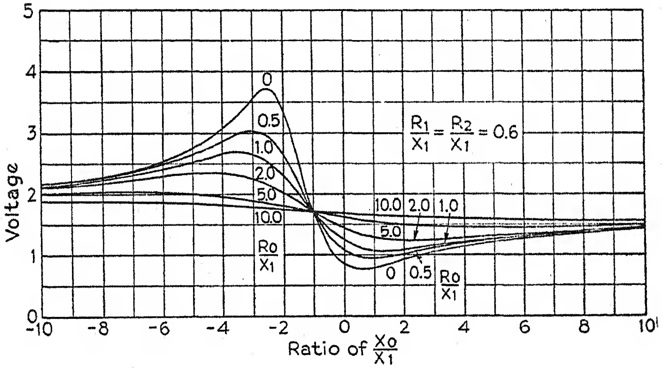


FIG. 4(*b*). Phase *c* voltage to ground at point of fault in per unit of normal voltage to neutral versus X_0/X_1 with conductor-to-ground fault on phase *a*.





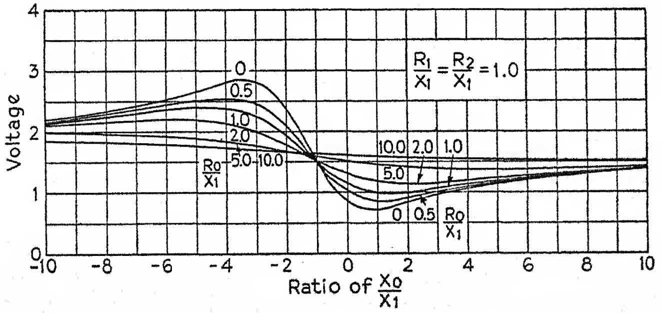


FIG. 4(*b*) *continued*

and transient voltages from switching and arcs to ground are not included. Only the voltages to ground which exist at the point in the system where the ground fault is located are given. In general, it may be stated that the voltages will be lower or higher at other points on the system, depending upon whether the system neutrals are grounded or isolated. The notation used is

$$Z_1 = Z_2 = R_1 + jX_1$$

$$Z_0 = R_0 + jX_0$$

where X_0 may be positive or negative. In an ungrounded system, the zero-sequence impedance viewed from the fault is a capacitive impedance, and X_0 is intrinsically negative. For an assumed ratio $R_1/X_1 = R_2/X_2$ with R_0/X_1 as parameter, X_0/X_1 is varied from -10 to $+10$ and the voltages of phases b and c plotted in per unit of V_f, the prefault line-to-neutral voltage at the point of fault F. With X_0/X_1 extended to $+\infty$ or $-\infty$, V_b and V_c in per unit of normal line-to-neutral voltage become 1.73. When the system neutral is grounded through a ground-fault neutralizer, the ratio of X_0/X_1 is very high, approaching infinity with the neutralizer tuned to the system.

Equations for V_b and V_c in terms of V_f, Z_1, and Z_0 without fault resistance can be obtained from Table I of Chapter IV if Z_f is equated to zero and Z_2 is replaced by its equivalent Z_1.

Fault Resistance. The curves of Figs. 4(*a*) and (*b*), as drawn, do not include fault resistance. In general, fault resistance, except in low-resistance systems, has a tendency to reduce the magnitude of the unbalanced voltage over that which would be obtained if the fault resistance were zero. The voltages that exist with resistance in the fault can be obtained from the curves by making the following substitutions:

The equations for the unbalanced voltages during a line-to-ground fault in Table I, Chapter IV, which include fault resistance may be written

$$V_a = \frac{V_f(3R_f)}{(2R_1' + R_0') + j(2X_1 + X_0)}$$

$$V_b = a^2 V_f - \frac{V_f[(R_0' - R_1') + j(X_0 - X_1)]}{(2R_1' + R_0') + j(2X_1 + X_0)}$$

$$V_c = a V_f - \frac{V_f[(R_0' - R_1') + j(X_0 - X_1)]}{(2R_1' + R_0') + j(2X_1 + X_0)}$$

where

$$R_1' = R_1 + R_f$$

$$R_0' = R_0 + R_f$$

$$R_f = \text{fault resistance}$$

The curves are expressed as functions of the positive resistance R_1 and the zero resistance R_0. However, if values of R_1'/X_1 and R_0'/X_1 are obtained and reference is made to curves of R_1/X_1 and R_0/X_1 which are numerically equal to those values obtained with fault resistance, these particular curves will give the voltages to be expected with resistance in the fault.

The curves of Figs. 4(*a*) and (*b*) not only are useful in reducing calculations, but they also indicate clearly the relative voltage trend under different conditions of system grounding. The voltages which occur during ground faults are of particular interest to application and operating engineers because they have a direct influence on the serviceableness of system insulation and connected apparatus and on the size of the neutral grounding impedance to be used. One example of this is the selection of the correct lightning arrester for a given system. These protective devices have a maximum permissible line-to-ground voltage rating which specifies the voltage across the arrester that must not be exceeded if arrester troubles are to be avoided. As most arresters are connected between the line conductors and ground, the dynamic voltage rating of the arresters should not be lower than the voltages that exist during ground faults at the proposed arrester locations.

Use of the Curves of Figs. 4(*a*) and (*b*). In Problem 4, with connection of the sequence networks for a line-to-ground fault at *F*, the positive- and zero-sequence impedances viewed from the fault are $Z_1 = 0 + j0.098$ and $Z_0 = 0 + j0.050$, respectively.

$$\frac{R_1}{X_1} = \frac{R_2}{X_2} = \frac{R_0}{X_1} = 0; \quad \frac{X_0}{X_1} = \frac{0.050}{0.098} = 0.51$$

From the first of the curves in Fig. 4(*a*) and in Fig. 4(*b*), V_b and V_c are both less than normal.

Problem 5. In Problem 4, and Fig. 3(*a*), assume that the breakers at *B* are open but the line-to-ground fault at *F* remains on the system. Determine the voltages to ground at *F* of phases *b* and *c*, assuming that the condenser does not lose synchronism with the large system while the breakers are closed and therefore its speed is not appreciably different from synchronous speed at the instant the breakers are opened. It will be assumed further that the transient reactance of the condenser and the voltage $E'_a = 0.85$ behind this reactance can be used to calculate rms symmetrical voltages, immediately following the opening of the breakers.

Solution. The diagram of Fig. 3(*b*) can be used if the three networks are all opened at B. There are two ways of determining the fault current and voltages V_b and V_c. One way is to determine the positive-sequence voltage V_f of phase a at F with the fault removed and the voltage $E'_a = 0.85$ in the condenser behind transient reactance. The fault is then applied and calculations are made as in Problem 4. A second method is to replace the fault in the positive sequence network by an equivalent circuit with impedance $Z_0 + Z_2$ between F and the zero-potential bus of the positive-sequence network. Figure 3(*b*) with the circuit breakers open at B satisfies this condition. The former method will be used; the latter is reserved for a problem. (See Problem 14.)

The positive-sequence impedance viewed from the condenser neutral with the fault *removed* and the breaker *open* is

$$j2.00 + \frac{-j15.2\,(0.38 - j13.93)}{0.38 - j29.13} = 0.10 - j5.28$$

The current I in the shunt at F with $E'_a = 0.85 \underline{/0^\circ}$ is

$$I = \frac{0.85}{0.10 - j5.28} \times \frac{-j15.2}{0.38 - j29.13} = 0.00 + j0.084$$

The voltage at F is

$$V_f = (-j15.2)(j0.084) = 1.27 \text{ per unit}$$

At P the voltage is 1.17. With the breaker open and no fault, there is a voltage rise through the condenser and along the line because of charging current flowing through reactance.

The sequence impedances viewed from the fault with the breaker open are Z''_1 and Z''_0, calculated in Problem 4.

$$Z_1 = Z_2 = 0.345 + j4.69; \quad R_1 = 0.345; \quad X_1 = 4.67$$

$$Z_0 = 0.44 - j11.44; \quad R_0 = 0.44; \quad X_0 = -11.44$$

$$\frac{R_1}{X_1} = 0.074; \quad \frac{R_0}{X_1} = 0.094; \quad \frac{X_0}{X_1} = -2.45$$

Read roughly from the curves of Figs. 4(*a*) and (*b*) with $R_1/X_1 = 0.2$ (an approximation) and $R_0/X_1 =$ between 0 and 0.5, but with $X_0/X_1 = -2.45$, V_b is higher than $3V_f$, and V_c higher than $4V_f$, where V_f as calculated above is 1.27 times normal line-to-neutral voltage. These are the approximate voltages which would exist at the fault if there were *no corona* on the line and *no saturation* in the Δ–Δ transformer banks along the line.

Voltages calculated with corona and saturation neglected if appreciably above corona starting voltage for the line (see Fig. 8, Appendix B) or rated voltages across transformer windings along the line, even though too high in magnitude, serve to indicate conditions and locations where high voltages are to be expected. This is the case for the system discussed in Problem 5. The calculations made in this problem indicate definitely that voltages to ground above $\sqrt{3}$ times line-to-neutral voltages will occur. To determine exactly how high the voltages will be requires a more comprehensive study or, preferably, field tests.

It is interesting to note the effect of zero-sequence resistance from Figs. 4(*a*) and (*b*). With $R_1/X_1 = 0.2$, $R_0/X_1 = 5.0$, and $X_0/X_1 = -2.45$, although in the resonance region of the curves, $V_b = 1.5$ and $V_c = 2.0$.

EQUIVALENT CIRCUITS FOR PARALLEL THREE-PHASE TRANSMISSION LINES

In Chapters XI and XII methods of calculating the self-inductive impedance and capacitive admittance of each line alone and the mutual inductive and capacitive impedances between parallel lines, taken two at a time, are discussed and equations given for determining them. Lines with and without ground wires are considered. Mutual impedances, both inductive and capacitive, between two parallel lines in the positive- or negative-sequence system are small relative to self-impedances and depend upon the arrangements of the phases of the two circuits in their tower positions. Unless a high degree of precision is required, each circuit can be replaced by its equivalent T or Π in the positive- or negative-sequence network without mutual coupling with parallel circuits. If it is desired to include mutual coupling, equivalent circuits similar to those which will be developed for use in the zero-sequence network can be used.

Zero-Sequence Equivalent Circuits. By definition, zero-sequence line currents and voltages to ground in the three phases at any system point are equal in magnitude and phase. Zero-sequence mutual impedances are therefore independent of the arrangements of the phases of the two circuits in their tower positions.

Circuits with Negligible Capacitance. Zero sequence mutual impedances between parallel lines on the same or adjacent towers may be as high as 50% or more of the self-impedance of either circuit alone. Except in special cases, zero-sequence mutual impedances between parallel lines cannot be neglected. Figures 9(*b*) and (*c*), 11(*b*), and 12(*c*) of Chapter I, which show equivalent circuits to replace two mutually coupled circuits, are directly applicable to two parallel transmission lines with negligible capacitance, bussed at both ends, at one end, and at neither end, respectively. For three parallel lines bussed at one end, the equivalent circuits of Fig. 13(*b*) or (*c*) of Chapter I are applicable, the former if two of the mutual impedances are equal and the latter if all three are equal. For three unequal mutual impedances, the four-terminal equivalent circuit, Fig. 5(*a*), may be used. This equivalent circuit is similar to Fig. 13(*b*) of Chapter I except for the mutual coupling between circuits *B* and *C* introduced because $Z_{bc} \neq Z_{ac}$. The mutual impedance $(Z_{bc} - Z_{ac})$ is introduced between circuits *B* and *C* by means of the mutual coupling circuit of Fig. 12(*c*), Chapter I, except that the positions of the terminals at one end are reversed. Figure 5(*a*) can be checked by following the current in each line with the other lines open to see if it meets an impedance equal to its self-impedance and induces the required voltages in the open lines. With negligible capacitance, a line is opened at any point by opening it at a terminal, without otherwise changing the equivalent circuit. With circuits *A* and *C* open at terminals *A* and *C*, the current I_b flows through $Z_{ac} + (Z_{ab} - Z_{ac}) + (Z_{bb} - Z_{ab}) = Z_{bb}$. I_b has two paths, both of zero impedance, through the equivalent circuit which couples circuits *B* and *C* through the mutual impedance $(Z_{bc} - Z_{ac})$. I_b induces a voltage drop $I_b(Z_{ac} + Z_{ab} - Z_{ac}) = I_bZ_{ab}$ in circuit *A* in the direction of I_b. In circuit *C* the induced voltage drop is $I_b(Z_{ac} + Z_{bc} - Z_{ac}) = I_bZ_{bc}$.

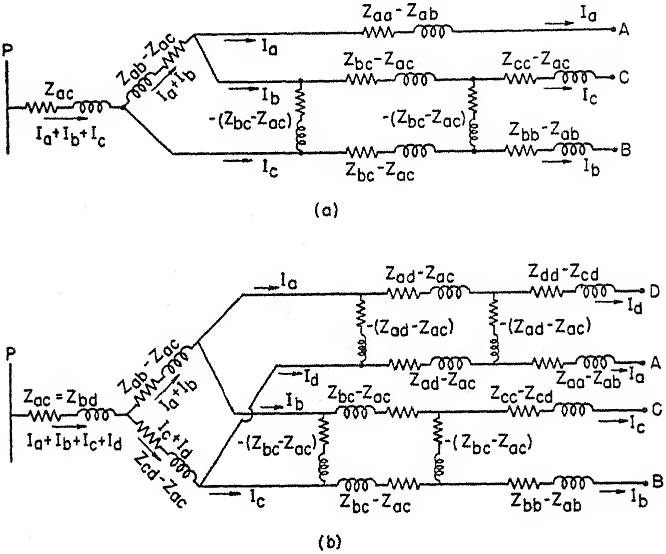

P
Zac
Ia+Ib+Ic
Zab-Zac
Ia+Ib
Ia
Ib
Ic
Zaa-Zab
Ia
A
Zbc-Zac
Zcc-Zac
C
Ic
-(Zbc-Zac)
-(Zbc-Zac)
Zbb-Zab
B
Ib
Zbc-Zac
(a)
P
Zac=Zbd
Ia+Ib+Ic+Id
Zab-Zac
Ia+Ib
Ic+Id
Zcd-Zac
Ia
Zad-Zac
Zdd-Zcd
D
Id
-(Zad-Zac)
-(Zad-Zac)
Id
Zad-Zac
Zaa-Zab
Ia
A
Ib
Zbc-Zac
Zcc-Zcd
C
Ic
-(Zbc-Zac)
-(Zbc-Zac)
B
Ic
Zbc-Zac
Zbb-Zab
Ib
(b)

FIG. 5. Equivalent circuits for parallel lines of negligible capacitance bussed at one end *P* for use in analytic calculations. (*a*) Three lines with unequal mutual impedances between them. (*b*) Four lines with $Z_{ac} = Z_{bd}$ but other mutual impedances unequal.

The equivalent circuit used in Fig. 5(*a*) can be extended to four or more parallel lines bussed at one end with unequal mutual impedances between them. For the case of four parallel circuits on the same right-of-way, if $Z_{ac} = Z_{bd}$ but the other mutual impedances are unequal, Fig. 5(*b*) can be used. If $Z_{ac} \neq Z_{bd}$, an additional mutual coupling circuit would be required to insert the mutual impedance $(Z_{bd} - Z_{ac})$ between circuits B and D.

When parallel lines are not bussed but are supplied through transformers connected Δ–Y, grounded on the line side, the transformers may be included in the zero-sequence self-impedances of the lines, and the lines plus transformers considered to have a ground point in common. The equivalent circuits for two, three, and four parallel lines discussed above can then be used in the zero-sequence network. The only limitation to such equivalent circuits is that zero-sequence voltages at the transformer terminals cannot be obtained directly; but they can be calculated from the transformer impedances and the zero-sequence currents flowing through them.

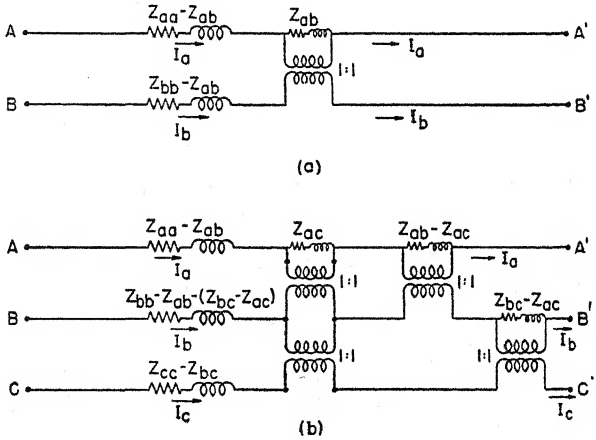


FIG. 6. Equivalent circuits for parallel lines with negligible capacitance for use on an a-c network analyzer. (*a*) Two lines. (*b*) Three lines.

Equivalent Circuits for Use with An A-C *Network Analyzer.* In equivalent circuits used on the network analyzer, mutual coupling transformers are usually employed to secure mutual coupling between circuits. The ideal mutual coupling transformer has infinite exciting impedance, zero resistance, and zero leakage reactance relative to the analyzer impedance units.

Figure 6 and Fig. 8(*d*) with the capacitive shunts to ground omitted show zero sequence equivalent circuits for two, three, and four parallel lines in which capacitance is negligible. If the circuits are so designated that Z_{ac} and Z_{ad}, with three and four parallel lines, respectively, have the lowest resistance components of mutual impedance, there need be no negative resistances in the equivalent circuits. In Fig. 6(*a*) the mutual impedance Z_{ab} is inserted in one circuit (here circuit A) with the terminals of one winding of a mutual coupling transformer of 1 : 1 turn ratio connected across it, the other transformer winding being connected in series with circuit B. The impedance met by I_a is $Z_{aa} - Z_{ab} + Z_{ab} = Z_{aa}$. The voltage drop induced in circuit B by I_a is I_aZ_{ab} in the direction of I_a.

In Fig. 6(*b*), two 1 : 1 turn ratio transformers are required to couple the three lines through a common mutual impedance Z_{ac}. An additional transformer is required to obtain the correct mutual impedances between circuits A and B, and another for the correct mutual impedance between circuits B and C. In Fig. 8(*d*) the method used in Fig. 6(*b*) has been extended to include four parallel lines. The method can be extended to include any number of parallel circuits, being limited only by the number of coupling transformers available.

Parallel Lines with Appreciable Capacitance. There are two ways in which the zero-sequence capacitances associated with parallel transmission lines may be defined. With each three-phase line represented on a per phase basis, the *capacitive admittance* to ground of any line and its mutual admittances with the other lines may be determined with all other lines *grounded* or with all other lines *open*. By the first method of determining the capacitive admittances of a line, the voltages of all other lines are equated to zero. By the second method, the currents in all other lines are equated to zero. The admittances of a line, determined with all other lines grounded, correspond to driving-point and transfer admittances defined in Chapter I under equation [33]. The admittances of a line, determined with the other lines open, are analogous to self- and mutual admittances between inductively coupled circuits. To avoid confusing capacitive admittances defined in two different ways, self- and mutual *capacitive impedances* instead of admittances will be used in this chapter when capacitances associated with a transmission line are defined as the capacitances determined with all other lines open.

Consider two three-phase circuits A and B, ℓ miles in length, which for the purpose of this discussion will be assumed symmetrical, with all conductors of each circuit equidistant from the conductors of the other circuit. (Unsymmetrical transmission circuits are discussed in Chapters XI and XII.) Three equivalent circuits are shown in Figs. 7(*a*), (*b*), and (*c*), each constructed on a per phase basis. Figure 7 (*a*) is a *capacitive admittance equivalent circuit* in which b_{aa} is the self-capacitive susceptance per mile of circuit A with circuit B grounded, b_{bb} the capacitive susceptance per mile of circuit B with circuit A grounded, and b_{ab} is the mutual capacitive susceptance per mile between circuits A and B, determined with either A or B grounded. Figure 7 (*b*) is a *capacitive impedance equivalent circuit* in which x_{aa} and x_{bb} are the self-capacitive reactances in ohms-miles of circuits A and B, respectively, each determined with no current in the other circuit; x_{ab} is the mutual capacitive reactance in ohms-miles between circuits A and B, determined with no current in one of the circuits.

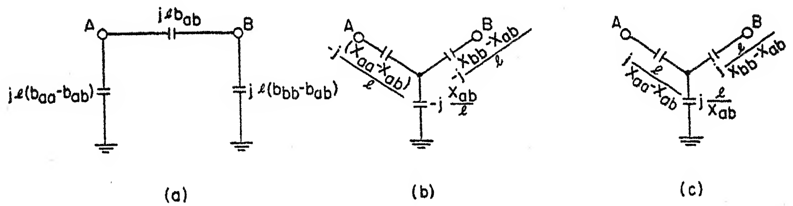


FIG. 7. Nominal equivalent capacitance circuits for two parallel lines of length l. (*a*) Admittance Δ; (*b*) Impedance Y; (*c*) Admittance Y.

Although calculated in different ways, either of these circuits may be obtained from the other, and either may be used in determining equivalent circuits for two parallel transmission lines.

Figure 7 (*c*) is a capacitive admittance Y-connected circuit, obtained from Fig. 7 (*b*) by replacing its capacitive impedance branches by reciprocal capacitive admittances. In an equivalent circuit composed of static branches, the impedances of all branches may be replaced by their reciprocal admittances to obtain an admittance circuit; likewise, an admittance circuit may be converted to an impedance circuit. Figure 7 (*a*) may be obtained by converting the Y of Fig. 7 (*c*) into a Δ, or directly from Fig. 7 (*b*) by means of the following equations, determined by inverting the fractions on the right-hand sides of equations [40], Chapter I, and simplifying:

$$
\begin{aligned}
b_{aa} &= \frac{x_{bb}}{x_{aa}x_{bb} - x_{ab}^2} \\
b_{bb} &= \frac{x_{aa}}{x_{aa}x_{bb} - x_{ab}^2} \qquad [29] \\
b_{ab} &= \frac{x_{ab}}{x_{aa}x_{bb} - x_{ab}^2}
\end{aligned}
$$

Figure 7(b) may be obtained from Fig. 7(a) by means of the equations

$$
\begin{aligned}
x_{aa} &= \frac{b_{bb}}{b_{aa}b_{bb} - b_{ab}^2} \\
x_{bb} &= \frac{b_{aa}}{b_{aa}b_{bb} - b_{ab}^2} \qquad [30] \\
x_{ab} &= \frac{b_{ab}}{b_{aa}b_{bb} - b_{ab}^2}
\end{aligned}
$$

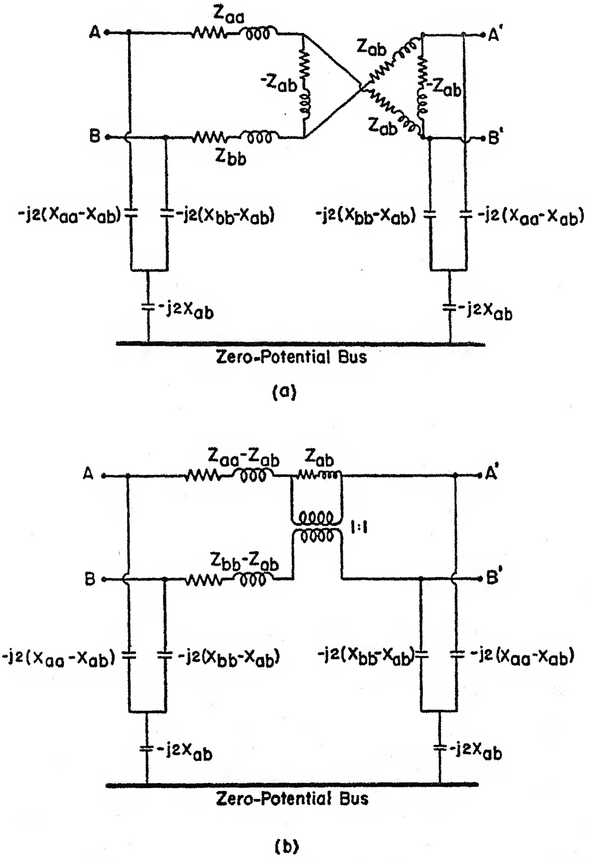
With one transmission line only, assumed symmetrical, the capacitive admittance $jb\ell$ in mhos and the capacitive impedance $-j(x/\ell)$ in ohms are reciprocals of each other. With two parallel lines, x_{aa}, x_{bb}, and x_{ab} are *not the reciprocals* of b_{aa}, b_{bb}, and b_{ab} as may be seen from equations [29] and [30]. Also, by definition, b_{aa} is determined with circuit B grounded, while $1/x_{aa}$ is determined with circuit B open.

With ground wires or more than two parallel transmission lines, self- and mutual capacitive impedances are more easily calculated than capacitive admittances. Curves are given in Chapter XII, from which the self-capacitive impedances of each circuit and the mutual capacitive impedances between circuits taken two at a time can be obtained. Capacitive impedances are also more convenient in developing equivalent circuits for three or more parallel lines than capacitive susceptance, and will be used in the work which follows.

Nominal Π *Equivalent Circuits.* Figures 8(a) and (b) show equivalent circuits for two parallel lines not bussed at either end represented by their *nominal* Π-*lines*. Figure 8(a) is for use in analytic calculations; Fig. 8(b) may be used on the a-c network analyzer. The architraves of the Π's consist of the inductive self-impedances Z_{aa} and Z_{bb} of the two circuits mutually coupled through the mutual impedance Z_{ab}. Twice the self- and mutual capacitive reactances are used in the capacitive impedance Y's at the terminals of the lines. These impedance Y's can be converted to admittance Y's or Δ's as explained above.

The nominal equivalent circuits of Figs. 8(a) and (b) for two parallel lines can be extended to three or more parallel lines if the self-capacitive impedances of each line alone and the mutual capacitive impedances between lines, taken two at a time with the other lines open, are given. Figure 8(c) shows a nominal equivalent circuit for three parallel lines bussed at one end; in this equivalent circuit, two of the mutual inductive impedances and also two of the mutual capacitive impedances are equal. Figure 8(d) shows the nominal equivalent circuit for use on an a-c calculating table for four parallel lines in which the identity of the terminals of the four lines is retained. In this circuit all mutual inductive and capacitive impedances are assumed unequal.

The nominal equivalent circuits of Fig. 8 are combinations of equivalent circuits. The self- and mutual inductive impedances of the system



A
A'
B
B'
Zaa
Zab
-Zab
Zab
-Zab
Zab
Zbb
-j2(Xaa-Xab)
-j2(Xbb-Xab)
-j2(Xbb-Xab)
-j2(Xaa-Xab)
-j2Xab
-j2Xab
Zero-Potential Bus
(a)
A
A'
Zaa-Zab
Zab
1:1
Zbb-Zab
B
B'
-j2(Xaa-Xab)
-j2(Xbb-Xab)
-j2(Xbb-Xab)
-j2(Xaa-Xab)
-j2Xab
-j2Xab
Zero-Potential Bus
(b)

FIG. 8(*a* and *b*). Nominal equivalent circuits for parallel lines in which each line is represented by its nominal Π with inductive and capacitive mutual impedances between them. (*a*) and (*b*) — Two parallel lines.

are represented by an equivalent circuit between the two ends of lines, constructed with capacitance neglected. Self- and mutual capacitive impedances to ground are represented by equivalent circuits at the ends of the lines, constructed with inductance neglected. As half the capacitance is in each shunt circuit, self- and mutual capacitive impedances are multiplied by two. With self-capacitances to ground and mutual capacitances between circuits expressed in terms of capacitive impedances, the construction of the equivalent capacitance circuits between circuit terminals and ground is similar to that of inductively coupled circuits.

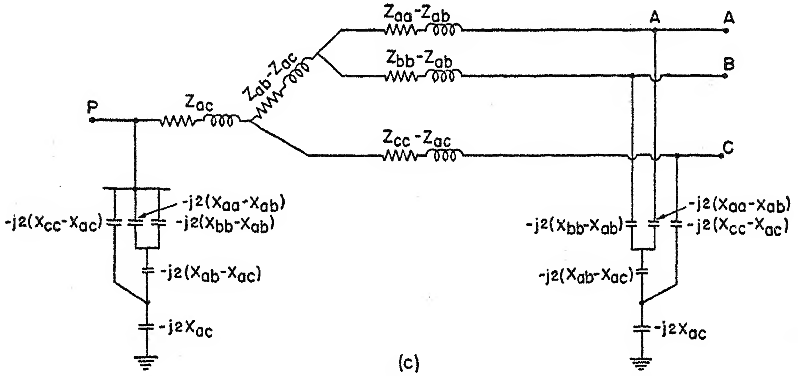


FIG. 8(*c*). Nominal equivalent circuit for three parallel lines bussed at one end P in which $Z_{ac} = Z_{bc}$ and $-jX_{ac} = -jX_{bc}$.

A fault on one of two or more parallel transmission lines may be considered to divide the parallel lines into two sections, each of which may be replaced by its equivalent circuit, the fault being located on the given circuit at the junction of the equivalent circuits.

NORMAL OPERATION OF A SYMMETRICAL THREE-PHASE TRANSMISSION CIRCUIT

With the power to be transmitted and the distance of transmission given, the *choice of transmission voltage and number of circuits* are influenced by (1) the voltage drop in the line, (2) the power loss under normal operating conditions, and (3) the requirements for power system stability during steady-state operation and under specified transient conditions. The subject of power system stability is not covered in this volume.

At a specified voltage, the choice of conductors is influenced by (1) the diameter of the conductors required to avoid corona under normal operating conditions and (2) the allowable temperature of the conductors when carrying maximum load current. Figure 8 of Appendix B gives approximate corona starting voltages versus geometric mean spacing between conductors for copper and A.C.S.R. conductors used in overhead transmission circuits. Figures 9 and 10 of Appendix B give temperature rise above ambient temperature versus current in the conductors for copper and A.C.S.R. conductors, respectively.

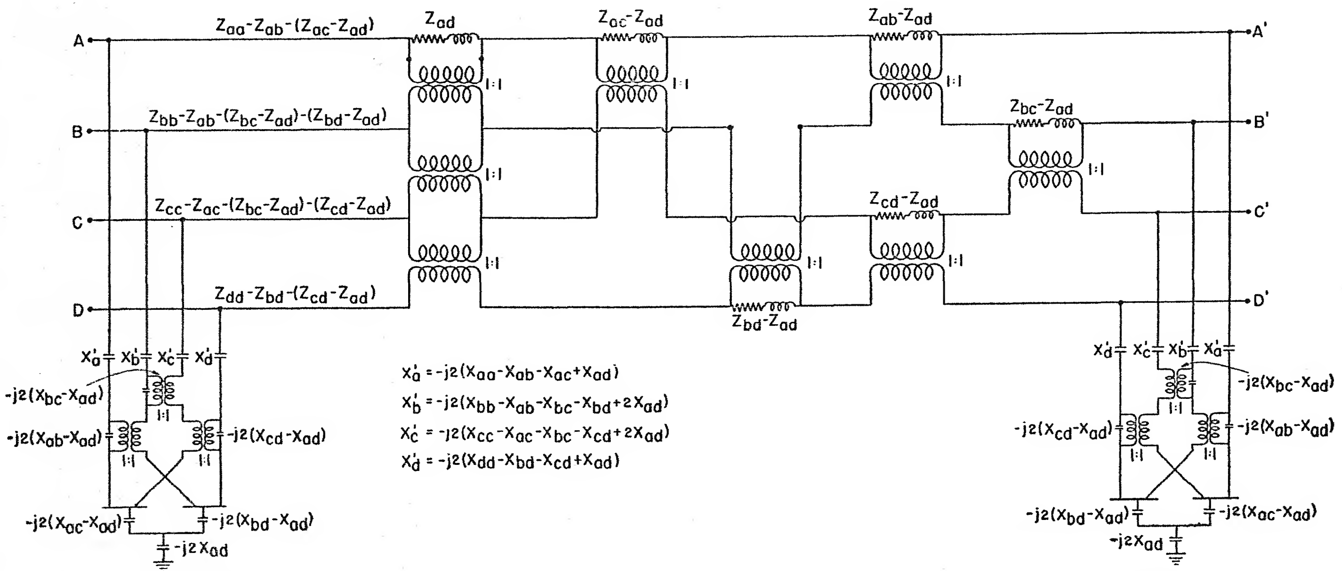

A
B
C
D
A'
B'
C'
D'
Zaa-Zab-(Zac-Zad)
Zbb-Zab-(Zbc-Zad)-(Zbd-Zad)
Zcc-Zac-(Zbc-Zad)-(Zcd-Zad)
Zdd-Zbd-(Zcd-Zad)
Zad
Zac-Zad
Zab-Zad
Zbc-Zad
Zcd-Zad
Zbd-Zad
1:1
X'a
X'b
X'c
X'd
-j2(Xbc-Xad)
-j2(Xab-Xad)
-j2(Xcd-Xad)
-j2(Xac-Xad)
-j2(Xbd-Xad)
-j2Xad
X'a = -j2(Xaa-Xab-Xac+Xad)
X'b = -j2(Xbb-Xab-Xbc-Xbd+2Xad)
X'c = -j2(Xcc-Xac-Xbc-Xcd+2Xad)
X'd = -j2(Xdd-Xbd-Xcd+Xad)

FIG. 8(*d*). Nominal equivalent circuit for four parallel lines in which all inductive and capacitive mutual impedances are unequal, and the identities of all terminals are retained.

During normal operation, the currents and voltages in a symmetrical three-phase power system are positive-sequence currents and voltages. The conditions in the transmission circuit at specified terminal conditions can be calculated from [13]–[16] of this chapter when positive-sequence resistance, reactance, and capacitive susceptance are known. Resistances and internal reactances of commonly used conductors are given in the wire tables of Appendix B; positive-sequence reactances and capacitive susceptances can be obtained from the curves of Appendix B.

Problem 7. Solve Problem 4 with the two Δ–Y transformer banks ungrounded.

Problem 8. Approximately what value of reactance in ohms placed in the neutrals of the transformers in Fig. 3(*a*) would neutralize the zero-sequence capacitance and keep the line from being cut out of service during a line-to-ground fault at *B* caused by lightning? Would it require more or less reactance with one-half in the neutral of each transformer, or all in the neutral of one transformer with the other neutral ungrounded? What would be the kva rating of the ground-fault neutralizer (or neutralizers), based on normal line-to-neutral voltage and the maximum current it would be required to carry during line-to-ground faults?

Problem 9. In Fig. 2, Chapter IV, consider that the per unit impedances in the sequence networks are based on a three-phase kva base of 100,000 kva and base line-to-line voltage of 115 kv in the transmission circuit. The lines *A* and *B* are identical, and are 50 miles long. The frequency is 60 cycles. The positive- and zero-sequence line constants for each line, determined with the other line open, are

$z_1 = 0.278 + j0.794$ ohm per mile; $y_1 = j5.2 \times 10^{-6}$ mho per mile

$z_0 = 0.55 + j2.40$ ohms per mile; $y_0 = j2.85 \times 10^{-6}$ mho per mile

The positive-sequence mutual inductive and capacitive impedances are negligible. The zero-sequence mutual impedance $z_{ab} = j1.30$ ohms per mile. The zero-sequence mutual capacitive impedance (determined with one circuit open) is $-jx_{ab} = -j0.125 \times 10^6$ ohm-miles.

Construct a nominal equivalent circuit for the two parallel lines, (*a*) not bussed at either end, (*b*) bussed at the sending end only, (*c*) bussed at both ends. (The capacitive admittance y_0 given for one line alone can be changed to capacitive impedance by taking its reciprocal, which is required in determining a capacitive impedance Y-connected circuit.)

Determine the fault current with a line-to-ground fault on phase *a* at *B*, as in Fig. 2 of Chapter IV, neglecting resistance. Compare this current with that obtained by using the sequence networks given in Figs. 2(*b*), (*c*), and (*d*) of Chapter IV, with capacitance neglected.

Problem 10. Assume that the transformer bank at *C* in Fig. 2(*a*), Chapter IV, is ungrounded and that circuit breakers at *B* have opened, leaving a line-to-ground fault on phase *a* of one of the lines at its sending end terminals. The equivalent excitation on machine *N* is 100% of base voltage referred to the line side of the transformer bank. Using the equivalent circuit for the two parallel lines determined in Problem 9, find the voltages to ground at *B* of the two unfaulted phases, neglecting resistance.

Problem 11. Construct the equivalent Π's for use in the positive- and zero-sequence networks for a transmission line 400 miles long operating at 50 cycles.

$z_1 = 0.12 + j0.67$ ohm per mile; $y_1 = j4.40 \times 10^{-6}$ mho per mile

$z_0 = 0.60 + j1.80$ ohms per mile; $y_0 = j2.40 \times 10^{-6}$ mho per mile

Problem 12. Construct an equivalent circuit consisting of two sections, each 200 miles long, using z_1 and y_1 given in Problem 11. By transformations reduce this equivalent circuit to a single Π and compare with the positive-sequence Π of Problem 11.

Problem 13. Derive the equivalent T and Π given in Figs. 1 (*d*) and (*e*) for a line with distributed constants from equations [13]–[16]. (Problem for mathematicians only.)

Problem 14. Solve Problem 5 by the second method discussed in Problem 5.

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CHAPTER VII

SIMULTANEOUS FAULTS ON SYMMETRICAL THREE-PHASE SYSTEMS — ANALYSIS BY THE METHOD OF SYMMETRICAL COMPONENTS

The term " fault " is here used to denote an accidental departure from normal operating conditions. A short circuit or an open conductor constitutes a fault. When a fuse opens one end of a short-circuited conductor without clearing the short circuit, the short circuit and open conductor are simultaneous faults on the same conductor. Simultaneous faults may consist of two or more short circuits on the same or on different circuits, open conductors in two or more circuits, or any combinations of short circuits and open conductors. Since each fault affects the voltages and currents resulting from the other, simultaneous faults cannot be treated independently. Simultaneous short circuits and a short circuit and open conductor on the same phase of a symmetrical three-phase system are discussed in this chapter. Simultaneous faults are further discussed in Chapter X.

TWO SIMULTANEOUS SHORT CIRCUITS

As with one short circuit, system currents and voltages during two simultaneous short circuits may be determined analytically or by means of a calculating table. Both methods will be given here. As the faults may occur on the same or on opposite sides of a Δ–Y transformer bank, both cases will be included. Grounded and ungrounded systems will be considered.

Grounded System — No Δ–Y Transformer Bank between Faults

Let the two fault points be C and D, with the conductors at C indicated by a, b, c and those at D by A, B, C, where A and a are of the first phase, B and b of the second phase, and C and c of the third phase in the order of their normal time sequence. Let V_a, V_b, V_c and V_A, V_B, V_C indicate the voltages to ground of conductors a, b, c at C and A, B, C at D, respectively, with the corresponding currents flowing from the conductors into the fault indicated by I_a, I_b, I_c and I_A, I_B, I_C, respectively. Figure 1 shows two faults at different points C and D, with the conductor voltages to ground and currents flowing into the fault indicated by their assigned symbols. Positive direction for currents and their components is taken *into* the faults, as indicated by the arrows.

The symmetrical components of V_a and I_a at C are V_{a1}, V_{a2}, V_{a0} and I_{a1}, I_{a2}, I_{a0}, respectively. The symmetrical components of V_A and I_A at D are V_{A1}, V_{A2}, V_{A0} and I_{A1}, I_{A2}, I_{A0}, respectively. With phase a as reference phase, these are the twelve unknowns to be determined. It will be shown that there are twelve independent equations connecting these twelve unknowns: three for each fault point, and two relating components of fault currents and voltages in each of the three sequence networks. These twelve equations are required in an analytic solution. When a calculating table is available, the sequence networks are connected to satisfy the equations which relate symmetrical components of currents and of voltages at both fault points; as explained later, the connections may be direct, through coupling transformers, or through phase shifters. As the six equations relating the components of voltage or current at the two fault points are required in both analytic solutions and in solutions on a calculating table, they will be developed first.

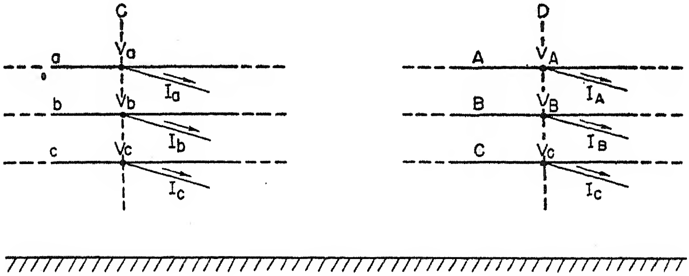


FIG. 1. Simultaneous short circuits at points C and D of a symmetrical three-phase system.

Equations Relating the Components of Fault Voltage or of Fault Current of Different Sequences. Although simultaneous short circuits cannot be treated independently, each may be considered separately to determine the equations relating the symmetrical components of current flowing into the fault, or of voltage to ground at the fault of the reference phase a. Such equations are determined in Chapter III for one short circuit involving various phases. Where there is only one fault on a symmetrical three-phase system, its location with respect to the reference phase may be arbitrarily chosen. With two faults, the location of one of them may be arbitrary, but that of the other will depend upon given fault conditions. For example, let it be stated that a double line-to-ground fault occurs at point C, and a single line-to-ground fault at point D on the normally leading phase of the two phases involved in the fault at point C; then, if the double line-to-ground fault at point C is arbitrarily located on phases b and c, the location of the line-to-ground fault at point D is of necessity on phase b (conductor B) for the assumed phase order abc.

TABLE I

FAULT EQUATIONS EXPRESSING RELATIONS BETWEEN THE SYMMETRICAL COMPONENTS OF I_a AND V_a

Phase a is reference phase. I_a is current flowing into the fault and V_a is voltage to ground at the fault.

Case A. Line-to-Ground Fault

(*a*) *Phase a*

$$I_{a0} = I_{a1} \qquad I_{a2} = I_{a1} \qquad V_{a1} = -(V_{a0} + V_{a2})$$

(*b*) *Phase b*

$$I_{a0} = a^2 I_{a1} \qquad I_{a2} = a I_{a1} \qquad V_{a1} = -(a V_{a0} + a^2 V_{a2})$$

(*c*) *Phase c*

$$I_{a0} = a I_{a1} \qquad I_{a2} = a^2 I_{a1} \qquad V_{a1} = -(a^2 V_{a0} + a V_{a2})$$

Case B. Line-to-Line Fault

(*a*) *Phases b and c*

$$I_{a0} = 0 \qquad I_{a2} = -I_{a1} \qquad V_{a2} = V_{a1}$$

(*b*) *Phases a and c*

$$I_{a0} = 0 \qquad I_{a2} = -a I_{a1} \qquad V_{a2} = a V_{a1}$$

(*c*) *Phases a and b*

$$I_{a0} = 0 \qquad I_{a2} = -a^2 I_{a1} \qquad V_{a2} = a^2 V_{a1}$$

Case C. Double Line-to-Ground Fault

(*a*) *Phases b and c*

$$I_{a1} = -(I_{a0} + I_{a2}) \qquad V_{a0} = V_{a1} \qquad V_{a2} = V_{a1}$$

(*b*) *Phases a and c*

$$I_{a1} = -(a I_{a0} + a^2 I_{a2}) \qquad V_{a0} = a^2 V_{a1} \qquad V_{a2} = a V_{a1}$$

(*c*) *Phases a and b*

$$I_{a1} = -(a^2 I_{a0} + a I_{a2}) \qquad V_{a0} = a V_{a1} \qquad V_{a2} = a^2 V_{a1}$$

Case D. Three-Phase Fault

(*a*) *Phases a, b, and c*

$$I_{a0} = 0 \qquad V_{a1} = 0 \qquad V_{a2} = 0$$

(*b*) *Phases a, b, c, and Ground*

$$V_{a1} = 0 \qquad V_{a2} = 0 \qquad V_{a0} = 0$$

Table I gives three equations connecting the symmetrical components of current flowing into the fault or of voltage to ground at the fault for short circuits involving various phases, phase a being reference phase. The equations given under cases $A(a)$, $B(a)$, $C(a)$, $D(a)$, and $D(b)$ are derived in Chapter III. Derivation of other equations of Table I is given in the following development:

Line-to-Line Fault between Phases a and b. The conditions at the fault are $V_a = V_b$; $I_a = -I_b$; $I_c = 0$. From these equations and those of Chapter II,

$$\begin{aligned} V_a - V_b &= V_{a1} + V_{a2} + V_{a0} - (a^2 V_{a1} + aV_{a2} + V_{a0}) \\ &= (1 - a^2)V_{a1} + (1 - a)V_{a2} = 0 \\ I_{a1} &= \tfrac{1}{3}(I_a + aI_b + a^2 I_c) = \tfrac{1}{3}I_a(1 - a) \\ I_{a2} &= \tfrac{1}{3}(I_a + a^2 I_b + aI_c) = \tfrac{1}{3}I_a(1 - a^2) \\ I_{a0} &= \tfrac{1}{3}(I_a + I_b + I_c) = 0 \end{aligned} \qquad [1]$$

From [1], the relations between the symmetrical components of V_a and of I_a are

$$V_{a2} = -V_{a1}\frac{1 - a^2}{1 - a} = -V_{a1}(1 + a) = a^2 V_{a1} \qquad [2]$$

$$I_{a2} = -a^2 I_{a1} \qquad [3]$$

Line-to-Ground Fault on Phase b. The conditions at the fault are $V_b = 0$; $I_a = 0$; $I_c = 0$. It therefore follows that

$$\begin{aligned} V_b &= a^2 V_{a1} + aV_{a2} + V_{a0} = 0 \\ I_{a1} &= \tfrac{1}{3}(aI_b) \\ I_{a2} &= \tfrac{1}{3}(a^2 I_b) \\ I_{a0} &= \tfrac{1}{3}(I_b) \end{aligned} \qquad [4]$$

From [4],

$$V_{a1} = -a^2 V_{a2} - aV_{a0} \qquad [5]$$

$$I_{a2} = aI_{a1} \qquad [6]$$

$$I_{a0} = a^2 I_{a1} \qquad [7]$$

Double Line-to-Ground Fault on Phases a and c. The conditions at the fault are $V_a = 0$; $V_c = 0$; $I_b = 0$. From the fault equations,

$$\begin{aligned} V_a - V_c &= V_{a1} + V_{a2} + V_{a0} - (aV_{a1} + a^2 V_{a2} + V_{a0}) \\ &= (1 - a)V_{a1} + (1 - a^2)V_{a2} = 0 \\ V_a + V_c &= V_{a1} + V_{a2} + V_{a0} + (aV_{a1} + a^2 V_{a2} + V_{a0}) \\ &= (1 + a)V_{a1} + (1 + a^2)V_{a2} + 2V_{a0} = 0 \\ I_b &= a^2 I_{a1} + aI_{a2} + I_{a0} = 0 \end{aligned} \qquad [8]$$

From [8],

$$V_{a2} = aV_{a1} \qquad [9]$$

$$V_{a0} = a^2V_{a1} \qquad [10]$$

$$I_{a1} = -a^2I_{a2} - aI_{a0} \qquad [11]$$

The relations between the symmetrical components of V_a and I_a for a line-to-line fault on phases a and c, a line-to-ground fault on phase c, and a double line-to-ground fault on phases a and b can be determined from [1], [4], and [8], respectively, if a and a^2 are interchanged in these equations.

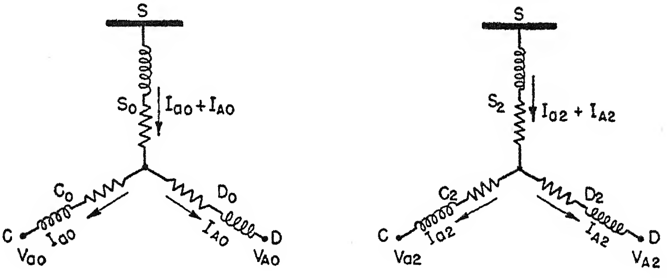
The three equations connecting the symmetrical components of I_a or of V_a at point C for various types of short circuits can be taken directly from Table I. If a, b, and c in Table I are replaced by A, B, and C, respectively, three equations relating the components of I_A or of V_A at point D are also obtained. Table I furnishes the six equations required when solutions are made on a calculating table. It also provides six of the twelve equations required in an analytic solution. These six equations give relations between the components of fault current or of fault voltage of different sequences and are independent of system impedances.

The other six equations needed in an analytic solution give relations between fault currents and voltages of the same sequence, and therefore depend upon the impedances in the three sequence networks of the symmetrical three-phase system.

Equations Relating Components of Fault Voltage and Fault Current of the Same Sequence. Equations [1]–[3] of Chapter IV express components of fault voltage in terms of components of fault current and the sequence impedances viewed from the fault, when there is but one short circuit. With two short circuits, there are two components of fault voltage and two components of fault current in each of the three sequence networks. If each sequence network is replaced[1] by an equivalent Δ or Y between the two fault points and the zero-potential bus for the network, two equations relating the components of fault currents and voltages of each sequence can be written (as explained below), thereby giving the six additional equations required in an analytic solution.

Zero-Sequence Network. In a symmetrical system, there are no generated zero-sequence voltages. The equivalent Y of the zero-sequence network between fault points C and D and the zero-potential bus for the network, here indicated by S, is shown in Fig. 2(a). The three branches of the equivalent Y are labeled C_0, D_0, and S_0 and are represented as inductive impedances. (This is true also for the equivalent Y's replacing the negative- and positive-sequence networks in

Figs. 2(*b*) and 3, respectively.) This is not intended to exclude capacitive impedances, or negative resistances which may result from the reduction of the sequence networks to equivalent Y's or Δ's.



(a) (b)

FIG. 2. Equivalent Y's to replace the sequence networks between fault points *C* and *D* and the zero-potential bus for the network, here indicated by *S*. (*a*) Equivalent Y of zero-sequence network. (*b*) Equivalent Y of negative-sequence network.

In Fig. 2(*a*), the zero-sequence impedance viewed from *C*, with no fault at *D*, is $C_0 + S_0$; that from *D*, with no fault at *C*, is $D_0 + S_0$. With positive direction for currents towards the faults, I_{a0} and I_{A0} flow from the zero-potential bus *S* through S_0, then I_{a0} flows through C_0 and I_{A0} through D_0. Before the simultaneous faults occurred there were no currents and no voltages in the zero-sequence network. Superposing the zero-sequence voltage rises from *S* to *C* and from *S* to *D* on the voltages at *C* and *D*, respectively, before the fault (or subtracting the voltage drops from zero), V_{a0} and V_{A0} are

$$V_{a0} = 0 - (I_{a0} + I_{A0})S_0 - I_{a0}C_0 = -I_{a0}(C_0 + S_0) - I_{A0}S_0 \quad [12]$$

$$V_{A0} = 0 - (I_{a0} + I_{A0})S_0 - I_{A0}D_0 = -I_{a0}S_0 - I_{A0}(D_0 + S_0) \quad [13]$$

Equations [12] and [13] express the zero-sequence components of voltage at the two points of fault in terms of the two zero-sequence currents flowing into the faults and the branch impedances of the equivalent Y which replaces the zero-sequence network between *C*, *D*, and *S*.

Solving [12] and [13], the currents I_{a0} and I_{A0} are expressed in terms of V_{a0} and V_{A0}:

$$I_{a0} = -V_{a0}\frac{D_0 + S_0}{\Delta_0} + V_{A0}\frac{S_0}{\Delta_0} \quad [14]$$

$$I_{A0} = V_{a0}\frac{S_0}{\Delta_0} - V_{A0}\frac{C_0 + S_0}{\Delta_0} \quad [15]$$

where $\Delta_0 = C_0D_0 + C_0S_0 + D_0S_0$

Equations [14] and [15] are not independent of [12] and [13]; there are only two independent equations connecting the four unknowns, V_{a0}, V_{A0}, I_{a0}, and I_{A0}, in the zero sequence network.

Negative-Sequence Network. In a symmetrical system, there are no generated negative-sequence voltages. The negative-sequence network, just as the zero-sequence network, can be replaced by an equivalent Y or Δ between the points of fault C and D and the zero-potential bus for the network, here indicated by S. The equivalent Y of the negative-sequence network between points C, D, and S is shown in Fig. 2(*b*). The branch impedances are indicated by C_2, D_2, and S_2. The negative-sequence impedance viewed from C, with no fault at D, is $C_2 + S_2$; that from D, with no fault at C, is $D_2 + S_2$.

From Fig. 2(*b*), with positive direction for currents towards the faults, the two negative-sequence voltages in terms of the two negative-sequence currents and the impedances of the Y are

$$V_{a2} = -I_{a2}(C_2 + S_2) - I_{A2}S_2 \qquad [16]$$

$$V_{A2} = -I_{a2}S_2 - I_{A2}(D_2 + S_2) \qquad [17]$$

Solving [16] and [17], the currents I_{a2} and I_{A2} are expressed in terms of V_{a2} and V_{A2}:

$$I_{a2} = -V_{a2}\frac{D_2 + S_2}{\Delta_2} + V_{A2}\frac{S_2}{\Delta_2} \qquad [18]$$

$$I_{A2} = V_{a2}\frac{S_2}{\Delta_2} - V_{A2}\frac{C_2 + S_2}{\Delta_2} \qquad [19]$$

where

$$\Delta_2 = C_2D_2 + C_2S_2 + D_2S_2$$

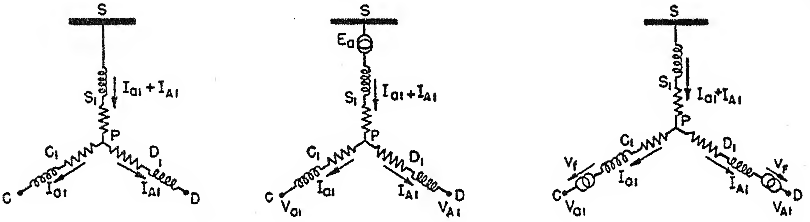
Equations [18] and [19] are not independent of [16] and [17]; there are only two independent equations connecting the four unknowns, V_{a2}, V_{A2}, I_{a2}, and I_{A2} in the negative-sequence network.

Positive-Sequence Network. The positive-sequence impedance network without generated voltages, just as the zero- and negative-sequence impedance networks, can be replaced by an equivalent Y or Δ between the zero-potential bus for the network, here indicated by S, and the two fault points C and D. The equivalent Y is shown in Fig. 3(*a*) with branch impedances S_1, C_1, and D_1. The positive-sequence impedance viewed from C, with no fault at D, is $C_1 + S_1$; that from D, with no fault at C, is $D_1 + S_1$. The positive-sequence network of a symmetrical system differs from the negative- and zero-sequence networks in that there are generated voltages in all synchronous machines.

If the system is operating at *no load*, neglecting charging currents, the per unit generated voltages and the voltages at all points in the positive-sequence network are equal and in phase. Let the voltage of phase a be E_a. For this case, the internal voltages of all the machines can be replaced by a voltage E_a between the zero-potential bus S for the network and the branch impedance S_1 of the equivalent Y, as in Fig. 3(*b*). From Fig. 3(*b*), subtracting the voltage drops resulting from the fault from the voltages at C and D before the fault,

$$V_{a1} = E_a - I_{a1}(C_1 + S_1) - I_{A1}S_1 \tag{20}$$

$$V_{A1} = E_a - I_{a1}S_1 - I_{A1}(D_1 + S_1) \tag{21}$$



(a) (b) (c)

FIG. 3. Positive-sequence equivalent Y's between the zero-potential bus S for the network and the fault points C and D. (*a*) All generated voltages equated to zero. (*b*) System operating at no load before the fault. (*c*) System under load with voltages V_f and V_F at C and D, respectively, before the fault.

If the system is operating *under load*, the voltages and currents throughout the system are determined by the given operating condition. Let the voltages of phase a at C and D before the faults be indicated by V_f and V_F, respectively. When the fault occurs, the positive-sequence voltages at C and D become V_{a1} and V_{A1}, respectively. The positive-sequence currents flowing from the system at C and D before the fault are zero; when the fault occurs, they become I_{a1} and I_{A1}, respectively. The changes in currents are, therefore, I_{a1} and I_{A1}. Subtracting the voltage drops (or superposing the voltage rises) caused by the fault currents flowing from the zero-potential bus through the network to the fault points from the voltages V_f and V_F at C and D, respectively, before the fault occurred, V_{a1} and V_{A1} are

$$V_{a1} = V_f - I_{a1}(C_1 + S_1) - I_{A1}S_1 \tag{22}$$

$$V_{A1} = V_F - I_{a1}S_1 - I_{A1}(D_1 + S_1) \tag{23}$$

Equations [22] and [23] are satisfied by the equivalent circuit shown in Fig. 3(*c*), where *series* voltages V_f and V_F are inserted between the impedances C_1 and D_1, and fault points C and D, respectively. V_f

and V_F represent voltage *rises* in the direction PC and PD, respectively, as indicated by the accompanying arrows. Figure 3(*c*) is an equivalent circuit for the positive-sequence network to be used for determining positive-sequence voltages at the faults and positive-sequence currents flowing into the faults. In this equivalent circuit, V_{a1}, V_{A1}, I_{a1}, and I_{A1} are unknown quantities which cannot be determined from the positive-sequence network alone; but the voltages V_{a1} and V_{A1} in terms of I_{a1}, I_{A1}, and the known quantities V_f, V_F, S_1, C_1, D_1 are given by [22] and [23]. If $V_F = V_f$, [22] and [23] are similar in form to [20] and [21], respectively, and therefore Fig. 3(*b*) can be used with V_f replacing E_a.

Analytic Determination of Currents and Voltages

It has been shown that there are twelve equations connecting the twelve unknown components of current and voltage of phase a at the two fault points, three for each fault point and two for each of the three sequence networks. It is proposed to eliminate the eight unknown negative- and zero-sequence components, leaving four equations in terms of the four positive-sequence components. Two of these four equations will be in terms of the negative- and zero-sequence system impedances, and will therefore correspond to [4] of Chapter IV for one short circuit. The other two equations, given by [22] and [23] or by [20] and [21], involve positive-sequence quantities only; they correspond to [1] of Chapter IV for one short circuit. The four positive-sequence equations will be solved for the two positive-sequence currents flowing into the faults and the two positive-sequence voltages at the fault. From the positive-sequence components at the faults and the relations between the symmetrical components at the two fault points, the negative- and zero-sequence components at the faults will be determined. From the symmetrical components of currents and voltages at the faults and the sequence networks, the symmetrical components of current and voltage throughout the system can be obtained.

The two positive-sequence equations in terms of the negative- and zero-sequence system impedances will be considered for the purpose of determining an equivalent circuit to replace the two faults in the positive-sequence network.

Solution of Ten Simultaneous Equations. The six equations for the two fault points from Table I, [12] and [13] from the zero-sequence network, and [16] and [17] from the negative-sequence network give the ten equations needed to eliminate the eight unknowns V_{a0}, V_{A0}, I_{a0}, I_{A0}, V_{a2}, V_{A2}, I_{a2}, and I_{A2}, so that the components of voltage

V_{a1} and V_{A1} may be expressed in terms of the components of current, I_{a1} and I_{A1}, and the known zero- and negative-sequence impedances of the system. When the ten equations are linear, the two resulting equations can be put in the form:

$$V_{a1} = kI_{a1} + mI_{A1} \tag{24}$$

$$V_{A1} = nI_{a1} + lI_{A1} \tag{25}$$

where k, l, m, and n depend upon the branch impedances of the equivalent Y's replacing the negative- and zero-sequence networks and the particular combination of conductors involved in the simultaneous faults. They are independent of positive-sequence impedances and of operating conditions, except as operating conditions affect negative- and zero-sequence impedances.

Equations [24] and [25] together with [20] and [21], or [22] and [23], give the four equations required for determining I_{a1}, I_{A1}, V_{a1}, and V_{A1}. Before solving these equations, the constants k, l, m, and n in [24] and [25] will be discussed.

Values of k, l, m, *and* n in terms of the branch impedances C_2, D_2, S_2, C_0, D_0, S_0 of the equivalent Y's which replace the negative- and zero-sequence networks (determined in reference 1) are tabulated in Table II for various combinations of phases which may be involved in simultaneous short circuits at two points of a grounded three-phase system. For some of the cases[2], equivalent Δ's and their admittances rather than equivalent Y's and their impedances would have given simpler equations and a reduction of work; for consistency, the equivalent Y's with impedance branches are used throughout.

The reduction of ten equations to two equations of the form given by [24] and [25], from which k, m, n, and l can be obtained, is given for the following case to illustrate the procedure.

Line-to-Ground Faults on Conductor A at D and Conductor b at C. From Table I, the three equations with the fault at D are

$$V_{A1} = -V_{A2} - V_{A0} \tag{26}$$

$$I_{A2} = I_{A1} \tag{27}$$

$$I_{A0} = I_{A1} \tag{28}$$

With the fault at C, they are

$$V_{a1} = -(a^2V_{a2} + aV_{a0}) \tag{29}$$

$$I_{a2} = aI_{a1} \tag{30}$$

$$I_{a0} = a^2I_{a1} \tag{31}$$

Equations [26]–[31], together with [12], [13], [16], and [17], are the ten equations to be solved. Replacing I_{A2} and I_{A0} by I_{A1}, I_{a2} and I_{a0}

by aI_{a1} and a^2I_{a1}, respectively, in [12], [13], [16], and [17], and then substituting [12] and [16] in [29] and [13] and [17] in [26],

$$\begin{aligned} V_{a1} &= I_{a1}(C_0 + S_0) + aI_{A1}S_0 + I_{a1}(C_2 + S_2) + a^2I_{A1}S_2 \\ &= I_{a1}(C_0 + S_0 + C_2 + S_2) + I_{A1}(aS_0 + a^2S_2) \qquad [32] \end{aligned}$$

$$\begin{aligned} V_{A1} &= a^2I_{a1}S_0 + I_{A1}(D_0 + S_0) + aI_{a1}S_2 + I_{A1}(D_2 + S_2) \\ &= I_{a1}(a^2S_0 + aS_2) + I_{A1}(D_0 + S_0 + D_2 + S_2) \qquad [33] \end{aligned}$$

From [32] and [33] and [24] and [25],

$k = (C_2 + S_2) + (C_0 + S_0) =$ sum of negative- and zero-sequence impedances viewed from C with no fault at D [34]

$l = (D_2 + S_2) + (D_0 + S_0) =$ sum of negative- and zero-sequence impedances viewed from D with no fault at C [35]

$$m = aS_0 + a^2S_2 = -\tfrac{1}{2}(S_0 + S_2) + j\frac{\sqrt{3}}{2}(S_0 - S_2) \qquad [36]$$

$$n = a^2S_0 + aS_2 = -\tfrac{1}{2}(S_0 + S_2) - j\frac{\sqrt{3}}{2}(S_0 - S_2) \qquad [37]$$

TABLE II

VALUES OF k, l, m, AND n TO BE SUBSTITUTED IN EQUATIONS [24] AND [25] FOR SIMULTANEOUS SHORT CIRCUITS AT TWO POINTS ON THE SAME SIDE OF A Δ–Y TRANSFORMER BANK

Let $Z_{cs} = C_0 + S_0 + C_2 + S_2$; $\Delta_0 = C_0D_0 + C_0S_0 + D_0S_0$

$Z_{ds} = D_0 + S_0 + D_2 + S_2$; $\Delta_2 = C_2D_2 + C_2S_2 + D_2S_2$

$Z_{cds} = (C_0 + C_2)(D_0 + D_2) + (C_0 + C_2)(S_0 + S_2) + (D_0 + D_2)(S_0 + S_2)$

Case A. *Single Line-to-Ground Faults at Two Points*

(*a*) Phases a and A	(*b*) Phases b and A	(*c*) Phases c and A
$k = Z_{cs}$	$k = Z_{cs}$	$k = Z_{cs}$
$n = S_0 + S_2$	$n = a^2S_0 + aS_2$	$n = aS_0 + a^2S_2$
$m = S_0 + S_2$	$m = aS_0 + a^2S_2$	$m = a^2S_0 + aS_2$
$l = Z_{ds}$	$l = Z_{ds}$	$l = Z_{ds}$

Case B. *Line-to-Line Faults at Two Points*

(*a*) Phases b, c and B, C	(*b*) Phases a, c and B, C	(*c*) Phases a, b and B, C
$k = C_2 + S_2$	$k = C_2 + S_2$	$k = C_2 + S_2$
$n = S_2$	$n = aS_2$	$n = a^2S_2$
$m = S_2$	$m = a^2S_2$	$m = aS_2$
$l = D_2 + S_2$	$l = D_2 + S_2$	$l = D_2 + S_2$

Case C. *Double Line-to-Ground Faults at Two Points*

(a) Phases b, c and B, C

$$k = \frac{\Delta_2(C_0 + S_0) + \Delta_0(C_2 + S_2)}{Z_{cds}}$$

$$n = \frac{\Delta_2 S_0 + \Delta_0 S_2}{Z_{cds}}$$

$$m = \frac{\Delta_2 S_0 + \Delta_0 S_2}{Z_{cds}}$$

$$l = \frac{\Delta_2(D_0 + S_0) + \Delta_0(D_2 + S_2)}{Z_{cds}}$$

(b) Phases a, c and B, C

$$k = \frac{\Delta_2(C_0 + S_0) + \Delta_0(C_2 + S_2)}{Z_{cds} + 3S_0S_2}$$

$$n = \frac{a^2S_0\Delta_2 + aS_2\Delta_0}{Z_{cds} + 3S_0S_2}$$

$$m = \frac{aS_0\Delta_2 + a^2S_2\Delta_0}{Z_{cds} + 3S_0S_2}$$

$$l = \frac{\Delta_2(D_0 + S_0) + \Delta_0(D_2 + S_2)}{Z_{cds} + 3S_0S_2}$$

(c) Phases a, b and B, C: Similar to C (b) with a and a^2 interchanged in n and m.

Case D. *Three-Phase Faults at Two Points:* $k = n = m = l = 0$.

Case E. *Line-to-Line Fault at C and Single Line-to-Ground Fault at D*

(a) Phases b, c, and A

$k = C_2 + S_2$
$n = -S_2$
$m = -S_2$
$l = Z_{ds}$

(b) Phases a, c, and A

$k = C_2 + S_2$
$n = -aS_2$
$m = -a^2S_2$
$l = Z_{ds}$

(c) Phases a, b, and A

$k = C_2 + S_2$
$n = -a^2S_2$
$m = -aS_2$
$l = Z_{ds}$

Case F. *Double Line-to-Ground Fault at C. Single Line-to-Ground Fault at D*

(a) Phases b, c, and A

$$k = \frac{(C_0 + S_0)(C_2 + S_2)}{Z_{cs}}$$

$$n = -\frac{S_0(C_2 + S_2) + S_2(C_0 + S_0)}{Z_{cs}}$$

$$m = -\frac{S_0(C_2 + S_2) + S_2(C_0 + S_0)}{Z_{cs}}$$

$$l = Z_{ds} - \frac{(S_0 - S_2)^2}{Z_{cs}}$$

(b) Phases a, c, and A

$$k = \frac{(C_0 + S_0)(C_2 + S_2)}{Z_{cs}}$$

$$n = -\frac{a^2S_0(C_2 + S_2) + aS_2(C_0 + S_0)}{Z_{cs}}$$

$$m = -\frac{aS_0(C_2 + S_2) + a^2S_2(C_0 + S_0)}{Z_{cs}}$$

$$l = Z_{ds} - \frac{S_0^2 + S_0S_2 + S_2^2}{Z_{cs}}$$

(c) Phases a, b, and A: Similar to F (b) with a and a^2 interchanged in n and m.

Case G. *Three-Phase Fault at C. Single Line-to-Ground at D on Phase A*

(a) Three-phase fault.

$$k = n = m = 0$$

$$l = D_0 + S_0 + D_2 + \frac{C_2S_2}{C_2 + S_2}$$

(b) Three-phase fault involving ground.

$$k = n = m = 0$$

$$l = D_0 + \frac{C_0S_0}{C_0 + S_0} + D_2 + \frac{C_2S_2}{C_2 + S_2}$$

The impedances k and l in [24] and [25] are effective self-impedances met by I_{a1} and I_{A1}, respectively, flowing from the positive-sequence network into the faults; m and n are effective mutual impedances associated with I_{A1} and I_{a1}, respectively. From a study of Table II, it may be seen that, with a line-to-line or a line-to-ground fault at one fault point, k or l at the other fault point is the equivalent circuit which would replace the fault in the positive-sequence network at that point if there were but one fault. (See equations [34] and [35].) With a double line-to-ground or a three-phase fault at one fault point, k or l at the other fault point is not so simply determined. In the solution of ten simultaneous equations, the conditions relating to negative- and zero-sequence currents and voltages imposed by the simultaneous faults are satisfied; therefore, one of the faults is not effectively removed by equating I_{A1} in [24] or I_{a1} in [25] to zero *unless* the corresponding negative- and zero-sequence components of fault current also become zero. This will be illustrated for the case of a three-phase fault not involving ground at C and a line-to-ground fault on conductor A at D. The fault equations at C are

$$V_{a1} = 0; \; V_{a2} = 0; \; I_{a0} = 0 \qquad [38]$$

Those at D are

$$I_{A1} = I_{A2} = I_{A0};$$
$$V_{A1} + V_{A2} + V_{A0} = 0 \qquad [39]$$

Equations [38] and [39] are satisfied by the connections indicated in Fig. 4, where the sequence networks are shown as equivalent Y's.

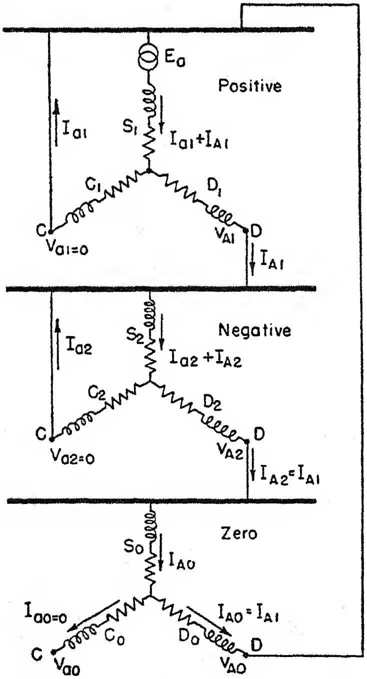


FIG. 4. Connection of the positive-, negative-, and zero-sequence equivalent Y's of a system operated at no load for a three-phase fault at C not involving ground and a line-to-ground fault on conductor A at D.

The effective self-impedance l met by I_{A1} flowing into the fault, calculated from Fig. 4, is

$$l = \frac{C_2 S_2}{C_2 + S_2} + D_2 + S_0 + D_0 \qquad [40]$$

The value of l in [40] is tabulated in Table II, Case $G(a)$. For this case, l is not the equivalent circuit which replaces the line-to-ground fault at D with the three-phase fault removed.

If the three-phase fault involves ground, from Table I, $V_{a0} = 0$ as well as $V_{a2} = 0$. The additional condition $V_{a0} = 0$ requires that point C in Fig. 4 be connected to the zero-potential bus in the zero-sequence network. With $V_{a2} = 0$ and $V_{a0} = 0$, I_{A1} flows through S_0 and C_0 in parallel as well as through S_2 and C_2 in parallel, the value of l met by I_{A1} being

$$l = \frac{C_2 S_2}{C_2 + S_2} + D_2 + \frac{C_0 S_0}{C_0 + S_0} + D_0$$

This value is tabulated in Table II, Case $G(b)$.

Solution of Four Positive-Sequence Equations and Determination of System Currents and Voltages. Equations [24] and [25] express V_{a1} and V_{A1} in terms of I_{a1} and I_{A1} and k, l, m, and n, which are functions of the negative- and zero-sequence system impedances. Equations [22] and [23] express V_{a1} and V_{A1} in terms of I_{a1} and I_{A1}, the impedances C_1, D_1, and S_1 of the positive-sequence equivalent Y, and V_f and V_F the voltages of phase a at C and D, respectively, before the fault. Eliminating V_{a1} and V_{A1} from these four equations,

$$V_f = I_{a1}(C_1 + S_1 + k) + I_{A1}(S_1 + m) \qquad [41]$$

$$V_F = I_{a1}(S_1 + n) + I_{A1}(D_1 + S_1 + l) \qquad [42]$$

Solving [41] and [42] for I_{a1} and I_{A1},

$$I_{a1} = \frac{V_f}{\Delta_1}(D_1 + S_1 + l) - \frac{V_F}{\Delta_1}(S_1 + m) \qquad [43]$$

$$I_{A1} = -\frac{V_f}{\Delta_1}(S_1 + n) + \frac{V_F}{\Delta_1}(C_1 + S_1 + k) \qquad [44]$$

where

$$\Delta_1 = (C_1 + S_1 + k)(D_1 + S_1 + l) - (S_1 + m)(S_1 + n) \qquad [45]$$

If $V_F = V_f$,

$$I_{a1} = \frac{V_f}{\Delta_1}(D_1 + l - m) \qquad [46]$$

$$I_{A1} = \frac{V_f}{\Delta_1}(C_1 + k - n) \qquad [47]$$

where Δ_1 is defined by [45].

I_{a1} and I_{A1} are given by [43] and [44] or [46] and [47] in terms of known voltages and impedances. Knowing I_{a1} and I_{A1}, V_{a1} and V_{A1} can be obtained from [24] and [25], respectively; and the negative- and zero-sequence components from the fault equations and [12], [13], [16], and [17]. The phase currents and voltages at the fault are calculated by substituting the symmetrical components of I_a and V_a in the equations of Chapter II. The negative- and zero-sequence system currents and voltages are determined by calculation from the negative- and zero-sequence components of fault currents and voltages and the complete negative- and zero-sequence networks, respectively. The *changes* in positive-sequence system currents and voltages resulting from the faults can be determined from the positive-sequence fault currents and the complete positive-sequence network with internal generated voltages equated to zero. The currents due to the faults superimposed upon load currents before the fault give total initial symmetrical rms positive-sequence currents; the voltage drops in the system due to the fault currents subtracted from the voltages at various system points before the fault give initial symmetrical rms positive-sequence system voltages.

When the positive-sequence network contains many loops, analytic determination of positive-sequence currents and voltages is simplified by the use of an equivalent circuit to replace both faults in the positive-sequence network.

Equivalent Circuits to Replace Both Faults in the Positive-Sequence Network. Equations [24] and [25] may be written

$$V_{a1} = (k - n)I_{a1} + \frac{m + n}{2}(I_{a1} + I_{A1}) + \frac{m - n}{2}(I_{A1} - I_{a1}) \quad [48]$$

$$V_{A1} = (l - m)I_{A1} + \frac{m + n}{2}(I_{a1} + I_{A1}) + \frac{m - n}{2}(I_{A1} - I_{a1}) \quad [49]$$

m and n Equal. When m and n are equal [48] and [49] become

$$V_{a1} = (k - m)I_{a1} + m(I_{a1} + I_{A1}) \quad [50]$$

$$V_{A1} = (l - m)I_{A1} + m(I_{a1} + I_{A1}) \quad [51]$$

The relations expressed in [50] and [51] are satisfied if the fault is replaced by an equivalent Y in the positive-sequence network with branch impedances $(k - m)$, $(l - m)$, and m, connecting the points C, D, and the zero-potential bus, here indicated by N. The equivalent circuit and the connection of this equivalent circuit at points C and D of the complete positive-sequence network are shown in Fig. 5. For simplicity, the network is represented as a rectangle with one side a heavy line to indicate the zero-potential bus for the network. The internal generated voltages of all synchronous machines of the system are understood to be present. The fault points *C* and *D* are shown as points within the rectangle with leads brought out to which the equivalent circuit is connected to satisfy fault conditions.

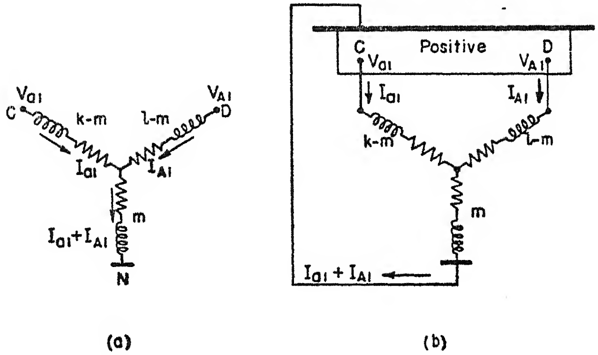


FIG. 5. (*a*) Equivalent Y to replace two short circuits in the positive-sequence network for special case of $m = n$ in [24] and [25]. (*b*) Positive-sequence network with equivalent circuit replacing two simultaneous short circuits when $m = n$ in [24] and [25].

The equivalent circuit in Fig. 5 is a function of the negative- and zero-sequence system impedances only; and, since these impedances are substantially the same for initial, transient, or steady-state operation, the equivalent circuit is a general one suitable for determining positive-sequence currents and voltages during initial, transient, or steady-state conditions.

Solution by Means of a Calculating Table

When a calculating table is used, it is desirable to set up the complete positive-, negative-, and zero-sequence networks and to connect them so that the relations between the symmetrical components of current and voltage of phase *a* at both fault points are satisfied. These relations are given in Table I for various types of short circuits, involving various phases.

Short Circuits Symmetrical with Respect to the Reference Phase. A line-to-ground fault on phase *a* and a line-to-line or a double line-to-ground fault on phases *b* and *c* are symmetrical with respect to the reference phase *a*. For those cases in which both faults are symmetrical with respect to phase *a*, direct connections of the sequence networks to satisfy the fault equations at one fault point (considered alone) can always be made. For the second fault point, direct connections can be made only if the three fault equations for both fault points are simultaneously satisfied. Direct connection of the sequence networks to satisfy the equations for both faults is illustrated in Fig. 4. When one of the faults is a three-phase fault, and also with line-to-line or double line-to-ground faults involving the same phases at both faults, the connections between the sequence networks made for each fault point are those which would be made if there were but one fault. These connections, shown in Fig. 6 where each sequence network is represented by a rectangle with points C and D within the rectangle and the zero-potential bus a heavy line, can be made on either an a-c or a d-c calculating table.

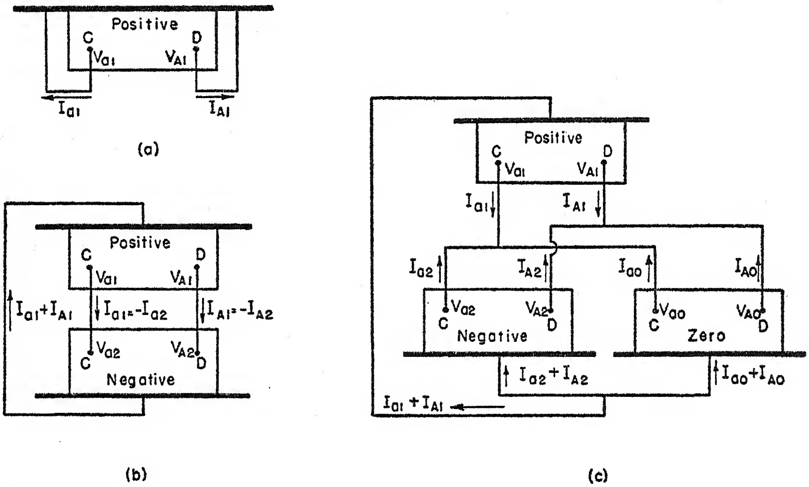


FIG. 6. Direct connections of the sequence networks for simultaneous faults. (*a*) Three-phase faults at C and D. (*b*) Line-to-line faults at C and D between phases b and c at both faults. (*c*) Double line-to-ground faults at C and D on phases b and c at both faults.

With one of the faults a line-to-ground fault on phase a and the other a line-to-ground, line-to-line, or double line-to-ground fault symmetrical with respect to phase a, direct connections of the sequence networks to satisfy voltage conditions at both faults may not (and in general do not) satisfy current conditions. Expressed in another way, voltage restrictions are introduced by direct connections for both faults which may not be true restrictions. For such cases, Dr. E. W. Kimbark[3] represents one fault by connecting the sequence networks through 1 : 1 turn ratio mutual coupling transformers; for the other fault, direct connection of the sequence networks is made. Figure 7 illustrates the connection of the sequence networks through 1 : 1 turn ratio transformers for a line-to-ground fault on phase a at C, direct connection of the sequence network being made for the line-to-ground fault on phase a at D. If the connections for the fault at C had been made in the same way as for the fault at D, the relations $V_{a1} = V_{A1}$, $V_{a2} = V_{A2}$, and $V_{a0} = V_{A0}$ would have been introduced. Except for fault points symmetrically located in all three sequence networks (for example, faults involving the same phase of two identical parallel lines bussed at both ends) these restricting equations are untrue.

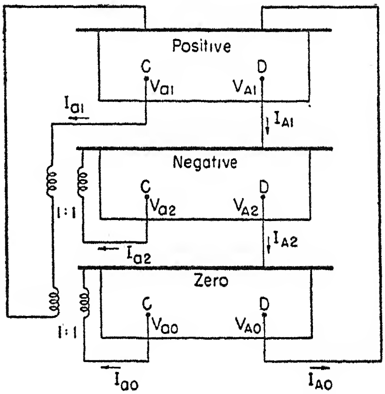


Fig. 7. Connection of the sequence networks through transformers of 1 : 1 turn ratio. Line-to-ground faults at C and D both on phase a.

One of Two Short Circuits Unsymmetrical with Respect to the Reference Phase. With the phases so named that the faulted phase or phases at one fault point are symmetrical with respect to phase a, while those at the other point are unsymmetrical, direct connections or connections by means of coupling transformers between the sequence networks can be made for the fault symmetrical with respect to phase a. Because of the operators a and a^2 in the fault equations of Table I which must be satisfied, direct connections or connections by means of coupling transformers cannot be made at the other fault point. Phase converters capable of rotating fault voltages and currents through 120° and 240° would be required.[3]

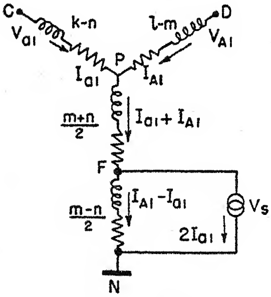


Fig. 8. A-c network analyzer equivalent circuit to replace two faults in the positive-sequence network when $m \neq n$ in [24] and [25].

An alternate method of solution[1] is by means of an equivalent circuit which replaces both faults in the positive-sequence network. By this method, the equivalent Y's to replace the negative- and zero-sequence networks are determined analytically or on the calculating table, and k, l, m, and n in terms of C_2, D_2, S_2, C_0, D_0, and S_0 read from Table II corresponding to the given fault conditions. Figure 8 shows a general equivalent circuit to replace the two short circuits in the positive-sequence network when an a-c calculating table is used. This equivalent circuit consists of a Y having branch impedances $k - n$, $l - m$, and $(m + n)/2$ connected between points C, D, and F; and between F and the zero-potential bus N, an impedance $(m - n)/2$ [or $(n - m)/2$] paralleled by an adjustable voltage V_s. Equations [48] and [49] will be satisfied if current $(I_{A1} - I_{a1})$ is made to flow through the impedance $(m - n)/2$, or if current $(I_{a1} - I_{A1})$ flows through the impedance $(n - m)/2$. If the voltage, V_s, is adjusted in phase and magnitude until the current through it is double I_{a1}, the current entering the fault at C, then the current $(I_{A1} - I_{a1})$ will flow in the impedance $(m - n)/2$. If the impedance $(n - m)/2$ is used, V_s must be adjusted until the current through it is double I_{A1}, the current entering the fault at D. (See reference 4.)

When a *d-c calculating table* is used and a direct connection between the sequence networks cannot be made for both faults, the equivalent circuit shown in Fig. 5 can replace both faults in the positive sequence network for those cases where $m = n$.

A study of Table II shows that *with resistance and capacitance neglected*, k and l have no resistance components, but are positive reactive impedances larger in magnitude than m and n. When m and n are equal, they also have no real components, but are positive or negative reactive impedances: $m = n = \pm jx$. When m and n are unequal, they have real components which are equal in magnitude and opposite in sign, while their reactive components are equal in magnitude and of the same sign: $m = \pm r \pm jx$, $n = \mp r \pm jx$. The error made by neglecting the real components of m and n will ordinarily be no greater than the error made by neglecting line resistances and capacitances. When the real components of m and n are neglected, $(m - n)/2 = 0$, and the equivalent circuit in Fig. 8 becomes an impedance Y connecting C, D, and the zero-potential bus N with branch impedances $(k - n)$, $(l - m)$ and $(m + n)/2$, as in Fig. 9. The impedances $(k - n)$, and $(l - m)$ will be positive reactive impedances and therefore can be represented on the d-c calculating table, while $(m + n)/2$ may be either positive or negative; if positive, it can also be represented on the d-c table. If $(m + n)/2$ is negative, the following procedure is suggested: The branch impedance $(m + n)/2$ which is connected to the zero-potential bus for the positive-sequence network (as are the neutrals of the generators) is in series with the genera-

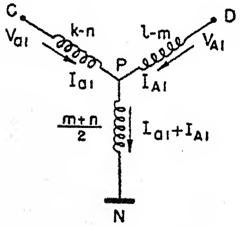


FIG. 9. Approximate equivalent circuit to replace two faults in the positive-sequence network when $m \neq n$ in [24] and [25].

tor reactances. If there is but one generating source, $(m + n)/2$ may be combined with its reactance. With several synchronous machines, or groups of machines, $(m + n)/2$ between P and N in Fig. 9 can be set to zero and the distribution of currents obtained, these currents to be increased by the ratio $X_p / \left(X_p + \frac{m+n}{2}\right)$, where X_p is the equivalent impedance between generator neutrals and P, and is found by dividing generator voltage by total current when P is shorted to N.

Simultaneous Faults on Opposite[3] Sides of a Δ–Y Transformer Bank

A Δ–Y transformer bank usually divides the zero-sequence system into two parts which have no connection with each other in the zero-sequence network. The positive- and negative-sequence equivalent circuits for three-phase power systems, used in analytic calculations or on a calculating table, are based on equivalent Y–Y transformer banks. With any circuit selected as reference circuit and any voltage or current vector in that circuit (or referred to that circuit) as reference vector, the positive-sequence currents and voltages in a circuit separated from the reference circuit by a Δ–Y transformer bank are correctly determined in magnitude and in phase relative to each other, but their phases relative to the system reference vector are given for an equivalent Y–Y transformer bank and not for the actual Δ–Y bank. To refer them to the system reference vector, a phase correction must be applied. This is true also for negative-sequence currents and voltages. The phase correction to be applied to negative-sequence currents and voltages to refer them to the system reference vector is not the same as that for positive-sequence currents and voltages. As explained in Chapter III, and illustrated in Figs. 19(*a*) and (*b*), Chapter III, there are two possible connections of a Δ–Y transformer bank. With either connection, the positive-sequence phase correction can be determined from the angular displacement at no load with magnetizing current neglected between the voltages to neutral of the reference phases on the two sides of the bank. Consider the transformer connection diagram given in Fig. 10(*a*). This diagram may also be used as a positive-sequence no-load voltage vector diagram. Let the circuit at the transformer terminals D with phases labeled ABC be arbitrarily chosen as the reference circuit, and phase A the reference phase. In the circuit at transformer terminals C, the phase to be selected as reference phase (that is the phase which determines the positive-sequence phase correction for the Δ–Y bank) is arbitrary; any one of the three may be chosen. Following the convention adopted in Chapter III, the reference phase is designated a and so chosen that the line-to-neutral voltage V_{a1} at no load and no faults is 90° out of phase with the line-to-neutral voltage V_{A1}. Whether V_{a1} leads or lags V_{A1} is determined from the transformer connection diagram. In Fig. 10(a), V_{a1} lags V_{A1} by 90°; and, as shown in Chapter III, V_{a2} leads V_{A2} by 90° for this connection. Positive-sequence currents and voltages in circuit C, determined from the positive-sequence network based on equivalent Y–Y transformers with the circuit at D as reference circuit, are therefore to be turned backward through 90° or multiplied by $-j$ to be referred to the system reference vector. Negative-sequence currents and voltages in circuit C determined from the negative-sequence network are to be turned forward through 90° or multiplied by j. In calculating system currents and voltages by means of symmetrical components when there is a single fault, the circuit in which the fault occurs is selected as the reference circuit and the

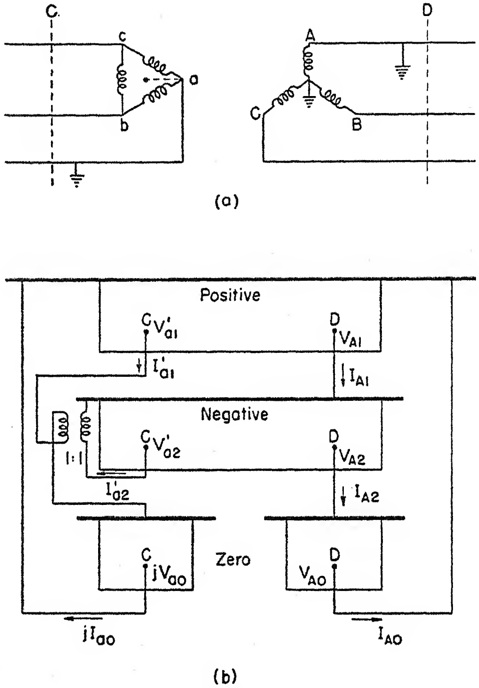


FIG. 10. (a) Connection diagram of a Δ–Y transformer bank with line-to-ground faults on conductors A and a as indicated. (b) Connections of the sequence networks for solution on an a-c network analyzer. Line-to-ground faults as indicated in (a). Circuit D is reference circuit. There is no connection between the zero-sequence impedances viewed from C and D.

sequence networks connected to satisfy fault equations. Positive- and negative-sequence currents and voltages throughout the system are determined in magnitude and phase based on equivalent Y–Y transformer banks. Before combining the components of current and voltage in circuits separated from the reference circuit by a Δ–Y transformer bank, it is necessary to apply the phase corrections resulting from the presence of Δ–Y transformer banks. This has been explained in Chapter III and illustrated in Problem 6 of that chapter.

With simultaneous faults at points C and D in circuits separated by a Δ–Y transformer bank, the positive- and negative-sequence equivalent circuits based on equivalent Y–Y transformer banks may be used, with either the circuit at D or that at C as reference circuit. With the circuit at D arbitrarily chosen, the equations at the fault point D can be taken from Table I, with subscripts ABC replacing abc, respectively, in these equations. Assuming the connection diagram of Fig. 10(a), let V_{a1}, V_{a2}, V_{a0} be the components of V_a the voltage to ground of conductor a at the fault point C and I_{a1}, I_{a2}, and I_{a0} the components of current I_a flowing into the fault, all referred to the *reference vector for the system.* Let these same symbols primed be the components of fault current and voltage at C referred to the circuit at D; i.e., the primed components are components which appear in the positive- and negative-sequence networks based on equivalent Y–Y transformer banks. With no connection between the zero sequence impedances viewed from C and D,

$$\begin{aligned} V_{a1} &= -jV'_{a1}; \quad I_{a1} = -jI'_{a1} \\ V_{a2} &= jV'_{a2}; \quad I_{a2} = jI'_{a2} \\ V_{a0} &= V'_{a0}; \quad I_{a0} = I'_{a0} \end{aligned} \qquad [52]$$

Line-to-Ground Faults on Conductor a at C and Conductor A at D. The fault equations are given by Case A (a) of Table I. At D,

$$I_{A1} = I_{A2} = I_{A0} \qquad [53]$$

$$V_{A1} + V_{A2} + V_{A0} = 0 \qquad [54]$$

The equations at C are the same with the subscript A replaced by a. Referring the components at C to the circuit at D by means of [52], the fault equations at C become

$$-jI'_{a1} = jI'_{a2} = I'_{a0}$$

$$-jV'_{a1} + jV'_{a2} + V'_{a0} = 0$$

Multiplying by j,

$$I'_{a1} = -I'_{a2} = jI'_{a0} = jI_{a0} \qquad [55]$$

$$V'_{a1} - V'_{a2} + jV'_{a0} = V'_{a1} - V'_{a2} + jV_{a0} = 0 \qquad [56]$$

Equations [53]–[56] are satisfied on an a-c network analyzer by the connections given in Fig. 10(b). Coupling transformers are used to satisfy the fault conditions at C, and direct connections of the networks are made at D. A comparison of Fig. 10(b) with Fig. 7 shows that the connections to the negative-sequence network in Fig. 10(b) made through a mutual coupling transformer are the reverse of those in Fig. 7. As the circuit at D is the reference circuit, the components of current and voltage at D are given directly. Components at C referred to the system reference vector are obtained by substituting values obtained from Fig. 10(b) in equations [52]. The fault current in the zero-sequence network at C in Fig. 10(b) is jI_{a0} and the fault voltage is jV_{a0}. Zero-sequence currents and voltages at C are obtained by multiplying the values obtained from Fig. 10(b) by $-j$.

With a *line-to-ground fault on conductor A at D and a line-to-ground fault on conductor b at C*, the fault equations at D are given in [53] and [54]. Those at C from Table I, case A (b), are

$$I_{a1} = a^2 I_{a2} = aI_{a0} \qquad [57]$$

$$V_{a1} + a^2 V_{a2} + aV_{a0} = 0 \qquad [58]$$

Replacing the components at C in [57] and [58] by their values from [52], and multiplying the equations by j,

$$I'_{a1} = -a^2 I'_{a2} = jaI'_{a0} \qquad [59]$$

$$V'_{a1} - a^2 V'_{a2} + jaV'_{a0} = 0 \qquad [60]$$

Equations [53] and [54] are satisfied by direct connections of the sequence networks. Equations [59] and [60] require phase converters to turn negative-sequence currents and voltages through $-a^2 = \underline{/60^\circ}$, and zero-sequence currents and voltages through $ja = \overline{/150^\circ}$. When there is no connection between the zero-sequence networks at C and D, the phase converter for the zero-sequence network may be omitted. Zero-sequence currents and voltages at C read on the a-c calculating table are then multiplied by $-ja^2$ to obtain V_{a0} and I_{a0} at C.

In an *analytic solution*, the components of fault currents and voltages referred to circuit D are calculated as for faults on the same side of a Δ–Y transformer bank, except that equations [59] and [60] instead of [57] and [58] are used. With I'_{a2}, V'_{a2}, I'_{a0}, and V'_{a0} replacing I_{a2}, V_{a2}, I_{a0}, and V_{a0}, respectively, equations [12] and [13] apply to the zero-sequence network, and [16] and [17] to the negative-sequence network. If I_{A2}, I_{A0}, I'_{a2}, and I'_{a0} in these equations are replaced by their values in terms of I_{A1} and I'_{a1} given in [53] and [59], and V_{A2} and V_{A0} then substituted in [54] and V'_{a2} and V'_{a0} in [60], the following equations will be obtained:

$$V'_{a1} = I'_{a1}(C_0 + S_0 + C_2 + S_2) + I_{A1}(jaS_0 - a^2S_2) \qquad [61]$$

$$V_{A1} = I'_{a1}(-ja^2S_0 - aS_2) + I_{A1}(D_0 + S_0 + D_2 + S_2) \qquad [62]$$

Comparing these equations with [24] and [25], with V_{a1} and I_{a1} replaced by V'_{a1} and I'_{a1}:

$$\begin{aligned} k &= C_0 + S_0 + C_2 + S_2 \\ l &= D_0 + S_0 + D_2 + S_2 \\ m &= jaS_0 - a^2S_2 \\ n &= -ja^2S_0 - aS_2 \end{aligned} \qquad [63]$$

When there is no connection in the zero-sequence network between the zero-sequence impedances viewed from C and D, S_0 in the above equations disappears.

I'_{a1} and I_{A1} can be calculated from [43] and [44], or [46] and [47], if E_a is referred to circuit D and I'_{a1} replaces I_{a1} in these equations. Knowing I'_{a1} and I_{A1}, the other components at the faults and in the system referred to circuit D are obtained. The components of currents and voltages at C referred to the reference vector for the system are obtained by substituting the components referred to circuit D in [52].

The procedure outlined above can be applied to faults of any type involving any phases. When the equivalent Y's representing the sequence networks can be obtained by inspection, an analytic solution is not difficult. Consider the system in Fig. 11(*a*). The equivalent Y's of the sequence networks are shown in Figs. 11(*b*), (*c*), and (*d*). For this case, no calculations are required to obtain the equivalent Y's. Each equivalent Y has one branch of zero impedance: $C_1 = C_2 = S_0 = 0$. In Figs. 11(*b*), (*c*), and (*d*), the components of fault currents and voltages at C are primed, indicating that they are referred to the circuit at D, i.e., based on an equivalent Y–Y transformer bank.

Ungrounded System — Simultaneous Ground Faults

In ungrounded systems of appreciable capacitance, currents and voltages resulting from two simultaneous faults can be determined just as in grounded systems. In the zero-sequence network the impedance to ground viewed from either fault point is a capacitive impedance, usually high relative to the inductive impedances of the system — but finite.

In ungrounded systems of negligible capacitance, there is little or no fault current during a single ground fault. In the equivalent Y representing the zero-sequence network, $S_0 = \infty$. With two simultaneous

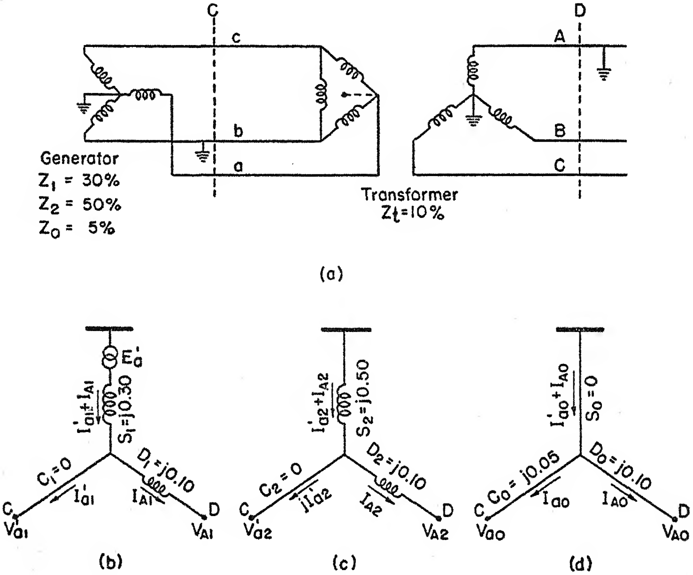

C
c
D
A
b
B
a
C
Generator
Z₁ = 30%
Z₂ = 50%
Z₀ = 5%
Transformer
Zt=10%
(a)
E'a
I'a1+IA1
S₁=j0.30
C₁=0
D₁=j0.10
I'a1
IA1
C
V'a1
D
VA1
(b)
I'a2+IA2
S₂=j0.50
C₂=0
D₂=j0.10
jI'a2
IA2
C
V'a2
D
VA2
(c)
I'a0+IA0
S₀=0
C₀=j0.05
D₀=j0.10
I'a0
IA0
C
Va0
D
VA0
(d)

FIG. 11. (*a*) Three-phase system with line-to-ground faults on conductor *A* at *D* and on conductor *b* at *C*, *D* and *C* on opposite sides of a Δ–Y transformer bank. (*b*), (*c*), and (*d*) Per unit equivalent Y's representing the positive-, negative-, and zero-sequence networks, respectively, based on an equivalent Y–Y transformer bank, with circuit *D* the reference circuit.

ground faults, there is a path for zero-sequence currents, out of the system into the ground at one fault point and from the ground back to the system at the other fault. The zero-sequence impedance, in terms of the impedances of the equivalent Y of the zero-sequence network, is $C_0 + D_0$. In an ungrounded loop, the impedance met by zero-sequence currents is the zero-sequence impedance of the two paths between *C* and *D* in parallel. Representing the zero-sequence impedance between *C* and *D* by z_0, the following equations replace [12] and [13] in the zero-sequence network:

$$I_{a0} = -I_{A0} \tag{64}$$

$$V_{a0} = V_{A0} - I_{a0}z_0 = V_{A0} + I_{A0}z_0 \tag{65}$$

where $z_0 = C_0 + D_0$ and $S_0 = \infty$.

The equations given in Table I are applicable to ungrounded as well as grounded systems. The equations in Table II are also applicable, but are indeterminate with $S_0 = \infty$. If k, l, m, and n from Table II are substituted in [43] and [44], or [46] and [47], these equations can be evaluated for $S_0 = \infty$. The procedure is to divide both numerator and denominator of fractions containing powers of S_0 by S_0 to the highest power appearing in the denominator after simplification. Then, letting $S_0 = \infty$, simple expressions for the two positive-sequence components of fault current are obtained. Components of fault current and voltage, and positive- and negative-sequence currents and voltages in the system are obtained as for grounded systems. In the zero-sequence system, equations [64] and [65] are used instead of [12] and [13].

Solution by Means of an A-C Network Analyzer. The diagrams showing connections between the sequence networks for simultaneous ground faults on grounded systems are also applicable to ungrounded systems of negligible capacitance, provided the zero-sequence impedance network in these diagrams has no direct connections to the zero-potential bus for the network. For example, in Fig. 6(c), which represents double line-to-ground faults on phases b and c at both faults, the rectangle enclosing C and D in the zero-sequence network is to be disconnected from the zero-potential bus to represent an ungrounded system. As in ground systems, zero-sequence voltages are referred to the zero-potential bus for the network, and positive direction for currents is from the network into the fault. In diagrams in which coupling transformers of 1 : 1 turn ratio are used, these transformers may be retained. To make Fig. 7 applicable to an ungrounded system, the rectangle enclosing C and D in the zero-sequence network must be disconnected from the zero-potential bus, but the coupling transformers of 1 : 1 turn ratio remain connected as for the grounded system. An alternate method which eliminates one transformer is to reverse connections to the fault point and the zero-potential bus in the zero-sequence network and replace zero-sequence currents and voltages by their negative values. Representing zero-sequence components of negative signs by primed symbols, let

$$I_{a0} = -I'_{a0}; \quad V_{a0} = -V'_{a0}; \quad I_{A0} = -I'_{A0}; \quad V_{A0} = -V'_{A0}$$

With *line-to-ground faults at C and D on the same phase* (phase a) of an ungrounded loop,

$$I_{a1} = I_{a2} = -I'_{a0}; \quad I_{A1} = I_{A2} = -I'_{A0}$$

$$V_{a1} + V_{a2} - V'_{a0} = 0; \quad V_{A1} + V_{A2} - V'_{A0} = 0$$

These equations are satisfied by the connections shown in Fig. 12. The connections to the zero-potential bus and fault points in the zero-sequence network made in Fig. 7 for the grounded system are reversed in Fig. 12. The zero-potential busses for the positive- and zero-sequence networks in Fig. 12 are directly connected, providing a reference for V'_{A0} and V'_{a0}. Actual zero-sequence currents and voltages are those obtained from the calculating table turned through 180°.

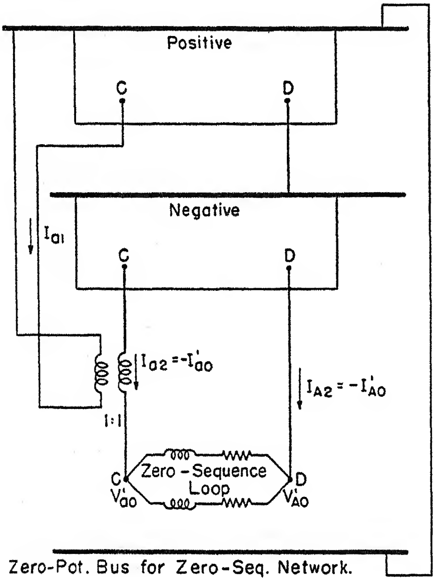


FIG. 12. Connections of the sequence networks for line-to-ground faults at *C* and *D* both on phase *a* of an ungrounded system, showing reversal of the connections to the zero-sequence network for use on an a-c calculating table.

Simultaneous faults in ungrounded systems are further discussed in Chapter X, where solutions for other cases are given.

Fault and Open Conductor Involving the Same Phase

Assume a line-to-ground fault at *C* and an open conductor at *D*, both involving phase *a* as in Fig. 13(*a*). The fault conditions at *C* from Table I are

$$I_{a1} = I_{a2} = I_{a0} \tag{66}$$

$$V_{a1} + V_{a2} + V_{a0} = 0 \tag{67}$$

The fault conditions at *D* from [40] and [41] of Chapter IV are

$$v_{A1} = v_{A2} = v_{A0} \tag{68}$$

$$I'_{A1} + I'_{A2} + I'_{A0} = 0 \tag{69}$$

Here v denotes series voltage drop across the opening and I' line current flowing across the opening.

With no Δ–Y transformer bank between *C* and *D*, the connections shown in Fig. 13(*b*) for use on an a-c calculating table satisfy equations [66]–[69]. The direct connections of the sequence networks at *D* is the same as that in Chapter IV, Fig. 14(*c*). The mutual coupling transformers used at *C* are connected just as in Fig. 7, although this may not be apparent at first glance. In Fig. 13(*b*), impedance is indicated between points C and D; the portions of the system to the left of C and to the right of D are represented as equivalent synchronous machines.

If C and D are on opposite sides of a Δ–Y transformer bank and there is no connection between the zero-sequence impedance viewed from C and D, the method of representing the open conductor in Fig. 13(*b*) may be retained. For the fault at C on phase a, referred to the circuit at D, the connections given in Fig. 10(*b*) can be used.

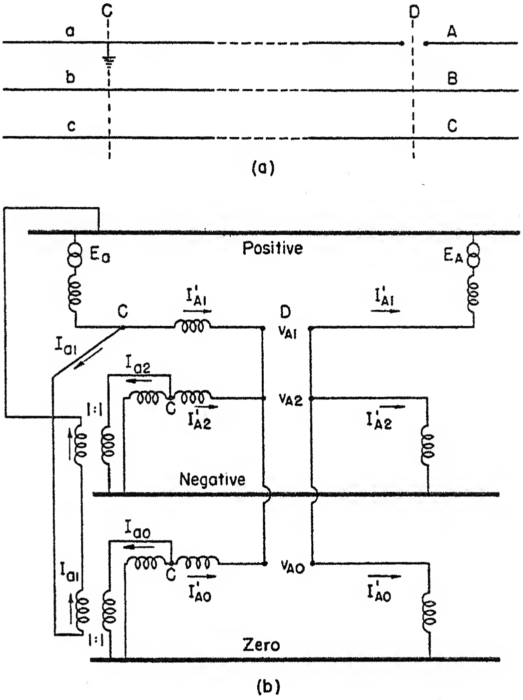


FIG. 13. (*a*) Line-to-ground fault and open conductor on the same phase. (*b*) Connections of the sequence networks to represent a fault and open conductor on the same phase for solution on an a-c network analyzer.

For all cases of simultaneous faults, where the positive- and negative-sequence impedances of rotating machines can be assumed equal, simpler solutions can be obtained by the use of α, β, and 0 components. These components are discussed in Chapter X and applied to the solution of problems involving simultaneous dissymmetries.

Problem 1. Check the equations of Tables I and II.

Problem 2. Draw a three-line diagram showing per unit phase currents and voltages to ground in the system shown in Fig. 11(*a*) with faults as indicated

Problem 3. A conductor breaks and one end falls to ground, the other end is isolated. Develop an equivalent circuit to replace[5] the fault and open conductor in the positive-sequence network. *Suggestion:* Set up an assumed equivalent Y between points *C*, *D*, and the zero-potential bus, where *C* and *D* are system points at the terminals of the double fault.

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CHAPTER VIII

UNSYMMETRICAL THREE-PHASE CIRCUITS — ANALYSIS BY THE METHOD OF SYMMETRICAL COMPONENTS

In a symmetrical three-phase system with balanced generated voltages, the currents and voltages under normal operation are balanced. During faults, symmetrical components of current flowing in a symmetrical circuit produce voltage drop of like sequence only. The impedances offered to currents of a given sequence are the same in the three phases and therefore each of the sequence systems can be represented by an equivalent circuit which has no mutual coupling with the equivalent circuits of the other two sequences. In an unsymmetrical three-phase circuit, the voltages and currents are unbalanced under normal operation. If the unbalance is small, it may be relatively unimportant. On the other hand, since it exists during normal operation, its effects may be serious. An example is the heating in a rotating machine resulting from double-frequency currents in the rotor induced by negative-sequence armature currents. Overheating is most likely to occur in the solid rotor of a turbine generator when supplying an unbalanced three-phase or a single-phase load. It is important that the negative-sequence current in a rotating machine does not exceed its allowable safe limit. (See Problem 2.)

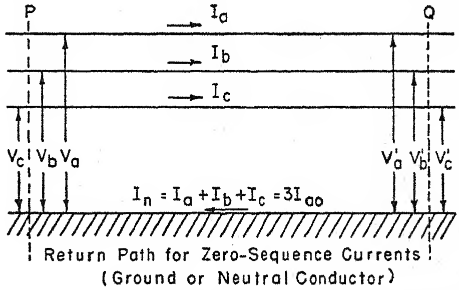
P
Q
I_a
I_b
I_c
V_c
V_b
V_a
V_a'
V_b'
V_c'
$I_n = I_a + I_b + I_c = 3I_{a0}$
Return Path for Zero-Sequence Currents
(Ground or Neutral Conductor)

FIG. 1. Three-phase series circuit between P and Q with negligible capacitance and no internal voltages.

Unsymmetrical circuits between two points in an otherwise symmetrical system, and unsymmetrical circuits connected at one point only of the system, will be considered. The simplest type of the unsymmetrical three-phase series circuit is one which provides a direct metallic connection between the two symmetrical parts of the system. In this type of circuit, phase currents flowing between the two symmetrical parts of the system enter the unsymmetrical circuit at one terminal P

and leave at the other terminal *Q* without change in magnitude or phase, as illustrated in Fig. 1. Examples of this type of circuit are an unsymmetrical three-phase transmission line, with capacitance neglected, and any three series impedances, such as series reactors, which happen to be unequal. Other types of unsymmetrical series circuits include unsymmetrical transformer banks with exciting currents neglected, where the connection between the two parts of the system is by electromagnetic induction rather than by direct metallic connection.

The simplest type of the unsymmetrical three-phase circuit, connected at one point only of an otherwise symmetrical system, is an unsymmetrical Y-connected circuit. The currents in the three phases of the Y are line currents which flow from the symmetrical part of the system into the Y-connected circuit or from the Y into the system, depending upon whether the unsymmetrical Y-connected circuit is receiving or delivering power. The unsymmetrical Y-connected circuit may be treated as a special case of the unsymmetrical three-phase metallically connected series circuit, obtained by connecting the terminals of the three phases at *P* or *Q* in Fig. 1 and disconnecting these terminals from the system. Unsymmetrical Y-connected generators, motors, and impedance loads belong in this class. Other unsymmetrical circuits connected at one point only of the system include unsymmetrical Δ-connected circuits and single-phase loads.

Sequence Impedances of the Three Phases of an Unsymmetrical Three-Phase Circuit. The impedances offered to positive-sequence currents in the three phases of a circuit will be defined as the ratios of the voltage drops in the three phases to the corresponding phase currents, with only positive-sequence currents flowing in the circuit. The impedances to positive-sequence currents in phases *a*, *b*, and *c* will be designated by Z_{a1}, Z_{b1}, and Z_{c1}, respectively. In a given circuit, they may be determined by causing positive-sequence currents only to flow in the circuit, or they may be calculated by assuming that only positive-sequence currents are flowing. Positive-sequence currents only can be made to flow in an unsymmetrical three-phase circuit without internal voltages by suitable adjustments of series impedances in the three phases, with positive-sequence voltages applied. The voltage drops in the three phases of the unsymmetrical circuit do not include the voltage drops through the adjustable impedances. A similar procedure can be used to cause only negative- or only zero-sequence currents to flow in an unsymmetrical three-phase circuit. Then, by definition,

$$Z_{a1} = \frac{v_a}{I_{a1}} \qquad Z_{b1} = \frac{v_b}{I_{b1}} \qquad Z_{c1} = \frac{v_c}{I_{c1}}$$

where I_{a1}, I_{b1}, and I_{c1} are positive-sequence currents and v_a, v_b, and v_c the series voltage drops in the three phases resulting from the flow of positive-sequence currents only in the circuit. Similarly, the impedances of the three phases to negative- (or zero-) sequence currents will be defined as the ratios of the voltage drops in the three phases to the corresponding currents with only negative- (or only zero-) sequence currents flowing in the circuit. The negative-sequence impedances of the three phases will be designated by Z_{a2}, Z_{b2}, and Z_{c2}, and the zero-sequence impedances by Z_{a0}, Z_{b0}, and Z_{c0}.

The definitions of the *sequence impedances of the three phases* of an unsymmetrical circuit given here are based on the flow of positive-, negative-, and zero-sequence currents in the circuit and the unbalanced voltage drops resulting from them.

The *sequence admittances of the three phases*, which will be defined in terms of applied positive-, negative-, and zero-sequence voltages and the resulting unbalanced currents are discussed later in this chapter. It is there pointed out that except in a symmetrical three-phase circuit, or in a solidly grounded unsymmetrical three-phase circuit which offers the same impedances to currents of all sequences, the *sequence admittances of the three phases are not the reciprocals of the corresponding sequence impedances.*

Reference Phase and Designation of Conductors. In a symmetrical circuit, the reference phase is conventionally designated *a* and may be any one of the three phases; when a fault involving one or more phases occurs, it is assumed to be located on the phase or phases which results in the simplest expressions for the sequence currents and voltages. In an unsymmetrical circuit, any phase may be designated the reference phase *a*; but with phase *a* specified, the fault or other dissymmetry must be located on the phase or phases relative to phase *a* which are actually involved. In the work of this and subsequent chapters where circuits are unsymmetrical, the reference phase *a* will be so selected as to simplify calculations.

Unsymmetrical Three-Phase Series Circuit without Internal Voltages. Let Fig. 1 represent a general three-phase series circuit without internal voltages which provides a direct electrical connection between points *P* and *Q* of the system. Currents I_a, I_b, and I_c in phases *a*, *b*, and *c*, respectively, enter the unsymmetrical circuit at *P* and leave at *Q*, positive direction for current flow being from *P* to *Q*, as indicated by arrows. If the three phases are connected at *P*, Fig. 1 can represent currents flowing out of an unsymmetrical Y-connected circuit; if at *Q*, current flowing into such a circuit; if connected at neither *P* nor *Q*, current flowing in an unsymmetrical series circuit.

Only the three phases and a return path for zero sequence currents are indicated in Fig. 1; but additional circuit elements can be understood to be present, so that the circuit will represent the desired three-phase circuit.

Phase voltages at any point in a grounded system are referred to ground at that point. If there is a neutral conductor and no ground on the system, phase voltages may be referred to the neutral conductor. Let V_a, V_b, and V_c be the phase voltages at P referred to ground (or some other reference for voltages) at P, and V'_a, V'_b, and V'_c those at Q referred to ground (or some other reference for voltages) at Q. Let the voltage drops between P and Q in the three phases be indicated by v_a, v_b, and v_c; then

$$v_a = V_a - V'_a \qquad v_b = V_b - V'_b \qquad v_c = V_c - V'_c$$

If the resistances and inductances associated with the circuit are constant, superposition can be applied (see Chapter I) and the voltage drop in each phase written as the sum of the voltage drops resulting from the phase current replaced by its symmetrical components. Replacing I_a by $(I_{a1} + I_{a2} + I_{a0})$, I_b by $(a^2I_{a1} + aI_{a2} + I_{a0})$, I_c by $(aI_{a1} + a^2I_{a2} + I_{a0})$, with currents of each sequence in each phase meeting their respective impedances, the voltage drops are

$$\begin{aligned} v_a &= V_a - V'_a = I_{a1}Z_{a1} + I_{a2}Z_{a2} + I_{a0}Z_{a0} \\ v_b &= V_b - V'_b = a^2I_{a1}Z_{b1} + aI_{a2}Z_{b2} + I_{a0}Z_{b0} \\ v_c &= V_c - V'_c = aI_{a1}Z_{c1} + a^2I_{a2}Z_{c2} + I_{a0}Z_{c0} \end{aligned} \qquad [1]$$

where v_a, v_b, and v_c are *series* voltage drops in phases a, b, and c, respectively, in the direction PQ. The lower-case letter v is here used to indicate a series voltage drop as distinguished from a voltage-to-ground, indicated by V. It is important to note that the voltages as defined are *voltage drops*, not voltage rises. (See Chapter I.)

Resolving v_a, v_b, and v_c in [1] into their symmetrical components by [10]–[12], Chapter II,

$$\begin{aligned} v_{a0} &= V_{a0} - V'_{a0} = \tfrac{1}{3}(v_a + v_b + v_c) = I_{a1}\frac{Z_{a1} + a^2Z_{b1} + aZ_{c1}}{3} \\ &\quad + I_{a2}\frac{Z_{a2} + aZ_{b2} + a^2Z_{c2}}{3} + I_{a0}\frac{Z_{a0} + Z_{b0} + Z_{c0}}{3} \\ v_{a1} &= V_{a1} - V'_{a1} = \tfrac{1}{3}(v_a + av_b + a^2v_c) = I_{a1}\frac{Z_{a1} + Z_{b1} + Z_{c1}}{3} \\ &\quad + I_{a2}\frac{Z_{a2} + a^2Z_{b2} + aZ_{c2}}{3} + I_{a0}\frac{Z_{a0} + aZ_{b0} + a^2Z_{c0}}{3} \end{aligned} \qquad [2]$$

$$v_{a2} = V_{a2} - V'_{a2} = \tfrac{1}{3}(v_a + a^2 v_b + a v_c) = I_{a1} \frac{Z_{a1} + aZ_{b1} + a^2 Z_{c1}}{3}$$
$$+ I_{a2} \frac{Z_{a2} + Z_{b2} + Z_{c2}}{3} + I_{a0} \frac{Z_{a0} + a^2 Z_{b0} + aZ_{c0}}{3}$$

With positive-sequence currents only flowing in the circuit, I_{a2} and I_{a0} are zero, and equations [2] become

$$\begin{aligned} v_{a0} &= I_{a1} \frac{Z_{a1} + a^2 Z_{b1} + aZ_{c1}}{3} \\ v_{a1} &= I_{a1} \frac{Z_{a1} + Z_{b1} + Z_{c1}}{3} \\ v_{a2} &= I_{a1} \frac{Z_{a1} + aZ_{b1} + a^2 Z_{c1}}{3} \end{aligned} \qquad [3]$$

Equations [3] show that, with positive-sequence currents only flowing in the circuit, voltage drops of all three sequences will occur between P and Q unless the coefficients of I_{a1} are zero. Likewise, with only negative- or only zero-sequence currents flowing in an unsymmetrical circuit, voltage drops of all three sequences may be obtained.

Sequence Self- and Mutual Impedances in Terms of the Sequence Impedances of the Phases. When currents of a given sequence produce voltage drops of unlike as well as like sequence, equations for the sequence voltage drops in the circuit are conveniently expressed in terms of *sequence self- and mutual impedances*. Replacing the coefficients of the currents in [2] by Z's with two subscripts,* the first subscript referring to the sequence of the voltage drop given by the equation and the second to the sequence of the current associated with the coefficient,

$$v_{a1} = V_{a1} - V'_{a1} = I_{a1}Z_{11} + I_{a2}Z_{12} + I_{a0}Z_{10} \qquad [4]$$

$$v_{a2} = V_{a2} - V'_{a2} = I_{a1}Z_{21} + I_{a2}Z_{22} + I_{a0}Z_{20} \qquad [5]$$

$$v_{a0} = V_{a0} - V'_{a0} = I_{a1}Z_{01} + I_{a2}Z_{02} + I_{a0}Z_{00} \qquad [6]$$

where the coefficients are defined below in [7].

$Z_{11} = \frac{1}{3}(Z_{a1} + Z_{b1} + Z_{c1})$ = self-impedance to positive-sequence currents

$Z_{22} = \frac{1}{3}(Z_{a2} + Z_{b2} + Z_{c2})$ = self-impedance to negative-sequence currents

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* Notation and expression "non-reciprocal," suggested by Dr. E. W. Kimbark's letter to Editor, *Electrical Engineering*, October, 1938, p. 431.

$Z_{00} = \frac{1}{3}(Z_{a0} + Z_{b0} + Z_{c0})$ = self-impedance to zero-sequence currents

$Z_{12} = \frac{1}{3}(Z_{a2} + a^2Z_{b2} + aZ_{c2})$ = ratio of the positive-sequence voltage drop produced by I_{a2} to I_{a2}

$Z_{10} = \frac{1}{3}(Z_{a0} + aZ_{b0} + a^2Z_{c0})$ = ratio of the positive-sequence voltage drop produced by I_{a0} to I_{a0} [7]

$Z_{21} = \frac{1}{3}(Z_{a1} + aZ_{b1} + a^2Z_{c1})$ = ratio of the negative-sequence voltage drop produced by I_{a1} to I_{a1}

$Z_{20} = \frac{1}{3}(Z_{a0} + a^2Z_{b0} + aZ_{c0})$ = ratio of the negative-sequence voltage drop produced by I_{a0} to I_{a0}

$Z_{01} = \frac{1}{3}(Z_{a1} + a^2Z_{b1} + aZ_{c1})$ = ratio of the zero-sequence voltage drop produced by I_{a1} to I_{a1}

$Z_{02} = \frac{1}{3}(Z_{a2} + aZ_{b2} + a^2Z_{c2})$ = ratio of the zero-sequence voltage drop produced by I_{a2} to I_{a2}

Equations [4]–[6] express the symmetrical components of voltage drop in an unsymmetrical three-phase series circuit in which there are no internal voltages in terms of the symmetrical components of current flowing through the circuit and the sequence self- and mutual impedances defined by [7]. Self-impedances are indicated by Z with two like subscripts, mutual impedances by Z with two unlike subscripts. Z_{11}, Z_{22}, Z_{00} represent the positive-, negative-, and zero-sequence self-impedances, respectively, of the circuit and are the impedances met by currents of positive, negative, and zero sequence flowing in their respective networks.

In an unsymmetrical circuit, positive-sequence current flowing in the positive-sequence network meets the positive-sequence self-impedance Z_{11} of the circuit and produces negative- and zero-sequence voltage drops in the negative- and zero-sequence networks. The positive-sequence network is therefore coupled with the other two sequence networks. The mutual or coupling impedances in the positive-sequence network are Z_{21} with the negative-sequence network and Z_{01} with the zero-sequence network. The negative-sequence voltage drop produced in the negative-sequence network by I_{a1} is $Z_{21}I_{a1}$ and the zero-sequence voltage drop produced in the zero-sequence network by I_{a1} is $Z_{01}I_{a1}$. Negative-sequence current flowing in the negative-sequence network meets the negative-sequence self impedance Z_{22} of the circuit and produces positive- and zero-sequence voltage drops in the positive and zero-sequence networks equal to $Z_{12}I_{a2}$ and $Z_{02}I_{a2}$, respectively. The mutual or coupling impedances between the negative-sequence network and the positive- and zero-sequence networks are Z_{12} and Z_{02}, respectively. Zero-sequence current flowing in the zero-sequence network meets the zero-sequence self-impedance Z_{00} of the circuit and produces positive- and negative-sequence voltage drops in the positive- and negative-sequence networks equal to $Z_{10}I_{a0}$ and $Z_{20}I_{a0}$, respectively. The mutual impedances between the zero-sequence network and the positive- and negative-sequence networks are Z_{10} and Z_{20}, respectively.

It is of interest to note that Z_{12} and Z_{21}, as defined in [7], are not equal in the general case. This means that, except in special cases, the ratio of the positive-sequence voltage drop to the negative-sequence current producing it is not the same as the ratio of the negative-sequence voltage drop to the positive-sequence current producing it. Also Z_{10} and Z_{01}, Z_{20} and Z_{02}, except in special cases, will be unequal. The mutual impedances between the sequence networks resulting from an unsymmetrical circuit are therefore *non-reciprocal* in the general case.

Unsymmetrical Three-Phase Series Circuit with Unbalanced Generated or Induced Voltages. Let E_a, E_b, and E_c be the internally generated, or externally induced, voltage rises in phases a, b, and c, respectively, of Fig. 1 in the direction PQ. These internal voltages, if unbalanced, can be resolved into their symmetrical components of voltage by [1], Chapter III. If the components of voltage drop in [4]–[6] are subtracted from the symmetrical components of voltage rise E_{a1}, E_{a2}, and E_{a0}, respectively, the resultant components of voltage rise from P to Q are given by the following equations:

$$\begin{aligned} v_{a1} &= E_{a1} - I_{a1}Z_{11} - I_{a2}Z_{12} - I_{a0}Z_{10} \\ v_{a2} &= E_{a2} - I_{a1}Z_{21} - I_{a2}Z_{22} - I_{a0}Z_{20} \\ v_{a0} &= E_{a0} - I_{a1}Z_{01} - I_{a2}Z_{02} - I_{a0}Z_{00} \end{aligned} \qquad [8]$$

where the sequence self- and mutual impedances are defined in [7]. If the applied voltages are balanced, E_{a2} and E_{a0} in [8] are zero.

An unsymmetrical Y-*connected synchronous machine*, with its neutral grounded through an impedance Z_n, is represented by Fig. 1 if the three phases are connected at P and grounded through an impedance Z_n, as in Fig. 2. P then represents the neutral of the machine and Q its terminals. The symmetrical components of the voltage to ground of phase a at the terminals of the machine, with positive direction for line currents from neutral towards the terminals, are given

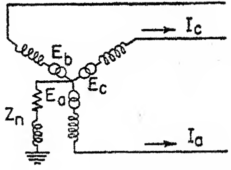


FIG. 2. Y-connected synchronous machine with neutral ground through Z_n.

by [8] if V_{a1}, V_{a2}, and V_{a0} replace v_{a1}, v_{a2}, and v_{a0} in these equations. If Z'_{00} represents the zero-sequence self-impedance between neutral and terminals, then

$$\begin{aligned} V_{a1} &= E_{a1} - I_{a1}Z_{11} - I_{a2}Z_{12} - I_{a0}Z_{10} \\ V_{a2} &= E_{a2} - I_{a1}Z_{21} - I_{a2}Z_{22} - I_{a0}Z_{20} \\ V_{a0} &= E_{a0} - I_{a1}Z_{01} - I_{a2}Z_{02} - I_{a0}(Z'_{00} + 3Z_n) \\ &= E_{a0} - I_{a1}Z_{01} - I_{a2}Z_{02} - I_{a0}Z'_{00} + V_n \end{aligned} \qquad [9]$$

where $V_n = -I_{a0}(3Z_n)$ is the voltage above ground of the neutral. If the neutral is solidly grounded, $Z_n = 0$ and $V_n = 0$.

If the *machine neutral is ungrounded*, $Z_n = \infty$ and $I_{a0} = 0$. If the zero-sequence impedances in the three phases between terminals and neutral are finite, as is the case in a Y-connected machine, the zero-sequence impedances Z'_{00}, Z_{10}, and Z_{20} in [9] are finite, and these equations become

$$\begin{aligned} V_{a1} &= E_{a1} - I_{a1}Z_{11} - I_{a2}Z_{12} \\ V_{a2} &= E_{a2} - I_{a1}Z_{21} - I_{a2}Z_{22} \\ V_{a0} &= E_{a0} - I_{a1}Z_{01} - I_{a2}Z_{01} + V_n \end{aligned} \qquad [10]$$

where V_{a0} and V_n, the zero-sequence voltages at the machine terminals and neutral, respectively, can be determined for a given problem when the zero-sequence diagram for the rest of the system is known.

Equations [9] and [10] give components of voltage to ground at the terminals of a synchronous machine in terms of the symmetrical components of line current flowing *from* the machine; if I_{a1}, I_{a2}, and I_{a0} are components of line current flowing *into* the machine, the negative signs in [9] and [10] become positive.

In an *unsymmetrical static load* with a path for zero-sequence currents, the symmetrical components of phase voltages to ground at the load terminals are given in terms of the positive-, negative-, and zero-sequence line currents flowing into the circuit and the sequence self- and mutual impedances of the circuit viewed from its terminals by the following equations:

$$\begin{aligned} V_{a1} &= I_{a1}Z_{11} + I_{a2}Z_{12} + I_{a0}Z_{10} \\ V_{a2} &= I_{a1}Z_{21} + I_{a2}Z_{22} + I_{a0}Z_{20} \\ V_{a0} &= I_{a1}Z_{01} + I_{a2}Z_{02} + I_{a0}Z_{00} \end{aligned} \qquad [11]$$

where the sequence self- and mutual impedances in [11] depend upon the characteristics of the unsymmetrical static load.

If *the load is* Y-*connected* with neutral ground through an impedance

Z_n, equations [11] can be written

$$\begin{aligned} V_{a1} &= I_{a1}Z_{11} + I_{a2}Z_{12} + I_{a0}Z_{10} \\ V_{a2} &= I_{a1}Z_{21} + I_{a2}Z_{22} + I_{a0}Z_{20} \\ V_{a0} &= I_{a1}Z_{01} + I_{a2}Z_{02} + I_{a0}(Z'_{00} + 3Z_n) \\ &= I_{a1}Z_{01} + I_{a2}Z_{02} + I_{a0}Z'_{00} + V_n \end{aligned} \tag{11a}$$

where $V_n = I_{a0}(3Z_n)$ is the voltage of the neutral of the load above ground, and Z'_{00} is the zero-sequence self-impedance between terminals and neutral.

If the *load is Y-connected with neutral ungrounded*, $I_{a0} = 0$. With finite zero-sequence impedances in the three phases between terminals and neutral, [11*a*] becomes

$$\begin{aligned} V_{a1} &= I_{a1}Z_{11} + I_{a2}Z_{12} \\ V_{a2} &= I_{a1}Z_{21} + I_{a2}Z_{22} \\ V_{a0} &= I_{a1}Z_{01} + I_{a2}Z_{02} + V_n \end{aligned} \tag{11b}$$

Δ-connected loads are discussed under Δ-connected circuits.

Determination of Sequence Self- and Mutual Impedances from Circuit Impedances

The sequence self- and mutual impedances in [4]–[6] are defined in [7] in terms of the *sequence impedances of the three phases*. The sequence impedances of the phases have been introduced to simplify calculations. It is the sequence self- and mutual impedances that are required, the sequence impedance of the phases being merely helpful in obtaining them. In certain unsymmetrical circuits, the sequence impedances of the phases can be obtained by inspection. This is the case in three-phase circuits with unequal self-impedances in the phases and no mutual impedances between phases or with other circuits. With static circuits having unequal circuit self- and mutual impedances, the introduction of the sequence impedances of the three phases allows the sequence self- and mutual impedances associated with the currents of each sequence to be calculated separately. This is especially useful in the calculation of zero-sequence impedances of overhead transmission lines with ground wires. (See Chapter XI.)

To avoid confusing the two different types of self- and mutual impedances and the two different types of impedances associated with the phases, these impedances will be redefined:

Sequence self- and mutual impedances refer to the self-impedances in the sequence networks (Z_{11}, Z_{22}, and Z_{00}) and the mutual impedances between these networks ($Z_{12}, Z_{21}, Z_{10}, Z_{01}, Z_{20}, Z_{02}$).

Sequence impedances of the three phases refer to the impedances in the three phases to the sequence currents, when currents of each sequence are applied separately ($Z_{a1}, Z_{b1}, Z_{c1};\ Z_{a2}, Z_{b2}, Z_{c2};\ Z_{a0}, Z_{b0}, Z_{c0}$).

Circuit or conductor self- and mutual impedances refer to the self-impedances of the elements or conductors of the actual circuit ($Z_{aa}, Z_{bb}, Z_{cc}, \cdots Z_{nn}$) and the mutual impedances between elements ($Z_{ab}, Z_{bc}, \cdots Z_{cn}$).

Three-Phase Static Circuit with Neutral Conductor — Presence of Earth Neglected. Let Fig. 1 represent an unsymmetrical, reciprocal, static circuit between P and Q in which capacitance is negligible, there are no internal voltages, and the return path for zero-sequence currents is through a neutral conductor with the presence of the earth neglected. Let the self-impedances of phases a, b, c be Z_{aa}, Z_{bb}, Z_{cc}, respectively, and the self-impedance of the neutral conductor n be Z_{nn}. Let the mutual impedances between phases be Z_{ab}, Z_{ac}, Z_{bc}, and between phases and neutral conductor be Z_{an}, Z_{bn}, Z_{cn}. In a reciprocal static circuit $Z_{ab} = Z_{ba}$, $Z_{bn} = Z_{nb}$, etc. (Formulas for the self-impedance of a linear conductor and the mutual impedance between two parallel linear conductors are given in Chapter XI, equations [1] and [2].)

The voltages referred to the neutral conductor of phases a, b, c at P are V_a, V_b, V_c; at Q they are V'_a, V'_b, V'_c. The phase voltage drops in the circuit are $v_a = V_a - V'_a$; $v_b = V_b - V'_b$; $v_c = V_c - V'_c$. If the three phases of the circuit are shorted to the neutral conductor at Q, $V'_a = V'_b = V'_c = 0$ and the voltage drops in the three phases between P and Q are the voltage drops in the three loop circuits, each consisting of a phase conductor with neutral conductor return. Shorting all three conductors at one terminal of a circuit to the reference point for voltages at that terminal is a convenient way of determining its impedances. The neutral conductor return is included because the voltages at P and Q are not referred to the same point of the neutral conductor.

The voltage drops in the three loop circuits, each consisting of a phase conductor and neutral conductor return, are

$$\begin{aligned}
v_a = V_a - V'_a &= (I_aZ_{aa} + I_bZ_{ab} + I_cZ_{ac} - I_nZ_{an}) \\
&\quad + (I_nZ_{nn} - I_aZ_{an} - I_bZ_{bn} - I_cZ_{cn}) \\
v_b = V_b - V'_b &= (I_aZ_{ab} + I_bZ_{bb} + I_cZ_{bc} - I_nZ_{bn}) \\
&\quad + (I_nZ_{nn} - I_aZ_{an} - I_bZ_{bn} - I_cZ_{cn}) \\
v_c = V_c - V'_c &= (I_aZ_{ac} + I_bZ_{bc} + I_cZ_{cc} - I_nZ_{cn}) \\
&\quad + (I_nZ_{nn} - I_aZ_{an} - I_bZ_{bn} - I_cZ_{cn})
\end{aligned} \qquad [12]$$

where $I_n = I_a + I_b + I_c$.

There are two ways of determining the sequence self- and mutual impedances of the circuit defined by [12]. The first method, already described, consists of determining the positive-, negative-, and zero-sequence impedances of the three phases of the circuit in accordance with the definitions given above, and from them the sequence self- and mutual impedances using [7]. In the second method, after I_n has been replaced by $I_a + I_b + I_c$ in [12], the procedure is to replace the phase currents by their symmetrical components of current ($I_a = I_{a1} + I_{a2} + I_{a0}, I_b = a^2 I_{a1} + aI_{a2} + I_{a0}$, $I_c = aI_{a1} + a^2 I_{a2} + I_{a0}$); then to resolve v_a, v_b, and v_c into their symmetrical components of voltage by [10]–[12], Chapter II; and finally to equate the coefficients of I_{a1}, I_{a2}, and I_{a0} in the resultant equations for v_{a1}, v_{a2}, and v_{a0} to the corresponding coefficients in [4]–[6], obtaining the sequence self- and mutual impedances in terms of the self- and mutual impedances of the conductors.

By the first method, the self- and mutual impedances of each sequence are determined separately. This tends to simplify the equations, but increases their number because of the introduction of the sequence impedances of the three phases. When the second method is used, the sequence impedances of the three phases are not required as the self- and mutual impedances of the sequence networks are expressed directly in terms of conductor self- and mutual impedances. The notation is therefore simpler and the number of equations less; the equations, however, are longer. The first method is employed in the following development. The second is reserved for an exercise (see Problem 7).

With positive-sequence currents only flowing in the three phases, $I_a = I_{a1}$, $I_b = a^2 I_{a1}$, $I_c = aI_{a1}$, and $I_n = 0$. Substituting these values in [12],

$$\begin{aligned} v_a &= V_a - V_a' \\ &= I_{a1}(Z_{aa} + a^2 Z_{ab} + aZ_{ac}) - I_{a1}(Z_{an} + a^2 Z_{bn} + aZ_{cn}) \\ v_b &= V_b - V_b' \\ &= I_{a1}(Z_{ab} + a^2 Z_{bb} + aZ_{bc}) - I_{a1}(Z_{an} + a^2 Z_{bn} + aZ_{cn}) \\ v_c &= V_c - V_c' \\ &= I_{a1}(Z_{ac} + a^2 Z_{bc} + aZ_{cc}) - I_{a1}(Z_{an} + a^2 Z_{bn} + aZ_{cn}) \end{aligned}$$

The positive-sequence impedances of the three phases by definition, using the above equations, are

$$Z_{a1} = \frac{v_a}{I_{a1}} = (Z_{aa} + a^2 Z_{ab} + aZ_{ac}) - (Z_{an} + a^2 Z_{bn} + aZ_{cn})$$

$$Z_{b1} = \frac{v_b}{a^2 I_{a1}} = (Z_{bb} + a^2 Z_{bc} + aZ_{ab}) - a(Z_{an} + a^2 Z_{bn} + aZ_{cn})$$

$$Z_{c1} = \frac{v_c}{aI_{a1}} = (Z_{cc} + a^2 Z_{ac} + aZ_{bc}) - a^2(Z_{an} + a^2 Z_{bn} + aZ_{cn})$$

Similar equations can be written for the voltage drops with only negative- and only zero-sequence currents flowing in the three phases and the negative- and zero-sequence impedances of the three phases determined.

The nine sequence impedances of the three phases of a static circuit with return for zero-sequence currents through a neutral conductor with the presence of the earth neglected are given in the following equations in terms of the self- and mutual impedances of the conductors:

$$\begin{aligned}
Z_{a1} &= (Z_{aa} + a^2 Z_{ab} + aZ_{ac}) - (Z_{an} + a^2 Z_{bn} + aZ_{cn}) \\
Z_{b1} &= (Z_{bb} + a^2 Z_{bc} + aZ_{ab}) - a(Z_{an} + a^2 Z_{bn} + aZ_{cn}) \\
Z_{c1} &= (Z_{cc} + a^2 Z_{ac} + aZ_{bc}) - a^2(Z_{an} + a^2 Z_{bn} + aZ_{cn}) \\[6pt]
Z_{a2} &= (Z_{aa} + aZ_{ab} + a^2 Z_{ac}) - (Z_{an} + aZ_{bn} + a^2 Z_{cn}) \\
Z_{b2} &= (Z_{bb} + aZ_{bc} + a^2 Z_{ab}) - a^2(Z_{an} + aZ_{bn} + a^2 Z_{cn}) \qquad [12a] \\
Z_{c2} &= (Z_{cc} + aZ_{ac} + a^2 Z_{bc}) - a(Z_{an} + aZ_{bn} + a^2 Z_{cn}) \\[6pt]
Z_{a0} &= Z_{aa} + Z_{ab} + Z_{ac} - 3Z_{an} + 3Z_{nn} - (Z_{an} + Z_{bn} + Z_{cn}) \\
Z_{b0} &= Z_{bb} + Z_{ab} + Z_{bc} - 3Z_{bn} + 3Z_{nn} - (Z_{an} + Z_{bn} + Z_{cn}) \\
Z_{c0} &= Z_{cc} + Z_{ac} + Z_{bc} - 3Z_{cn} + 3Z_{nn} - (Z_{an} + Z_{bn} + Z_{cn})
\end{aligned}$$

The sequence self- and mutual impedances of a reciprocal three-phase static circuit with no internal voltages, having a return path for zero-sequence currents through a neutral conductor with the presence of the earth neglected, obtained by substituting [12*a*] in [7], are

$$\begin{aligned}
Z_{11} &= Z_{22} = \tfrac{1}{3}(Z_{aa} + Z_{bb} + Z_{cc}) - \tfrac{1}{3}(Z_{ab} + Z_{ac} + Z_{bc}) \\
Z_{00} &= \tfrac{1}{3}(Z_{aa} + Z_{bb} + Z_{cc}) + \tfrac{2}{3}(Z_{ab} + Z_{ac} + Z_{bc}) \\
&\qquad + 3Z_{nn} - 2(Z_{an} + Z_{bn} + Z_{cn}) \\
Z_{12} &= \tfrac{1}{3}(Z_{aa} + a^2 Z_{bb} + aZ_{cc}) + \tfrac{2}{3}(aZ_{ab} + a^2 Z_{ac} + Z_{bc}) \\
Z_{21} &= \tfrac{1}{3}(Z_{aa} + aZ_{bb} + a^2 Z_{cc}) + \tfrac{2}{3}(a^2 Z_{ab} + aZ_{ac} + Z_{bc}) \qquad [13] \\
Z_{10} &= Z_{02} = \tfrac{1}{3}(Z_{aa} + aZ_{bb} + a^2 Z_{cc}) - \tfrac{1}{3}(a^2 Z_{ab} + aZ_{ac} + Z_{bc}) \\
&\qquad - (Z_{an} + aZ_{bn} + a^2 Z_{cn}) \\
Z_{20} &= Z_{01} = \tfrac{1}{3}(Z_{aa} + a^2 Z_{bb} + aZ_{cc}) - \tfrac{1}{3}(aZ_{ab} + a^2 Z_{ac} + Z_{bc}) \\
&\qquad - (Z_{an} + a^2 Z_{bn} + aZ_{cn})
\end{aligned}$$

Equations [13] are evaluated in terms of circuit dimensions in Chapter XI, equations [12]–[14] and [7]–[9].

Unsymmetrical Static Y-Connected Circuit with Mutual Impedances between Phases. If the three-phase circuit with mutual impedances between phases has a return for zero-sequence currents through a neutral grounding self-impedance Z_n and the length of the ground path is insignificant, the mutual impedances Z_{an}, Z_{bn}, and Z_{cn} in [12] and [13] disappear and Z_{nn} becomes Z_n in these equations.

Symmetrical Three-Phase Static Circuit. In a symmetrical three-phase static circuit,

$$Z_{aa} = Z_{bb} = Z_{cc}, \qquad Z_{ab} = Z_{ac} = Z_{bc}, \qquad Z_{an} = Z_{bn} = Z_{cn}$$

The sequence impedances of the three phases from [12] are

$$\begin{aligned} Z_{a1} &= Z_{b1} = Z_{c1} = (Z_{aa} - Z_{ab}) = Z_1 \\ Z_{a2} &= Z_{b2} = Z_{c2} = (Z_{aa} - Z_{ab}) = Z_2 \\ Z_{a0} &= Z_{b0} = Z_{c0} = Z_{aa} + 2Z_{ab} + 3Z_{nn} - 6Z_{an} = Z_0 \end{aligned} \qquad [14]$$

The sequence self- and mutual impedances from [13] are

$$\begin{aligned} Z_{11} &= Z_{22} = (Z_{aa} - Z_{ab}) = Z_1 = Z_2 \\ Z_{00} &= Z_{aa} + 2Z_{ab} + 3Z_{nn} - 6Z_{an} = Z_0 \\ Z_{12} &= Z_{21} = Z_{10} = Z_{02} = Z_{20} = Z_{01} = 0 \end{aligned} \qquad [15]$$

Substituting these impedances in [4]–[6],

$$\begin{aligned} v_{a1} &= I_{a1}Z_1 \\ v_{a2} &= I_{a2}Z_2 \\ v_{a0} &= I_{a0}Z_0 \end{aligned} \qquad [16]$$

From [14], in a symmetrical static circuit, the impedances to currents of a given sequence are the same for the three phases, and the impedances to positive- and negative-sequence currents are equal. From [15] and [16], there is no mutual coupling between the sequence networks of a symmetrical circuit and currents of a given sequence produce voltage drops of like sequence only. If voltages of a given sequence are applied to the circuit, currents of like sequence only will flow. This follows because the voltage drops must equal the applied voltages, and voltage drops of a given sequence in a symmetrical circuit can be produced only by currents of that sequence.

Equal Self-Impedances in the Three Phases and Mutual Impedances of Two Phases Equal. If $Z_{aa} = Z_{bb} = Z_{cc}$, $Z_{ab} = Z_{ac}$, and $Z_{bn} = Z_{cn}$, the sequence self- and mutual impedances from [13] are

$$\begin{aligned} Z_{11} &= Z_{22} = Z_{aa} - \tfrac{1}{3}(Z_{bc} + 2Z_{ab}) \\ Z_{00} &= Z_{aa} + \tfrac{2}{3}(Z_{bc} + 2Z_{ab}) + 3Z_{nn} - 2(Z_{an} + 2Z_{bn}) \\ Z_{12} &= Z_{21} = \tfrac{2}{3}(Z_{bc} - Z_{ab}) \\ Z_{10} &= Z_{01} = Z_{20} = Z_{02} = -\tfrac{1}{3}(Z_{bc} - Z_{ab}) - (Z_{an} - Z_{bn}) \end{aligned} \qquad [17]$$

In [17] the mutual impedances between the sequence networks are reciprocal.

Self-Impedance Circuits. When the impedances of the three phases of the circuit are unequal self-impedances without mutual coupling between phases or with other circuits, the impedance of any phase to positive-, negative-, and zero-sequence currents is the same. Let the phase impedances be Z_a, Z_b, and Z_c; then $Z_{a1} = Z_{a2} = Z_{a0} = Z_a$; $Z_{b1} = Z_{b2} = Z_{b0} = Z_b$; $Z_{c1} = Z_{c2} = Z_{c0} = Z_c$. The self- and mutual impedances defined in [7] for use in [4]–[6] become

$$\begin{aligned} Z_{11} &= Z_{22} = Z_{00} = \tfrac{1}{3}(Z_a + Z_b + Z_c) \\ Z_{12} &= Z_{20} = Z_{01} = \tfrac{1}{3}(Z_a + a^2 Z_b + aZ_c) \\ &= \tfrac{1}{3}\left[Z_a - \frac{Z_b + Z_c}{2} - j\frac{\sqrt{3}}{2}(Z_b - Z_c)\right] \\ Z_{10} &= Z_{21} = Z_{02} = \tfrac{1}{3}(Z_a + aZ_b + a^2 Z_c) \\ &= \tfrac{1}{3}\left[Z_a - \frac{Z_b + Z_c}{2} + j\frac{\sqrt{3}}{2}(Z_b - Z_c)\right] \end{aligned} \qquad [18]$$

In the general self-impedance circuit, $Z_{12} \neq Z_{21}$, $Z_{10} \neq Z_{01}$, $Z_{20} \neq Z_{02}$, and therefore the mutual impedances between the sequence networks are non-reciprocal.

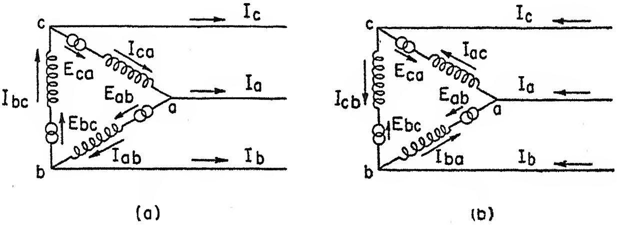
Self-Impedances Equal in Two Phases. If the self-impedances of two of the phases are equal, let these phases be b and c. Then $Z_b = Z_c$ and [18] become

$$\begin{aligned} Z_{11} &= Z_{22} = Z_{00} = \tfrac{1}{3}(Z_a + 2Z_b) \\ Z_{12} &= Z_{21} = Z_{10} = Z_{01} = Z_{20} = Z_{02} = \tfrac{1}{3}(Z_a - Z_b) \end{aligned} \qquad [19]$$

In the special case of two equal circuit self-impedances, the sequence mutual impedances are reciprocal.

Unsymmetrical Δ-Connected Circuits with Unbalanced Generated Voltages

Figure 3 shows two unsymmetrical Δ-connected circuits with unbalanced generated voltages. In part (a), line current flows from the Δ; in part (*b*), towards the Δ. In each case it is required to express the positive- and negative-sequence voltages to neutral at the circuit terminals in terms of the positive- and negative-sequence line currents and the sequence impedances of the Δ-connected windings. There are no zero-sequence components of current flowing from the Δ into the line or from the line into the Δ, but there may be zero-sequence currents circulating in the Δ. There are no zero-sequence components of voltage in the line-to-line voltages (since their sum is zero) but there may be zero-sequence voltages in the voltages to ground at the circuit terminals.



(a) (b)

Fig. 3. General unsymmetrical Δ-connected circuits with unbalanced generated voltages. (*a*) Line currents flowing from the Δ. (*b*) Line currents flowing towards the Δ.

Δ currents and voltages will be indicated by V and I, respectively, with two subscripts, the voltages by $V_{bc} = V_c - V_b$, $V_{ca} = V_a - V_c$, and $V_{ab} = V_b - V_a$, the currents by I_{bc}, I_{ca}, and I_{ab} or I_{cb}, I_{bc}, and I_{ac}, where positive direction of current flow is from the point indicated by the first subscript towards the point indicated by the second.

E_{bc}, E_{ca}, and E_{ab} are the generated voltage rises in the three phases of the Δ in the directions *bc*, *ca*, and *ab*, respectively. The positive-sequence impedances of phases *bc*, *ca*, and *ab* are Z_{bc1}, Z_{ca1}, and Z_{ab1}, respectively; the negative-sequence impedances are Z_{bc2}, Z_{ca2}, and Z_{ab2}, respectively; the zero-sequence impedances are Z_{bc0}, Z_{ca0}, and Z_{ab0}, respectively. The positive-sequence impedances are defined as the ratios of voltage drops in the impedances in the three phases to the corresponding phase currents with only positive-sequence currents flowing. The negative- and zero-sequence impedances are similarly defined. With $V_{bc} = V_c - V_b$ as reference phase,

$$V_{bc1},\ V_{ca1} = a^2 V_{bc1},\ V_{ab1} = a V_{bc1} \text{ are the positive-sequence line-to-line voltages of the } \Delta$$

$$V_{bc2},\ V_{ca2} = a V_{bc2},\ V_{ab2} = a^2 V_{bc2} \text{ are the negative-sequence line-to-line voltages of the } \Delta$$

The notation for the positive- and negative-sequence currents corresponds to that for the voltages. The zero-sequence circulating current in the Δ will be indicated by I_{bc0}.

If superposition can be applied, the phase voltages of the Δ which are line-to-line voltages will be the generated Δ voltages minus the voltage drops caused by the symmetrical components of current flowing through their respective impedances. From Fig. 3(*a*) or 3(*b*),

$$\begin{aligned} V_{bc} &= E_{bc} - I_{bc1}Z_{bc1} - I_{bc2}Z_{bc2} - I_{bc0}Z_{bc0} \\ V_{ca} &= E_{ca} - a^2 I_{bc1}Z_{ca1} - aI_{bc2}Z_{ca2} - I_{bc0}Z_{ca0} \\ V_{ab} &= E_{ab} - aI_{bc1}Z_{ab1} - a^2 I_{bc2}Z_{ab2} - I_{bc0}Z_{ab0} \end{aligned} \tag{20}$$

Resolving the line-to-line voltages into their symmetrical components of voltage, by equations analogous to [10]–[12] of Chapter II, the resultant equations are

$$\begin{aligned} V_{bc1} &= \tfrac{1}{3}(V_{bc} + aV_{ca} + a^2 V_{ab}) = E_{bc1} - I_{bc1}Z_{\Delta 11} - I_{bc2}Z_{\Delta 12} \\ &\qquad - I_{bc0}Z_{\Delta 10} \\ V_{bc2} &= \tfrac{1}{3}(V_{bc} + a^2 V_{ca} + aV_{ab}) = E_{bc2} - I_{bc1}Z_{\Delta 21} - I_{bc2}Z_{\Delta 22} \\ &\qquad - I_{bc0}Z_{\Delta 20} \\ V_{bc0} &= \tfrac{1}{3}(V_{bc} + V_{ca} + V_{ab}) = 0 = E_{bc0} - I_{bc1}Z_{\Delta 01} - I_{bc2}Z_{\Delta 02} \\ &\qquad - I_{bc0}Z_{\Delta 00} \end{aligned} \tag{21}$$

where

$$\begin{aligned} E_{bc1} &= \tfrac{1}{3}(E_{bc} + aE_{ca} + a^2 E_{ab}) \\ E_{bc2} &= \tfrac{1}{3}(E_{bc} + a^2 E_{ca} + aE_{ab}) \\ E_{bc0} &= \tfrac{1}{3}(E_{bc} + E_{ca} + E_{ab}) \end{aligned} \tag{22}$$

and

$$\begin{aligned} Z_{\Delta 11} &= \tfrac{1}{3}(Z_{bc1} + Z_{ca1} + Z_{ab1}) \\ Z_{\Delta 22} &= \tfrac{1}{3}(Z_{bc2} + Z_{ca2} + Z_{ab2}) \\ Z_{\Delta 00} &= \tfrac{1}{3}(Z_{bc0} + Z_{ca0} + Z_{ab0}) \\ Z_{\Delta 12} &= \tfrac{1}{3}(Z_{bc2} + a^2 Z_{ca2} + aZ_{ab2}) \\ Z_{\Delta 10} &= \tfrac{1}{3}(Z_{bc0} + aZ_{ca0} + a^2 Z_{ab0}) \\ Z_{\Delta 21} &= \tfrac{1}{3}(Z_{bc1} + aZ_{ca1} + a^2 Z_{ab1}) \\ Z_{\Delta 20} &= \tfrac{1}{3}(Z_{bc0} + a^2 Z_{ca0} + aZ_{ab0}) \\ Z_{\Delta 01} &= \tfrac{1}{3}(Z_{bc1} + a^2 Z_{ca1} + aZ_{ab1}) \\ Z_{\Delta 02} &= \tfrac{1}{3}(Z_{bc2} + aZ_{ca2} + a^2 Z_{ab2}) \end{aligned} \tag{23}$$

Equations [21] express the positive-, negative-, and zero-sequence components of the phase voltages of the Δ, or the line-to-line voltages across the Δ, in terms of the sequence components of voltages generated in the Δ, the sequence components of currents flowing in the Δ, and the sequence self- and mutual impedances of the Δ-connected circuit. Equations [22], which define the sequence generated Δ voltages in terms of the voltages generated in the three phases of the Δ, are similar to [1], Chapter III. Equations [23] are similar to [7], the sequence impedances in the three phases of the Δ replacing the sequence impedances in the three phases of the series circuit. The Δ impedances in [21], defined in [23], have an additional subscript Δ to indicate that they are the impedances of a Δ-connected circuit.

Solving the last equation of [21] for I_{bc0},

$$I_{bc0} = \frac{1}{Z_{\Delta 00}} (E_{bc0} - I_{bc1}Z_{\Delta 01} - I_{bc2}Z_{\Delta 02}) \tag{24}$$

Substituting I_{bc0} from [24] in the first two equations of [21],

$$\begin{aligned} V_{bc1} &= \left(E_{bc1} - E_{bc0}\frac{Z_{\Delta 10}}{Z_{\Delta 00}}\right) - I_{bc1}\left(Z_{\Delta 11} - \frac{Z_{\Delta 10}Z_{\Delta 01}}{Z_{\Delta 00}}\right) \\ &\quad - I_{bc2}\left(Z_{\Delta 12} - \frac{Z_{\Delta 10}Z_{\Delta 02}}{Z_{\Delta 00}}\right) \\ V_{bc2} &= \left(E_{bc2} - E_{bc0}\frac{Z_{\Delta 20}}{Z_{\Delta 00}}\right) - I_{bc1}\left(Z_{\Delta 21} - \frac{Z_{\Delta 20}Z_{\Delta 01}}{Z_{\Delta 00}}\right) \\ &\quad - I_{bc2}\left(Z_{\Delta 22} - \frac{Z_{\Delta 20}Z_{\Delta 02}}{Z_{\Delta 00}}\right) \end{aligned} \tag{25}$$

If currents, voltages, and impedances are expressed in amperes, voltages, and ohms, respectively, or in per unit on common kva and voltage bases, from [53], Chapter III,

$$\begin{aligned} V_{bc1} &= j\sqrt{3}V_{a1} \\ V_{bc2} &= -j\sqrt{3}V_{a2} \end{aligned} \tag{26}$$

With direction of line currents away from the Δ as in Fig. 3(a), from [60], Chapter III,

$$\begin{aligned} I_{bc1} &= \frac{jI_{a1}}{\sqrt{3}} \\ I_{bc2} &= -\frac{jI_{a2}}{\sqrt{3}} \end{aligned} \tag{27}$$

Substituting [26] and [27] in [25] and [27] in [24] and simplifying, V_{a1} and V_{a2}, the positive- and negative-sequence components of phase voltage to ground at the Δ terminals, and I_{bc0}, the zero-sequence current in the Δ in the direction *bca*, are expressed in terms of I_{a1} and I_{a2}, the positive- and negative-sequence components of line current flowing *from* the Δ terminals, the Δ-generated voltages, and sequence self- and mutual impedances of the Δ by the following equations:

$$V_{a1} = \frac{-j}{\sqrt{3}}\left(E_{bc1} - E_{bc0}\frac{Z_{\Delta 10}}{Z_{\Delta 00}}\right) - I_{a1}\left(\frac{Z_{\Delta 11}}{3} - \frac{Z_{\Delta 10}Z_{\Delta 01}}{3Z_{\Delta 00}}\right) + I_{a2}\left(\frac{Z_{\Delta 12}}{3} - \frac{Z_{\Delta 10}Z_{\Delta 02}}{3Z_{\Delta 00}}\right) \qquad [28]$$

$$V_{a2} = \frac{j}{\sqrt{3}}\left(E_{bc2} - E_{bc0}\frac{Z_{\Delta 20}}{Z_{\Delta 00}}\right) + I_{a1}\left(\frac{Z_{\Delta 21}}{3} - \frac{Z_{\Delta 20}Z_{\Delta 01}}{3Z_{\Delta 00}}\right) - I_{a2}\left(\frac{Z_{\Delta 22}}{3} - \frac{Z_{\Delta 20}Z_{\Delta 02}}{3Z_{\Delta 00}}\right) \qquad [29]$$

$$I_{bc0} = \frac{1}{Z_{\Delta 00}}\left(E_{bc0} - j\frac{I_{a1}}{\sqrt{3}}Z_{\Delta 01} + j\frac{I_{a2}}{\sqrt{3}}Z_{\Delta 02}\right) \qquad [30]$$

V_{a0} is indeterminate but can be evaluated in any given problem when the zero-sequence network for the system is known. If the generated line-to-line voltages are balanced, $E_{bc2} = E_{bc0} = 0$. With per unit quantities based on a common kva base, but base voltages in the Δ and line in the ratio of $\sqrt{3}$ to 1, the factors $1/\sqrt{3}$ and $\frac{1}{3}$, which change line-to-line generated voltages and Δ impedances to line-to-neutral voltages and impedances, respectively, disappear in [28]–[30].

Unsymmetrical Δ-Connected Synchronous Machine with Unbalanced Generated Voltages. Equations [28]–[30] give the positive- and negative-sequence components of line-to-neutral voltage at the machine terminals in terms of the positive- and negative-sequence components of line currents flowing *from* the Δ, the sequence self- and mutual impedances of the Δ-connected circuit, and the symmetrical components of generated Δ voltages. (See Fig. 3(*a*).) If I_{a1} and I_{a2} are components of line current flowing *towards* the Δ, as in Fig. 3(*b*), the signs of the coefficients of I_{a1} and I_{a2} in [28]–[30] are reversed.

Unsymmetrical Δ-Connected Static Circuit. If the generated voltages in [28]–[30] are equated to zero and the signs of I_{a1} and I_{a2} reversed, the following equations apply to the unsymmetrical Δ-connected static load:

$$V_{a1} = I_{a1}\left(\frac{Z_{\Delta 11}}{3} - \frac{Z_{\Delta 10}Z_{\Delta 01}}{3Z_{\Delta 00}}\right) - I_{a2}\left(\frac{Z_{\Delta 12}}{3} - \frac{Z_{\Delta 10}Z_{\Delta 02}}{3Z_{\Delta 00}}\right) \qquad [31]$$

$$V_{a2} = -I_{a1}\left(\frac{Z_{\Delta 21}}{3} - \frac{Z_{\Delta 20}Z_{\Delta 01}}{3Z_{\Delta 00}}\right) + I_{a2}\left(\frac{Z_{\Delta 22}}{3} - \frac{Z_{\Delta 20}Z_{\Delta 02}}{3Z_{\Delta 00}}\right) \qquad [32]$$

$$I_{bc0} = \frac{j}{\sqrt{3}}\left(I_{a1}\frac{Z_{\Delta 01}}{Z_{\Delta 00}} - I_{a2}\frac{Z_{\Delta 02}}{Z_{\Delta 00}}\right) \qquad [33]$$

where I_{a1} and I_{a2} are the symmetrical components of line current flowing towards the Δ, and I_{bc0} is the zero sequence current circulating in the Δ, in the direction *bca*.

Equations [31]–[33], as well as [28]–[30], apply where currents, voltages, and impedances are expressed in amperes, volts, and ohms, respectively, or in per unit on common kva and voltage bases. With all quantities in per unit based on a common kva, but base voltages in the Δ and line in the ratio of $\sqrt{3} : 1$, the factors $\frac{1}{3}$ in [31] and [32] and $1/\sqrt{3}$ in [33] disappear.

Δ-Connected Self-Impedance Circuit. For a self-impedance Δ-connected circuit without mutual impedances between phases, $Z_{bc1} = Z_{bc2} = Z_{cb0} = Z_{bc}$; $Z_{ca1} = Z_{ca2} = Z_{ca0} = Z_{ca}$; $Z_{ab1} = Z_{ab2} = Z_{ab0} = Z_{ab}$ From [23],

$$\begin{aligned} Z_{\Delta 11} &= Z_{\Delta 22} = Z_{\Delta 00} = \tfrac{1}{3}(Z_{bc} + Z_{ca} + Z_{ab}) \\ Z_{\Delta 12} &= Z_{\Delta 20} = Z_{\Delta 01} = \tfrac{1}{3}(Z_{bc} + a^2 Z_{ca} + aZ_{ab}) \qquad [34] \\ Z_{\Delta 21} &= Z_{\Delta 10} = Z_{\Delta 02} = \tfrac{1}{3}(Z_{bc} + aZ_{ca} + a^2 Z_{ab}) \end{aligned}$$

For the Δ-connected self-impedance circuit, [34] may be substituted in [31]–[33], or an equivalent Y can replace the Δ-connected self-impedance circuit for calculating current and voltage conditions at its terminals. (See [39], Chapter I.) The first two equations of [11*b*] then apply, where the sequence self- and mutual impedances Z_{11}, Z_{22}, Z_{12}, and Z_{21} are defined in [18] in terms of the self-impedances of the equivalent Y. It should be noted that it is the self-impedance Δ only which can be replaced by a single equivalent Y. The zero-sequence current circulating in the unsymmetrical Δ cannot be determined from the equivalent Y, but the total currents in the three branches of the Δ can be obtained by dividing the calculated line-to-line voltages at the terminals of the equivalent Y by the corresponding phase impedances of the Δ. One-third the sum of these Δ currents is the zero-sequence current which circulates in the Δ.

DETERMINATION OF CURRENTS AND VOLTAGES IN A SYSTEM CONTAINING AN UNSYMMETRICAL CIRCUIT BY MEANS OF EQUATIONS

As pointed out, the mutual impedances between the sequence networks are non-reciprocal for the general unsymmetrical circuit. This is true for the general unsymmetrical circuit with mutual impedances between phases (see [13]) and also for the general unsymmetrical circuit composed of self-impedances only (see [18]). With non-reciprocal mutual impedances between the sequence networks, there is no equivalent static circuit to replace the actual circuit in these networks. Solution, however, can be obtained by means of equations. On an a-c network analyzer, a phase shifter is required. Equations will be developed from which the currents and voltages can be determined in a system, symmetrical except for (1) a general unsymmetrical static series circuit, (2) a general unsymmetrical circuit connected at one point only of the system. When the mutual impedances are reciprocal, equivalent circuits can be developed to replace the actual circuit in the sequence networks. Such circuits will be developed for special unsymmetrical series and shunt circuits.

General Unsymmetrical Series Circuit. When the system except for the unsymmetrical series circuit is symmetrical and the parts of the system on the two sides of the unsymmetrical circuit are connected only through that circuit, the system can be represented as in

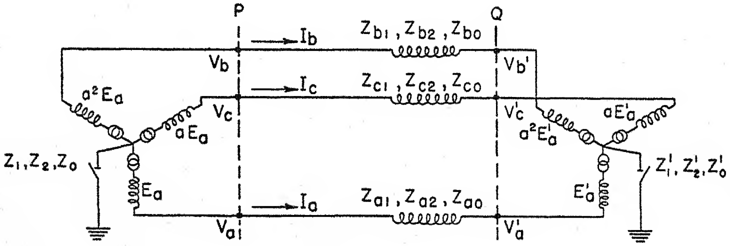


FIG. 4. Unsymmetrical series circuit between points P and Q in an otherwise symmetrical system in which symmetrical parts of the system are represented as equivalent synchronous machines.

Fig. 4, where the points P and Q divide the system into three parts. The parts of the system to the left of P and to the right of Q are symmetrical, the three phases offering the same impedances to currents of a given sequence, while the part between P and Q is unsymmetrical. The symmetrical parts of the system can be replaced by equivalent machines with sequence impedances Z_1, Z_2, Z_0 and Z_1', Z_2', Z_0', which are the sequence impedances of the symmetrical parts of the system viewed from P and Q, respectively, with the unsymmetrical series circuit open. E_a, a^2E_a, aE_a and E'_a, $a^2E'_a$, aE'_a are the balanced internal voltages of the two equivalent machines replacing the symmetrical parts of the system. They are the balanced voltages which would exist at P and Q, respectively, if the three phases were opened at P and Q, all rotors on the system retaining their relative angular positions. E_a and E'_a depend upon operating conditions. For example, if the unsymmetrical circuit is supplied at P through a transformer bank with kva rating so small relative to the kva capacity of the system to the left of P in Fig. 4 that the voltages on the primary side of the bank remain balanced, Z_1, Z_2, and Z_0 will be the positive-, negative-, and zero-sequence impedances of the transformer bank, and the internal voltages of the equivalent machine will be the balanced normal operating voltages on the primary side of the transformer bank. When the current taken at P by the unsymmetrical circuit is appreciable relative to the current in the system to the left of P, the balanced internal voltages of the equivalent machine at P must be high enough to allow for the voltage drop in the symmetrical part of the system caused by the unsymmetrical load. In many problems involving unsymmetrical circuits, the given conditions are such that E_a and E'_a are not required. For example, if the positive-sequence current flowing through an unsymmetrical series circuit in an otherwise symmetrical system is given and the problem is to find the negative-sequence current when the impedance to zero-sequence currents is infinite, it is only necessary to determine the ratio of the negative- to the positive-sequence current. (See [43].) In cases where E_a and E'_a are required, they can usually be satisfactorily estimated. For the present it will be assumed that they can be estimated or will be eliminated in the solution of a given problem. When the symmetrical part of the system to the right of Q is a symmetrical three-phase static circuit, $E'_a = 0$.

In Fig. 4, the line-to-ground voltages of the three phases at P will be designated by V_a, V_b, and V_c, and at Q by V'_a, V'_b, and V'_c and the corresponding line currents which flow in all three circuits by I_a, I_b, and I_c, positive direction for current flow being from P to Q, as indicated by arrows. If the symmetrical components of voltages to ground at P and Q, V_{a1}, V_{a2}, V_{a0}, and V'_{a1}, V'_{a2}, V'_{a0}, respectively, are expressed in terms of the symmetrical components of current flowing from P to Q, there will be three sets of equations, two sets in terms of the impedances of the symmetrical parts of the system and one set in terms of the impedances of the unsymmetrical part. By equating symmetrical components of voltages at P and at Q in these equations, the symmetrical components of current can be determined, and from them the currents and voltages of the system.

From the symmetrical part of the system to the left of P, the symmetrical components of voltage at P from [8]–[10], Chapter III, are

$$\begin{aligned} V_{a1} &= E_a - I_{a1}Z_1 \\ V_{a2} &= -I_{a2}Z_2 \\ V_{a0} &= -I_{a0}Z_0 \end{aligned} \qquad [35]$$

From the symmetrical part of the system to the right of Q, the symmetrical components of voltage at Q are

$$\begin{aligned} V'_{a1} &= I_{a1}Z'_1 + E'_a \\ V'_{a2} &= I_{a2}Z'_2 \\ V'_{a0} &= I_{a0}Z'_0 \end{aligned} \qquad [36]$$

The sequence voltage drops between P and Q, $(V_{a1} - V'_{a1})$, $(V_{a2} - V'_{a2})$, $(V_{a0} - V'_{a0})$, are given by [4]–[6].

Substituting [35] and [36] in [4]–[6], and simplifying,

$$E_a - E'_a = I_{a1}(Z_{11} + Z_1 + Z'_1) + I_{a2}Z_{12} + I_{a0}Z_{10} \qquad [37]$$

$$0 = I_{a1}Z_{21} + I_{a2}(Z_{22} + Z_2 + Z'_2) + I_{a0}Z_{20} \qquad [38]$$

$$0 = I_{a1}Z_{01} + I_{a2}Z_{02} + I_{a0}(Z_{00} + Z_0 + Z'_0) \qquad [39]$$

Using determinants (see Appendix A), the solution of the simultaneous equations [37]–[39] gives

$$\begin{aligned} I_{a1} &= (E_a - E'_a)\frac{(Z_{22} + Z_2 + Z'_2)(Z_{00} + Z_0 + Z'_0) - Z_{02}Z_{20}}{\Delta} \\ I_{a2} &= -(E_a - E'_a)\frac{Z_{21}(Z_{00} + Z_0 + Z'_0) - Z_{01}Z_{20}}{\Delta} \\ &= -I_{a1}\frac{Z_{21}(Z_{00} + Z_0 + Z'_0) - Z_{01}Z_{20}}{(Z_{22} + Z_2 + Z'_2)(Z_{00} + Z_0 + Z'_0) - Z_{02}Z_{20}} \\ I_{a0} &= -(E_a - E'_a)\frac{Z_{01}(Z_{22} + Z_2 + Z'_2) - Z_{21}Z_{02}}{\Delta} \\ &= -I_{a1}\frac{Z_{01}(Z_{22} + Z_2 + Z'_2) - Z_{21}Z_{02}}{(Z_{22} + Z_2 + Z'_2)(Z_{00} + Z_0 + Z'_0) - Z_{02}Z_{20}} \end{aligned} \qquad [40]$$

where

$$\begin{aligned} \Delta = (Z_{11} + Z_1 + Z'_1)&[(Z_{22} + Z_2 + Z'_2)(Z_{00} + Z_0 + Z'_0) - Z_{02}Z_{20}] \\ &- Z_{12}[Z_{21}(Z_{00} + Z_0 + Z'_0) - Z_{01}Z_{20}] \\ &+ Z_{10}[Z_{21}Z_{02} - Z_{01}(Z_{22} + Z_2 + Z'_2)] \end{aligned}$$

Equations [40] express I_{a1}, I_{a2}, and I_{a0} in terms of $(E_a - E'_a)$; I_{a2} and I_{a0} are also expressed in terms of I_{a1}. When I_{a1} is known or can be assumed, I_{a2} and I_{a0} are most simply calculated from [40] in terms of I_{a1}.

In circuits where the dissymmetry is but slight and therefore the negative- and zero-sequence currents are known to be small, the negative-sequence voltage drop $I_{a0}Z_{20}$ in [38] produced by I_{a0} and the zero-sequence voltage drop $I_{a2}Z_{02}$ in [39] produced by I_{a2} can be neglected with but slight error. Neglecting these terms in [38] and [39], and solving for I_{a2} and I_{a0} in terms of I_{a1},

$$I_{a2} = -I_{a1}\frac{Z_{21}}{Z_{22} + Z_2 + Z'_2} = -I_{a1}\frac{Z_{21}}{z_{22}}$$

$$I_{a0} = -I_{a1}\frac{Z_{01}}{Z_{00} + Z_0 + Z'_0} = -I_{a1}\frac{Z_{01}}{z_{00}} \qquad [40a]$$

where z_{22} and z_{00} are the negative- and zero-sequence self-impedances met by I_{a2} and I_{a0}, respectively. Equations [40*a*] could have been obtained from [40] by neglecting the products of two mutual impedances; when the dissymmetry is slight, these products are small relative to the product of one self- and one mutual impedance or two self-impedances. Equations [40*a*] are used in Problems 6 and 7 of Chapter XI to determine the effect of unsymmetrical transmission circuits upon system currents and voltages during normal operation.

Finite Zero-Sequence Impedances. When Z_0, Z'_0, and Z_{00} are all finite, zero sequence currents will flow, and the symmetrical components of current are given by [40] or [40*a*]. The symmetrical components of voltage at P and Q can be obtained from [35] and [36], respectively, when I_{a1}, I_{a2}, and I_{a0} are known. Substituting the symmetrical components of voltage and current in equations [7]–[9] and [19]–[21], Chapter II, respectively, line currents and voltages to ground at P and Q can be obtained.

Infinite Zero-Sequence Impedances. If any one of the zero-sequence impedances Z_0, Z'_0, or Z_{00} is infinite, there will be no zero-sequence currents; and $I_{a0}Z_{10}$ and $I_{a0}Z_{20}$, which represent positive- and negative-sequence voltage drops produced by zero-sequence currents, will be zero. With $I_{a0} = 0$, [37] and [38] become

$$E_a - E'_a = I_{a1}(Z_{11} + Z_1 + Z'_1) + I_{a2}Z_{12}$$

$$0 = I_{a1}Z_{21} + I_{a2}(Z_{22} + Z_2 + Z'_2) \qquad [41]$$

Solving these equations for I_{a1} and I_{a2},

$$I_{a1} = (E_a - E'_a)\frac{(Z_{22} + Z_2 + Z'_2)}{(Z_{11} + Z_1 + Z'_1)(Z_{22} + Z_2 + Z'_2) - Z_{21}Z_{12}} \qquad [42]$$

$$I_{a2} = -(E_a - E'_a)\frac{Z_{21}}{(Z_{11} + Z_1 + Z'_1)(Z_{22} + Z_2 + Z'_2) - Z_{21}Z_{12}}$$

$$= -I_{a1}\frac{Z_{21}}{Z_{22} + Z_2 + Z'_2} \qquad [43]$$

Knowing I_{a1} and I_{a2}, the positive- and negative-sequence components of the voltages at P and Q can be obtained from [35] and [36], respectively.

With $I_{a0} = 0$ and $(Z_{00} + Z_0 + Z'_0) = \infty$, $V_{a0} - V'_{a0}$ is indeterminate. Zero-sequence voltages at P and Q, however, can be determined from the zero-sequence diagram of the system. For example:

If Z_{00} is infinite and Z_0 and Z'_0 both finite, the zero-sequence voltages at both P and Q, determined from [35] and [36] with $I_{a0} = 0$, are zero.

If Z_{00} is finite, and Z_0 or Z'_0 is infinite, $I_{a0}Z_{00} = 0 \cdot Z_{00} = 0$, and [6] becomes

$$v_{a0} = V_{a0} - V'_{a0} = I_{a1}Z_{01} + I_{a2}Z_{02} \qquad [44]$$

where v_{a0} is the zero-sequence voltage drop between P and Q produced by positive- and negative-sequence currents flowing in the unsymmetrical circuit of finite zero-sequence self-impedance. If there is a ground in the part of the system connected at P, but none in the part connected at Q, Z_0 is finite and Z'_0 infinite. With $I_{a0} = 0$ and Z_0 finite, the zero-sequence voltage V_{a0} at P from [35] will be zero. By definition, v_{a0} is the zero-sequence voltage drop between P and Q. The zero-sequence voltage V'_{a0} at Q will therefore be V_{a0}, the zero-sequence voltage at P, minus the zero-sequence voltage drop given by [44]; with $V_{a0} = 0$,

$$V'_{a0} \text{ (at } Q) = 0 - v_{a0} = -I_{a1}Z_{01} - I_{a2}Z_{02} \qquad [45]$$

With no ground on the part of the system connected at Q, the zero-sequence voltage at Q will also be the zero-sequence voltage at all points in the section of the system where the connection to Q is a direct metallic one; or stated more comprehensively, at all points where the connection to Q is through finite zero-sequence impedance.

With Z_{00} finite, and the part of the system connected at Q grounded and the part connected at P ungrounded, Z'_0 is finite and Z_0 infinite. From [36], the zero-sequence voltage at Q will be zero and that at P will be given by [44]. The zero-sequence voltage at P will also be the zero-sequence voltage at all points in the section of the system connected at P where the connection to P is through finite zero-sequence impedance.

Unsymmetrical Circuit Connected at One Point Only of the System. The procedure for this case is similar to that used for the unsymmetrical series circuit. If the unsymmetrical circuit is a generator supplying power to the symmetrical part of the system, the system exclusive of the unsymmetrical generator is replaced by an equivalent synchronous motor with balanced sequence impedances and internal voltages. If the unsymmetrical circuit is receiving power from the symmetrical part of the system, the system exclusive of the unsymmetrical circuit is replaced by an equivalent synchronous generator with balanced sequence impedances and internal voltages. In each case, there are two sets of equations giving the symmetrical components of phase voltages to ground at the junction point of the symmetrical and unsymmetrical parts of the system in terms of the symmetrical components of line current flowing in both parts of the system.

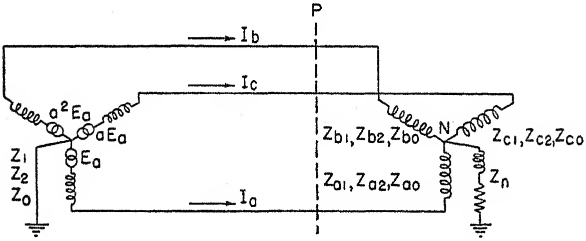


FIG. 5. Unsymmetrical Y-connected load with neutral grounded through Z_n, supplied at P from an otherwise symmetrical system, represented as an equivalent synchronous generator.

Y-Connected Static Circuit. A Y-connected static circuit grounded through a neutral impedance Z_n and supplied from an otherwise symmetrical system is shown in Fig. 5. Equating V_{a1}, V_{a2}, and V_{a0} in [11*a*] and [35] and solving for I_{a1}, I_{a2}, and I_{a0},

$$I_{a1} = E_a \frac{(Z_2 + Z_{22})(Z'_{00} + Z_0 + 3Z_n) - Z_{02}Z_{20}}{\Delta}$$

$$I_{a2} = -E_a \frac{Z_{21}(Z'_{00} + Z_0 + 3Z_n) - Z_{01}Z_{20}}{\Delta} \qquad [46]$$

$$I_{a0} = -E_a \frac{Z_{01}(Z_{22} + Z_2) - Z_{21}Z_{02}}{\Delta}$$

where

$$\begin{aligned}\Delta = (Z_1 + Z_{11})[(Z_2 + Z_{22})(Z'_{00} + Z_0 + 3Z_n) - Z_{02}Z_{20}]\\ - Z_{12}[Z_{21}(Z_{00} + Z_0 + 3Z_n) - Z_{01}Z_{20}]\\ + Z_{10}[Z_{21}Z_{02} - Z_{01}(Z_{22} + Z_2)]\end{aligned}$$

When the symmetrical components of current have been determined, the symmetrical components of voltage to ground at P can be obtained from either [35] or [11a], the former being simpler.

Neutral of Load Ungrounded. With the load ungrounded, $V_n = \infty$ and $I_{a0} = 0$. With the symmetrical part of the system grounded, the zero-sequence voltage at P in Fig. 5 from [35] is zero. Equating V_{a1}, V_{a2}, V_{a0} in [35] and [11b] and, simplifying,

$$\begin{aligned}E_a &= I_{a1}(Z_1 + Z_{11}) + I_{a2}Z_{12}\\ 0 &= I_{a1}Z_{21} + I_{a2}(Z_2 + Z_{22})\\ 0 &= I_{a1}Z_{01} + I_{a2}Z_{02} + V_n\end{aligned}$$

Solving the above equations for I_{a1}, I_{a2}, and V_n,

$$\left.\begin{aligned}I_{a1} &= \frac{E_a(Z_2 + Z_{22})}{(Z_1 + Z_{11})(Z_2 + Z_{22}) - Z_{12}Z_{21}}\\ I_{a2} &= \frac{-E_aZ_{21}}{(Z_1 + Z_{11})(Z_2 + Z_{22}) - Z_{12}Z_{21}} = -I_{a1}\frac{Z_{21}}{Z_2 + Z_{22}}\end{aligned}\right\} \quad [47]$$

and

$$V_n = -E_a\frac{Z_{01}(Z_2 + Z_{22}) - Z_{02}Z_{21}}{(Z_1 + Z_{11})(Z_2 + Z_{22}) - Z_{12}Z_{21}} \quad [48]$$

where V_n is the voltage at the neutral of the load. This is the voltage rise from P to N caused by positive- and negative-sequence currents flowing through an unsymmetrical circuit with finite zero-sequence impedance between terminals and neutral.

The positive- and negative-sequence components of voltage at P can be obtained by substituting I_{a1} and I_{a2} from [47] in [35].

If the neutral of the load is grounded but the *system ungrounded*, Z_0 in Fig. 5 is infinite and $I_{a0} = 0$. I_{a1} and I_{a2} are given by [47] and V_{a1} and V_{a2} by substituting these values of I_{a1} and I_{a2} in [35]. The positive- and negative-sequence currents and voltages at P are the same as those for the case of the system grounded and neutral of load ungrounded. With the neutral of the load grounded, $V_n = 0$. The zero-sequence voltage at P, determined from [11b], with V_n replaced by zero, is

$$V_{a0} = E_a \frac{Z_{01}(Z_2 + Z_{22}) - Z_{02}Z_{21}}{(Z_1 + Z_{11})(Z_2 + Z_{22}) - Z_{12}Z_{21}} \qquad [48a]$$

This is the zero-sequence voltage drop from P to N caused by positive- and negative-sequence currents flowing in the unsymmetrical circuit of finite zero-sequence impedance. V_{a0} in [48a] is equal in magnitude and opposite in sign to V_n given by [48]. As there is no zero-sequence current in the system supplying the load, the zero-sequence voltage given by [48a] will exist at P and at all other points in the system replaced by the equivalent generator where the connection to P is through finite zero-sequence impedance.

Self-Impedance Y-Connected Circuits. The sequence self- and mutual impedances for use in [46] for the grounded Y, and in [47] for the ungrounded Y (or the Δ replaced by its equivalent Y), are given by [18] in terms of the phase impedances of the Y. For the special case of *equal self-impedances in two phases* of the Y, the sequence self- and mutual impedances in terms of the phase impedances of the Y are given by [19].

Self-Impedances Equal in Two Phases of the Grounded Y. Assuming a grounded system and substituting [19] in [46],

$$\begin{aligned} I_{a1} &= E_a \frac{(Z_2 + Z_{11})(Z_0 + Z_{11} + 3Z_n) - Z_{12}^2}{\Delta} \\ I_{a2} &= -E_a \frac{Z_{12}(Z_0 + Z_{11} + 3Z_n) - Z_{12}^2}{\Delta} \\ I_{a0} &= E_a \frac{Z_{12}^2 - Z_{12}(Z_2 + Z_{11})}{\Delta} \end{aligned} \qquad [49]$$

where

$$\begin{aligned} \Delta = {} & (Z_1 + Z_{11})(Z_2 + Z_{11})(Z_0 + Z_{11} + 3Z_n) \\ & - Z_{12}^2(Z_1 + Z_2 + Z_0 + 3Z_{11} + 3Z_n - 2Z_{12}) \\ Z_{11} = {} & \tfrac{1}{3}(Z_a + 2Z_b) \\ Z_{12} = {} & \tfrac{1}{3}(Z_a - Z_b) \end{aligned}$$

Symmetrical components of voltage at P can be obtained by substituting I_{a1}, I_{a2}, and I_{a0} from [49] in [35].

Self-Impedances Equal in Two Phases of the Ungrounded Y, *or* Δ *Replaced by Its Equivalent* Y. Assuming a grounded system supplying the ungrounded load, and substituting [19] in [47] and [48],

$$\begin{aligned}
I_{a1} &= E_a \frac{9Z_2 + 3Z_a + 6Z_b}{(3Z_1 + Z_a + 2Z_b)(3Z_2 + Z_a + 2Z_b) - (Z_a - Z_b)^2} \\
I_{a2} &= -E_a \frac{3(Z_a - Z_b)}{(3Z_1 + Z_a + 2Z_b)(3Z_2 + Z_a + 2Z_b) - (Z_a - Z_b)^2} \\
&= -I_{a1} \frac{(Z_a - Z_b)}{3Z_2 + Z_a + 2Z_b} \\
V_n &= -E_a \frac{3(Z_a - Z_b)(Z_2 + Z_b)}{(3Z_1 + Z_a + 2Z_b)(3Z_2 + Z_a + 2Z_b) - (Z_a - Z_b)^2}
\end{aligned} \tag{50}$$

V_{a1}, V_{a2}, and V_{a0} at P are obtained from [35] by replacing I_{a1} and I_{a2} by their values from [50], and I_{a0} by zero.

Equal Self-Impedances in Two Phases of the Ungrounded Y *and Infinite Impedance in the Third Phase.* Equations [50] are evaluated for this case by dividing the numerators and denominators of the fractions by $3Z_a$, and then allowing Z_a to approach infinity. The following equations are obtained:

$$\begin{aligned}
I_{a1} &= \frac{E_a}{Z_1 + Z_2 + 2Z_b} \\
I_{a2} &= \frac{-E_a}{Z_1 + Z_2 + 2Z_b} = -I_{a1} \\
V_n &= -\frac{E_a(Z_2 + Z_b)}{Z_1 + Z_2 + 2Z_b}
\end{aligned} \tag{51}$$

where V_n is the voltage at the junction of Z_b and $Z_c = Z_b$; there is no Y and no neutral with $Z_a = \infty$ and phase a therefore open.

The following problem illustrates the procedure for determining the currents in an unsymmetrical load when the phase impedances of the Y are known.

Problem 1. Given a generator with internal voltage $E_a = 1.0$ and impedances $Z_1 = Z_2 = Z_0 = 0$, i.e., infinite bus, supplying an unsymmetrical resistance load connected in Y, $Z_a = 1.0$, $Z_b = 1.5$, $Z_c = 0.5$. The neutral of the load is ungrounded. Find the three line currents and the voltage to ground at the neutral of the load.

Solution. From [18],

$$Z_{11} = Z_{22} = \tfrac{1}{3}(1 + 1.5 + 0.5) = 1$$

$$Z_{12} = Z_{01} = \tfrac{1}{3}(1 + 1.5a^2 + 0.5a) = -j\frac{1}{2\sqrt{3}} = -j0.289$$

$$Z_{21} = Z_{02} = \tfrac{1}{3}(1 + 1.5a + 0.5a^2) = j\frac{1}{2\sqrt{3}} = j0.289$$

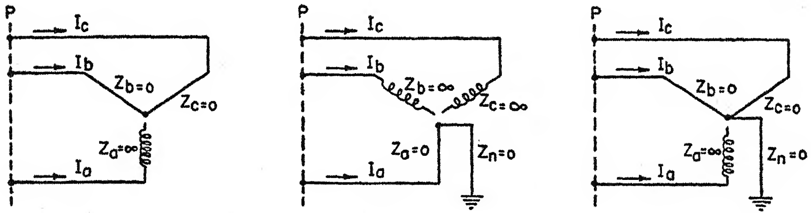
Substituting $E_a = 1.0$, $Z_1 = Z_2 = 0$ and the above self- and mutual impedances in [47] and [48],

$$\begin{aligned} I_{a1} &= 1.091 \\ I_{a2} &= -j0.315 \\ V_n &= -\ 0.091 + j0.315 \end{aligned}$$

$$\begin{aligned} I_a &= 1.091 - j0.315 \\ I_b &= -0.272 - j0.788 \\ I_c &= -0.819 + j1.103 \end{aligned}$$

For check: $I_a = \dfrac{1 - V_n}{1} = 1.091 - j0.315.$

Faults Considered Unsymmetrical Y-Connected Loads. Line-to-line, line-to-ground, and double line-to-ground faults may all be solved by applying the equations for an unsymmetrical Y-connected load supplied at P by a system represented as an equivalent generator with positive-, negative-, and zero-sequence impedances Z_1, Z_2, and Z_0, respectively, and generated voltage E_a.



(a) (b) (c)

FIG. 6. Faults through zero impedance represented as unsymmetrical shunt loads. (*a*) Line-to-line fault. (*b*) Line-to-ground fault. (*c*) Double line-to-ground fault.

Figure 6(*a*) represents a *line-to-line fault through zero impedance*, if $Z_b = Z_c = 0$ and $Z_a = \infty$. With $Z_b = 0$ in [51],

$$I_{a1} = \frac{E_a}{Z_1 + Z_2} \quad \text{and} \quad I_{a2} = -\frac{E_a}{Z_1 + Z_2} = -I_{a1}$$

$$V_n = -\frac{E_a Z_2}{Z_1 + Z_2} = V_b = V_c$$

A *line-to-ground fault* on phase a is shown in Fig. 6(*b*) as an unbalanced Y-connected load with solidly grounded neutral, and $Z_a = Z_n = 0$, $Z_b = Z_c = \infty$. The current equations can be obtained from [49] by substituting zero for Z_a and Z_n, then dividing numerators and denominators of the fractions by Z_b to the highest power found in the denomi-